8. Estimate how long it would take for $1000 to grow to $16 000 at each interest rate, compounded annually.

a) 6%   b) 12%

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$1000 \xrightarrow{\times 2} 2000 \xrightarrow{\times 2} 4000 \xrightarrow{\times 2} 8000 \xrightarrow{\times 2} 16000$

a) $t = \dfrac{72}{\text{Rate}}$

$t = \dfrac{72}{6}$

$t = 12 \text{ years}$

TOTAL → 12×4

48 years

12. Lenny has $5000 to invest and is looking at different GICs, as shown in the table to the left. These GICs cannot be redeemed until their maturity.
  a) Why do you think the interest rates increase as the term increases?
  b) Lenny cannot decide whether to invest $5000 for 10 years or to invest $5000 for 5 years and then reinvest for another 5 years.
  i) Compare the future values of each option. What assumptions are you making? — annual rate, rate fixed
  ii) What are the advantages and disadvantages of each option?

Term (years)	Rate (%)
1	1.35
2	1.65
3	1.90
4	2.15
5	2.65
6	2.70
7	2.85
8	2.90
9	3.00
10	3.25

a) Money in longer means more opportunities for loaning and making $.

10 years...

5000(1+0.0325/1)
^10
        6884.471519

$6884.47

Best

5 years & Re invest for 5 years...

5000(1+0.0265)^5
        5698.555375
5698.56(1+0.0265
)^5
$ 6494.711944

$6494.71

6. Claire wants a down payment of $17 500 to buy a house in 10 years, when she turns 30. Her bank offers her an investment with 5.6% interest, compounded semi-annually. What present value will she need to invest now?

$$P = \frac{17500}{\left(1 + \frac{0.056}{2}\right)^{2 \times 10}}$$

$$P = \boxed{\begin{array}{l} \texttt{17500/(1+0.056)\^{}} \\ \texttt{20} \\ \qquad \texttt{5885.17911} \\ \$5885.18 \end{array}}$$

8.5

Investments Involving Regular Payments

GOAL

Determine the future value of an investment that earns compound interest involving regular payments.
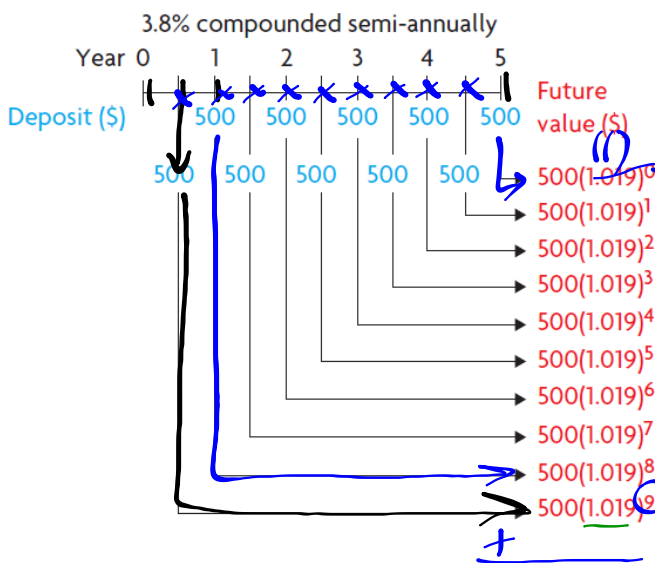
EXAMPLE 1
p. 485

Determining the future value of an investment involving regular deposits

Darva is saving for a trip to Australia in 5 years. She plans to work on a student visa while she is there, so she needs only enough money for a return flight and her expenses until she finds a job. She deposits $500 into her savings account at the end of each 6-month period from what she earns as a server. The account earns 3.8%, compounded semi-annually. How much money will be in the account at the end of 5 years? How much of this money will be earned interest?

SOLUTION BY HAND...

I drew a timeline to show the future value of each of the $500 deposits that I made at the end of each 6-month period for 5 years.

3.8% compounded semi-annually

Year 0   1   2   3   4   5

Deposit ($)   500   500   500   500   500   Future value ($)

500   500   500   500   500

$500(1.019)^0$
$500(1.019)^1$
$500(1.019)^2$
$500(1.019)^3$
$500(1.019)^4$
$500(1.019)^5$
$500(1.019)^6$
$500(1.019)^7$
$500(1.019)^8$
$500(1.019)^9$

I could see that I needed to do 10 calculations and then determine the sum.

$$A = 500\left(1 + \frac{0.038}{2}\right)^{10}$$

```
N=10
I%=3.8
PV=0
PMT=-500
FV=5449.890878
P/Y=2
C/Y=2
PMT:END BEGIN
```

a) $FV = \$5449.90$ PMT × n

b) $I = A - P$
$I = 5449.90 - 500(10)$
$I = 449.90$

Notes - TVM Solver.pdf

INSTRUCTIONS on using the TVM-Solver...

