6.4 - Volume and Capacity of Spheres, Cones and Pyramids

MATH ON THE JOB

Andrew Dolan, a Red Seal–certified steel fabricator and boilermaker, grew up in Fairvale, New Brunswick, and attended Kennebecasis Valley High School. He studied his trade at New Brunswick Community College Moncton through the Boilermakers Union Local 73.

He currently works for Lorneville Mechanical in Saint John, NB. One of the projects he worked on involved manufacturing a stainless steel tank for the Irving Pulp and Paper Mill to contain "black liquor," a pulp mill by-product that is used as a fuel. Building the tank involved fitting and welding the two halves of the cylinder together, attaching these to a base, and installing a roof on the cylinder. When finished, the 32-ton tank was lifted into place with a crane.

If the cylindrical part of the tank is 16 feet tall and has a =diameter of 33 feet, how much black liquor can the tank hold in litres? (Hint: 1 L = 1000 cm^3)

"I use math every day on the job, whether it's adding and subtracting fractions for cut measurements, or converting degrees, minutes, and seconds into inches to orientate nozzles on round vessels and tanks. There is also a lot of math involved in rigging large lifts with cranes. Good math skills are essential in my trade," says boilermaker Andrew Dolan.

SOLUTION

Convert the dimensions of the tank to centimetres.

1 m = 3.2808 ft

16 ft ÷ 3.2808 ft/m ≈ 4.877 m

4.877 m = 487.7 cm

33 ft ÷ 3.2808 ft/m ≈ 10.059 m

10.059 m = 1005.9 cm

Calculate the volume.

$V = \pi r^2 h$

$V = \pi\left(\frac{d}{2}\right)^2 h$

$V = \pi\left(\frac{1005.9}{2}\right)^2 (487.7)$

$V \approx 387\,571\,874\ cm^3$

To find the volume in litres, divide by 1000.

387 571 874 ÷ 1000 = 387 571.874 L

The tank can hold 387 571.874 L of black liquor.

$V = \pi r^2 h$

$= \pi (16.5)^2 16$

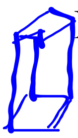
$= 13\,684.78\ ft^3$

$13\,684.78\ ft^3 \times \left(\frac{12\ in}{ft}\right)^3 \times \left(\frac{2.54\ cm}{in}\right)^3 \times \frac{1\ l}{1000\ cm^3}$

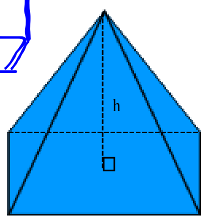
$387\,509.82\ l$

VOLUME FORMULAS...

$V = lwh$



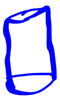
Pyramid



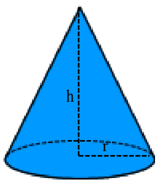
$$V_{pyramid} = \frac{A_{base} \times height}{3}$$

$V = \frac{lwh}{3}$

$V = \pi r^2 h$



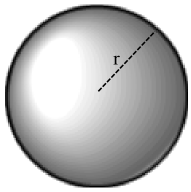
Cone



$$V_{cone} = \frac{A_{base} \times height}{3}$$
$$= \frac{\pi r^2 h}{3}$$

$V = \frac{\pi r^2 h}{3}$

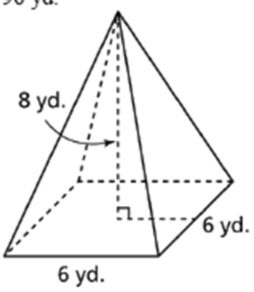
Sphere



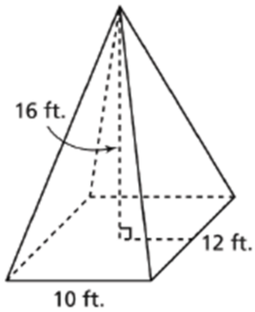
$$V_{sphere} = 4/3\ \pi\ r^3$$

EXERCISE: Find the volume of each of the following pyramids...

a) 96 yd.^{3}



b) 640 ft.^{3}



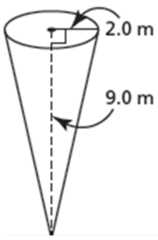
ERASE to get solution

$$V = \frac{A_{base} \times h}{3} = \frac{(6)(6) \times 8}{3} = 96\,yd^3$$

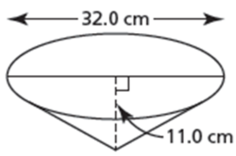
$$V = \frac{A_{base} \times height}{3} = \frac{(10)(12)(16)}{3} = 640\,ft^3$$