1.) On the TI-83, press 2nd, then FINANCE, then select 1:TVM Solver. On the TI-83 plus and TI-84, press APPS, then 1:FINANCE, then 1:TVM Solver. You should see the screen below:

```
N=█
I%=0
PV=0
PMT=0
FV=0
P/Y=1
C/Y=1
PMT:END BEGIN
```

2.) Now, suppose you are taking out a 5-year loan on $25000 at 6% annual interest compounded monthly and you want to know the monthly payment. Fill in the values on the TVM Solver screen as shown:

```
N=60
I%=6
PV=25000
PMT=█
FV=0
P/Y=12
C/Y=12
PMT:END BEGIN
```

3.) Now, move the cursor to PMT, press the green ALPHA key, then ENTER. Your payment will show up as a negative number:

```
N=60
I%=6
PV=25000
▪PMT=-483.32003...
FV=0
P/Y=12
C/Y=12
PMT:END BEGIN
```

NOTE: a **negative** number means that the money is coming 'out of your pocket'

4.) Suppose you know you can afford a $250 per month payment on a 60 month loan at 6% annual interest compounded monthly. Fill out the TVM Solver screen as shown:

```
N=60
I%=6
PV=0
PMT=-250
FV=0
P/Y=12
C/Y=12
PMT:END BEGIN
```

5.) To find how much you can afford to borrow, move the cursor to PV, press the green ALPHA key, then ENTER. The amount you can afford to borrow is shown:

```
N=60
I%=6
▪PV=12931.39019
PMT=-250
FV=0
P/Y=12
C/Y=12
PMT:END BEGIN
```

SOLUTION WITH TI-84 (Finance APP)...

```
N=█
I%=0
PV=0
PMT=0
FV=0
P/Y=1
C/Y=1
PMT:END BEGIN
```

Total number of payments
Yearly interest rate (as a percent)
Present Value (money invested/borrowed)
Payment
Future Value (money at the end of the term)
{ Number of payments/year
{ Number of times interest gets compounded/year

Payment is given at the beginning/end of pay period

NOTES — $PV = 0$ when invest
— PMT will be negative

$$P = PMT \times N$$

Formula vs APP
Payments

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

↑
once

EXAMPLE 2
p. 487

Comparing a regular payment investment with a sing
payment investment

Adam → App
Blake → formula

Adam made a $200 payment at the end of each year into an investment
that earned 5%, compounded annually. Blake made a single investment at
5%, compounded annually. At the end of 5 years, their future values were
equal.

a) What was their future value?
b) What principal amount did Blake invest 5 years ago?
c) Who earned more interest? Why?

a) Adam... $1105.13

```
N=5
I%=5
PV=0
PMT=-200
■FV=1105.12625
P/Y=1
C/Y=1
PMT:END BEGIN
```

$P = 200(5)$
$P = 1000$

b) $P = \dfrac{A}{(1 + \frac{s}{n})^{nt}}$

$P = \dfrac{1105.13}{(1 + \frac{0.05}{1})^5}$

$P = \$865.90$

$I = 1105.13 - 865.90$
$I = \$239.23$

more

c) $I = \dfrac{1105.13 - 1000}{}$
$I = \$105.13$

EXAMPLE 3 p. 489	Determining the interest rate of a regular payment investment

Jeremiah deposits $750 into an investment account at the end of every 3 months. Interest is compounded quarterly, the term is 3 years, and the future value is $10 059.07. What annual rate of interest does Jeremiah's investment earn?

```
N=12
■ I%=8.000019121
 PV=0
 PMT=-750
 FV=10059.07
 P/Y=4
 C/Y=4
 PMT:END  BEGIN
```

HOMEWORK...

p. 493: #3, 5, 6, & 9

NOTE: When using the TI-84...

Each question must have the following completed for homework
AND beginning of class tomorrow you will be given time to solve.

```
N=
I%=
PV=
PMT=
FV=
P/Y=
C/Y=
PMT:END BEGIN
```

Attachments

Notes - TVM Solver.pdf