EXERCISE: Find the volume of each of the following cones...

a) 37.7 m^3



b) 2948.9 cm^3



ERASE to get solution

$$V = \frac{\pi r^2 \times h}{3}$$

$$= \frac{\pi (16)^2 \times 11}{3}$$

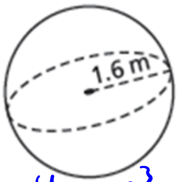
$$= 938.\overline{6}\,\pi$$

$$= 2948.9\,cm^3$$

$$V = \frac{A_{base} \times h}{3} = \frac{\pi r^2 h}{3} = \frac{\pi (2)^2 (9)}{3} = 37.7\,m^3$$

EXERCISE: Find the volume of each of the following spheres...

1) $17\ m^3$

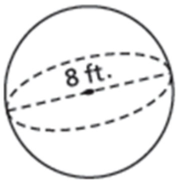


$V = \frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi(1.6)^3$

$= 17.2\ m^3$

2) $268\ ft.^3$



ERASE to get solution

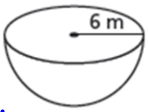
$V = \frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi(4)^3$

$= 268.08\ ft^3$

EXERCISE: Find the volume of each of the following hemispheres...

1) $452\ m^3$

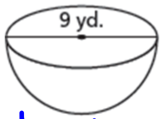


$V = \frac{4}{6}\pi r^3$

$= \frac{4}{6}\pi(6)^3$

$= 452.4\ m^3$

2) $191\ yd.^3$



ERASE to get solution

$V = \frac{4}{6}\pi r^3$

$= \frac{4}{6}\pi(4.5)^3$

$= 190.85\ yd^3$

A fitness ball is delivered in a flat package with a hand pump. The pump inflates the ball at a rate of 280 cm^3 per pump, to a diameter of 28 cm. How many pumps are needed to inflate the ball? Justify your answer.

$V = \frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi(14)^3$

$= 11494\ cm^3$

$\#pumps = \frac{11494\ cm^3}{280\ cm^3/pump}$

$= 41.05$

42 pumps

19. 42 pumps

1.6 Surface Area and Volume of a Sphere

A pail of cookie dough is cylindrical, with diameter 17 cm and height 13 cm. A scoop makes a sphere of cookie dough with diameter 5 cm. How many cookies can be made from this pail of dough?

20. 45 cookies

$$V_{pail} = A_{base} \times h$$
$$= \pi r^2 h$$
$$= \pi (8.5)^2 (13)$$
$$= 2950.7 \text{ cm}^3$$

$$V_{cookie} = \frac{4}{3} \pi r^3$$
$$= \frac{4}{3} \pi (2.5)^3$$
$$= 65.4 \text{ cm}^3$$

$$\# \text{Cookies} = \frac{2950.7}{65.4}$$
$$= 45.1 \quad 45 \text{ cookies}$$

1.6 Surface Area and Volume of a Sphere

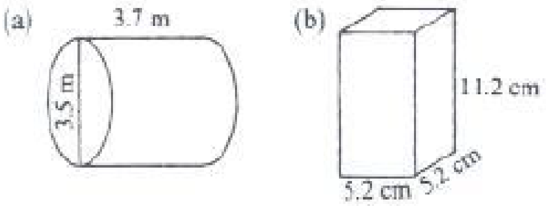
HOMEWORK...

Worksheet - Volume of Cones_Pyramids_Spheres.pdf

3. a) 823 b) 998 c) 1970

4. a) 60

4 Find the surface area of each shape.



b) $A = 2lw + 2lh + 2wh$

$= 287 cm^2$

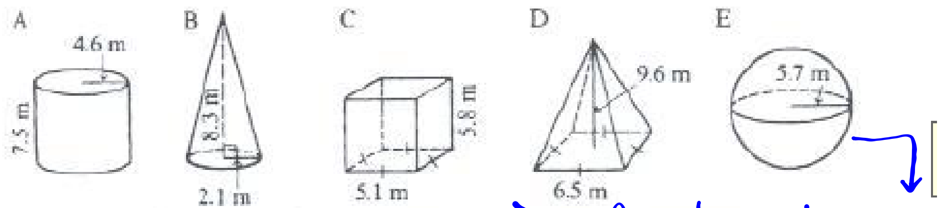
c) $A = \pi r^2 + \pi r s$

$= \pi(29)^2 + \pi(29)85$

$= 10\,386 mm^2$

5 (a) Predict which container holds the most.

(b) Arrange the containers in order from the one that holds the most to the one that holds the least.



A: $V = \pi r^2 h$

$= \pi (4.6)^2 7.5$

$= 498.6 m^3$

B: $V = \frac{\pi r^2 h}{3}$

$= \frac{\pi (2.1)^2 (8.3)}{3}$

$= 38.3 m^3$

C: $V = A_{base} \times h$

$= (5.1)(5.1)(5.8)$

$= 150.9 m^3$

D: $V = \frac{A_{base} \times h}{3}$

$= \frac{(6.5)(6.5)(9.6)}{3}$

$= 135.2 m^3$

E: $V = \frac{4\pi r^3}{3}$

$= \frac{4\pi(5.7)^3}{3}$

$= 775.7 m^3$

EADCB

B 6 Find each volume.

(a)

(b)

a) $V = \frac{\pi r^2 h}{6}$

$= \frac{\pi (2.4)^2 (5.8)}{6}$

$= 17.5 cm^3$

$5.2 - 3.7 = 1.5$

b) $V = V_{bottom} + V_{top}$

$= \pi r^2 h + \frac{\pi r^2 h}{2}$

$= \pi (3.5)^2 (3.7) + \frac{\pi (3.5)^2 (1.5)}{2}$

$= 45.325\pi + 9.2\pi$

$= 54.525\pi$

$= 171.3 m^2$

7 A rectangular pool has dimensions 10.6 m long, 6.2 m wide and 2.1 m deep.

(a) The water level is 22.5 cm below the edge of the pool. Calculate the volume of water in the pool.

(b) Each day it costs 3.29¢/m³ to maintain the pool. Calculate the total cost of maintaining the pool from May 24 to September 18.

a) Depth $2.1 - 0.225$

$= 1.875 m$

$V = lwh$

$= (10.6)(6.2)(1.875)$

$= 123.225 m^3$

May $31 - 24 = 7 + 1 = 8$
June 30
July 31
Aug 31
Sept 18
118 days

$\$3.29/m^3 \times 123.225 m^3 \times 118$ days

$= \$47838.41$

8 The cylindrical container is full of liquid plastic.
(a) Calculate the volume of liquid plastic.
(b) Billiard balls, with a diameter of 5.7 cm, are made from the plastic. How many can be made from one container of liquid plastic?

$\frac{0.057m}{2} = 0.0285$

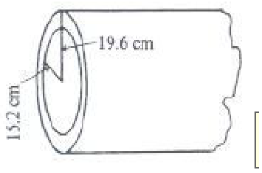
$V = \pi r^2 h$
$= \pi (0.8)^2 (1.2)$
$= 2.41 m^3$

$V_{ball} = \frac{4}{3}\pi r^3$
$= \frac{4}{3}\pi (0.0285)$
$= 0.000097$

Divide $= \frac{2.41}{0.000097}$

$= 24\ 875$ balls.

9 A concrete drainage pipe has an inner radius of 15.2 cm and an outer radius of 19.6 cm.
(a) Calculate the number of litres of water that a pipe 6.0 m in length can hold.
(b) If the material used to make the pipe has a mass of 12.6 g/cm³ find the mass of the pipe in (a).



a) $V = \pi r^2 h$
$= \pi (15.2)^2 (600)$
$= 435\ 500.14 cm^3$

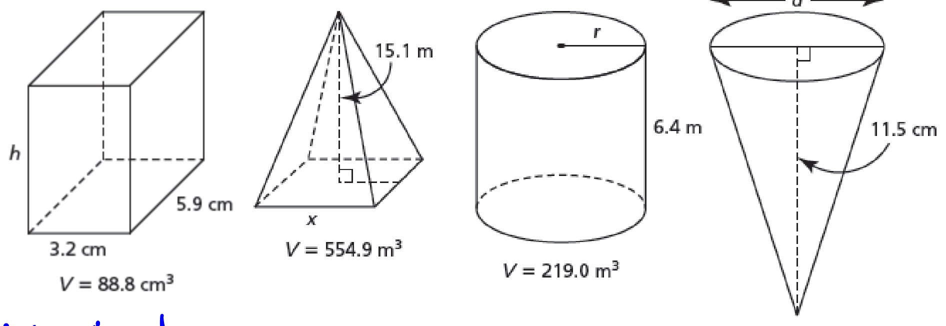
$435\ 500.14 cm^3 \times \frac{1\ ml}{1\ cm^3} \times \frac{1\ l}{1000 ml} = 435.5 L$

b) $V_{cement} = V_{outside} - V_{inside}$
$= \pi r^2 h - \pi r^2 h$
$= \pi (19.6)^2 (600) - 435\ 500.14$
$= 288\ 624.4 cm^3$

$12.6 g/cm^3 \times 288\ 624.4 cm^3 \times \frac{1 kg}{1000 g} = 3636.7 Kg$

For each object, its volume, V, and some dimensions are given.
Calculate the dimension indicated by the variable.
Write each answer to the nearest tenth of a unit.

a) right rectangular prism b) right square pyramid c) right cylinder d) right cone



$V = lwh$

$88.8 = (3.2)(5.9)h$

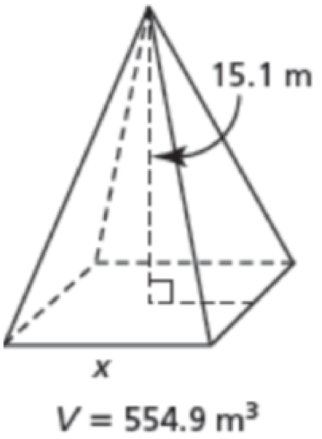
$\frac{88.8}{(3.2)(5.9)} = h$

18. a) 4.7 cm b) 10.5 m
c) 3.3 m d) 7.4 cm

1.5 Volumes of Right Pyramids and Right Cones

$h = 4.7\text{ cm}$

b) right square pyramid



$V = \frac{A_{base}}{3} \times h$

$554.9 = \frac{x \cdot x \cdot 15.1}{3}$

$\frac{3(554.9)}{15.1} = x^2$

$x^2 = 110.2450331$

$x = 10.5\text{ m}$

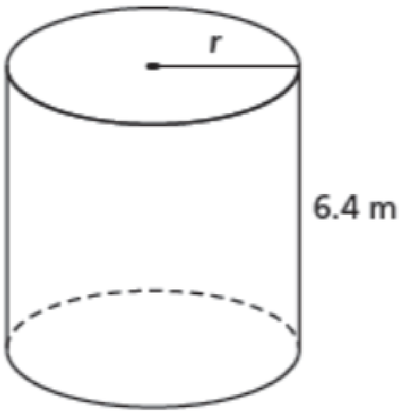
$x^2 = 36$

$x = \sqrt{36}$

$= 36^{\frac{1}{2}}$

$36^{0.5}$

c) right cylinder



$V = 219.0\ m^3$

$V = \pi r^2 h$

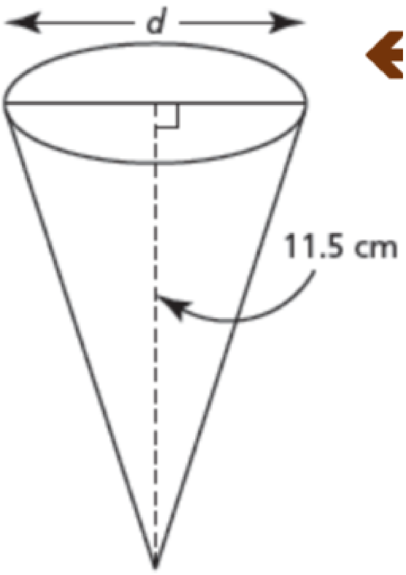
$219.0 = \pi r^2 (6.4)$

$\frac{219.0}{\pi(6.4)} = r^2$

$r^2 = 10.89\ldots$

$r = 3.3m$

d) right cone



$V = 164.9\ cm^3$

$V = \frac{A_{base} \times h}{3}$

$V = \frac{\pi r^2 \times h}{3}$

$164.9 = \frac{\pi r^2 \times 11.5}{3}$

$\frac{3(164.9)}{\pi(11.5)} = r^2$

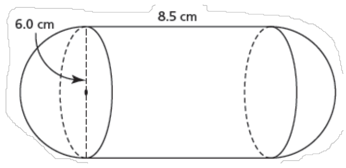
$r^2 = 13.6928\ldots$

$r = 3.7$

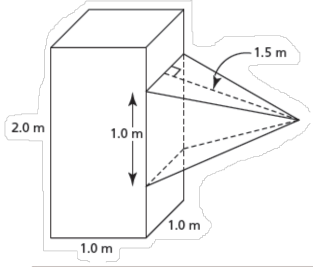
$d = 7.4 cm$

Determine the surface area and volume of each composite object. Write the answers to the nearest tenth of a unit.

a) right cylinder and hemispheres



b) right square prism and right square pyramid



$A = 2\pi rh + 4\pi r^2$

$= 2\pi(3)(8.5) + 4\pi(3)^2$

$= 51\pi + 36\pi$

$= 87\pi$

$= 273.3\,cm^2$

5. a) 273.3 cm², 353.4 cm³ b) 12.0 m², 2.5 m³

1.7 Solving Problems Involving Objects

$V = V_{cylinder} + V_{sphere}$

$= \pi r^2 h + \frac{4}{3}\pi r^3$

$= \pi(3)^2(8.5) + \frac{4}{3}\pi(3)^3$

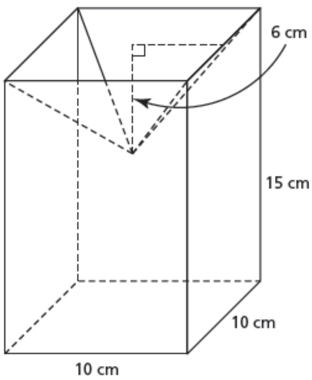
$= 76.5\pi + 36\pi$

$= 112.5\pi$

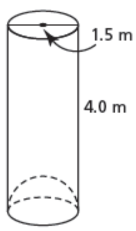
$= 353.4\,cm^3$

Determine the volume of each object to the nearest tenth of a cubic unit.

a) a right square prism with a right square pyramid removed



b) a right cylinder with a hemisphere removed



$V_{total} = V_{box} - V_{pyramid}$

$= lwh - \frac{lwh}{3}$

$= (10)(10)(15) - \frac{(10)(10)(6)}{3}$

$= 1500 - 200$

$= 1300\,cm^3$

$V = V_{cylinder} - V_{hemisphere}$

$= \pi r^2 h - \frac{4}{6}\pi r^3$

$= \pi(0.75)^2(4) - \frac{4}{6}\pi(0.75)^3$

$= 2.25\pi - 0.28125\pi$

$= 1.96875\pi$

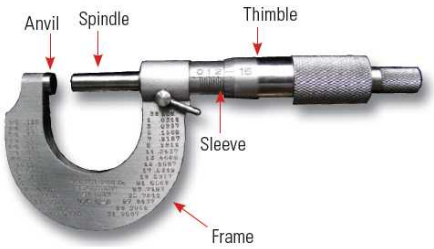
$= 6.2\,m^3$

1.7 Solving Problems Involving Objects

ACTIVITY 6.6
USING MICROMETERS AND CALIPERS

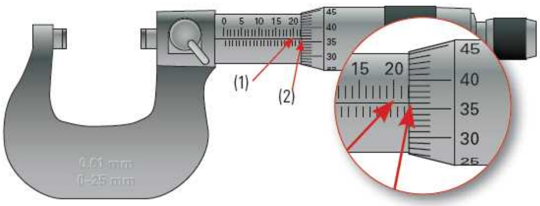
Some common measurement tools used to measure lengths are a ruler and a tape measure. When measuring the diameter of a cylindrical or spherical object, you can use two other tools: a caliper and a micrometer.

A micrometer can be used to measure the diameter of a sphere or a cylinder by reading the measurements on the sleeve and the thimble:



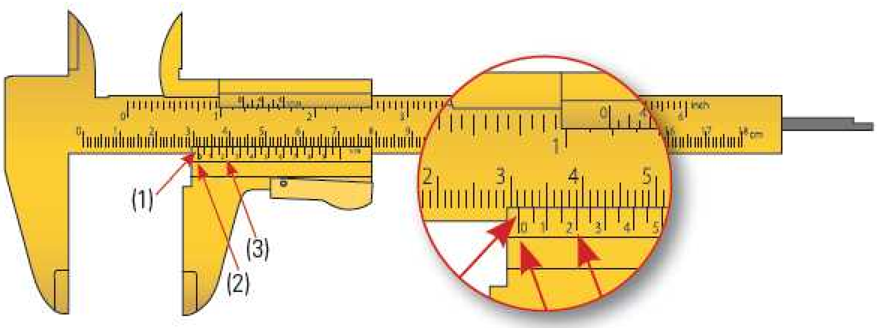
The value on the sleeve represents millimetres and the value on the thimble represents hundredths of a millimetre.

Using the reading below, look at the value on the sleeve first (1), which in this case is 22 millimetres. Then, look at the value on the thimble (2), which is 36, or 0.36 mm. That means the length you've measured is 22.36 mm.



A caliper can be used to measure the diameter of an object, inside or outside the object. For example, a paper towel roll is hollow, so the caliper can be used to measure the inside diameter.

The top ruler measures the number of centimetres and millimetres. For this measurement, find the nearest millimetre reading to the left of the 0 on the lower ruler, which in this case is 31 mm (1). The lower ruler is in hundredths of a centimetre. From the 0 on the bottom ruler (2), find the line that matches up perfectly with a line on the top ruler (3). This is at 0.02 cm, so the measurement is 3.12 cm.



HOMEWORK...

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6.4 - Build Your Skills Solutions.pdf

READY FOR THE TEST???

- Page 268: #1 - 10

Chapter 6 Surface Area, Volume, and Capacity, Practice Your New Skills.pdf

1. a) $\frac{V}{SA} = \frac{lwh}{6lw} = \frac{l^3}{6l^2} = \frac{l}{6}$

b) $\frac{V}{SA} = \frac{\frac{4}{3}\pi r^3}{4\pi r^2} = \frac{r}{3}$

c) $4\pi r^2 = 6l^2$

$6l^2 = 4\pi r^2$

$l = \sqrt{\frac{4}{6}\pi}\, r$

$= 1.45r$

1. If you have a solid rectangular metal prism with dimensions 34 m by 45m by 11m what would be the radius of a sphere that it was melted to form.

$V = lwh$

$= (34)(45)(11)$

$= 16\,830 m^3$

$V_{sphere} = \frac{4}{3}\pi r^3$

$16830 = \frac{4}{3}\pi r^3$

$\frac{16830(3)}{4\pi} = r^3$

$r = \left(\frac{16830(3)}{4\pi}\right)^{\frac{1}{3}}$

$r = \sqrt[3]{\frac{16830(3)}{4\pi}}$

$= 15.9 m$

2. How does doubling the dimensions of a rectangular prism affect the volume?

$V = lwh$

$= (2)(2)(2)$

$= 8$ times greater

SURFACE AREA, VOLUME, AND CAPACITY

Now that you have finished this chapter, you should be able to:

- Explain, using examples, the difference between volume and surface area.
- Explain, using examples and nets, the relationship between area and surface area.
- Estimate and calculate the surface area and volume of a three-dimensional object.
- Explain, using examples, the difference between volume and capacity.
- Convert a volume in one unit of measure, such as cm^3, to another unit of measure, such as m^3.
- Determine the volume of a three-dimesional object using a variety of measuring tools, such as rulers, tape measures, micrometers, and calipers.
- Determine the capacity of a three-dimensional object using a variety of measuring tools and methods, such as graduated cylinders, measuring cups, measuring spoons, and displacement.
- Describe the relationship between the volumes of cones and cylinders with the same base and height.
- Describe the relationship between the volumes of pyramids and prisms with the same base and height.
- Explain the effect a change in dimensions of a three-dimensional object has on its surface area and volume.

READY FOR THE TEST???

- Page 268: #1 - 10

Chapter 6 Surface Area, Volume, and Capacity, Practice Your New Skills.pdf

Attachments

6.4 - Build Your Skills Solutions.pdf

Chapter 6 Surface Area, Volume, and Capacity, Practice Your New Skills.pdf

Chapter 6 Sample Test.pdf

Chapter 6 Sample Test Answers.pdf

Worksheet - Volume of Cones_Pyramids_Spheres.pdf