

1. Evaluate each of the following limits, if they exist:

$$(a) \lim_{x \rightarrow -2} \frac{x^4 - 16}{x^3 + 8}$$

$$\begin{aligned} & \text{l.i.m}_{x \rightarrow -2} \frac{(x^2 - 4)(x^2 + 4)}{(x+2)(x^2 - 2x + 4)} \\ &= \frac{-4(4)}{12} \\ &= \frac{-32}{12} \\ &= \frac{-8}{3} \end{aligned}$$

$$(c) \lim_{x \rightarrow 7^+} \frac{|7-x|}{x^2 - 49}$$

$$\begin{aligned} & \text{l.i.m}_{x \rightarrow 7^+} \frac{|7 - 7.000 \dots 1|}{(x-7)(x+7)} \\ &= \frac{1.000 \dots 1}{(8.00-1)(14)} \\ &= \frac{1}{14} \end{aligned}$$

$$(b) \lim_{x \rightarrow 0} \frac{2x^3 - 6x^4}{\sin^3 5x} \quad [24]$$

$$\begin{aligned} & \text{l.i.m}_{x \rightarrow 0} \frac{2x^3(1-3x)}{\sin^3 5x} \\ &= 2 \left(\frac{5x}{\sin 5x} \right)^3 \frac{1-3x}{125} \\ &= 2(1)^3 \left(\frac{1}{125} \right) \\ &= \frac{2}{125} \end{aligned}$$

$$(d) \lim_{x \rightarrow \infty} \frac{(3x^4 - 2x^3)^2}{3x(5x^3 - 2x^7)}$$

$$\begin{aligned} & \text{l.i.m}_{x \rightarrow \infty} \frac{9x^8 - 12x^7 + 4x^6}{15x^4 - 6x^8} \\ &= \frac{9 - 0 + 0}{0 - 6} \\ &= -\frac{3}{2} \end{aligned}$$

$$(e) \lim_{x \rightarrow 0} \frac{5 - \sqrt{3x+10}}{x-5} \quad \left(\frac{5 + \sqrt{3x+10}}{5 + \sqrt{3x+10}} \right)$$

$$\text{l.i.m}_{x \rightarrow 0} \frac{25 - (3x+10)}{(x-5)(5 + \sqrt{3x+10})}$$

$$\begin{aligned} & \text{l.i.m}_{x \rightarrow 0} \frac{15 - 3x}{(x-5)(5 + \sqrt{3x+10})} \\ &= \frac{-3}{5 + \sqrt{10}} \end{aligned}$$

$$= -\frac{3}{5 + \sqrt{10}}$$

$$= -0.3675$$

$$(f) \lim_{x \rightarrow 3} \frac{\frac{3}{x-3}}{x-3}$$

$$\text{l.i.m}_{x \rightarrow 3} \frac{3(x+1)-3(y)}{4(x+1)} \cdot \frac{1}{x-3}$$

$$\text{l.i.m}_{x \rightarrow 3} \frac{3x+3-12}{4(x+1)(x-3)}$$

$$\text{l.i.m}_{x \rightarrow 3} \frac{3(x-3)}{4(x+1)(x-3)}$$

$$= \frac{3}{4(4)}$$

$$= \frac{3}{16}$$

2. Determine the equation of the TANGENT to the curve $f(x) = 3x^2 + x - 1$ at $x = -1$.
(Differentiation Rules Not Permitted)

[6]

$$f(x+h) = 3(x+h)^2 + (x+h) - 1$$

$$f(x+h) = 3(x^2 + 2xh + h^2) + x + h - 1$$

$$= 3x^2 + 6xh + 3h^2 + x + h - 1$$

$$\lim_{h \rightarrow 0} \frac{(3x^2 + 6xh + 3h^2 + x + h - 1) - (3x^2 + x - 1)}{h}$$

$$\lim_{h \rightarrow 0} \frac{x(6x + 3h + 1)}{h}$$

$$m = 6x + 1$$

$$\text{at } x = -1$$

$$m = 6(-1) + 1$$

$$= -5$$

$$y = 3(-1)^2 + (-1) - 1$$

$$= 3 - 2$$

$$= 1 \quad (-1, 1)$$

$$y - 1 = -5(x + 1)$$

$$y - 1 = -5x - 5$$

$$\cancel{5x + y + 4 = 0}$$

3. (a) Find the sum of the first fifteen terms of the arithmetic sequence where $t_7 = -14$ and $t_{25} = -68$.

[6]

$$\begin{aligned} a + 6d &= -14 \\ a + 24d &= -68 \\ 18d &= -54 \\ d &= -3 \end{aligned}$$

$$\begin{aligned} a + 6(-3) &= 14 \\ a &= 4 \end{aligned}$$

$$S_{15} = \frac{15}{2} [2(4) + (14)(-3)]$$

$$= \frac{15}{2} (8 - 42)$$

$$= 15(-34)$$

$$= -255$$

- (b) Determine the sum of the following series: $2 + 6 + 18 + 54 + \dots + 86\,093\,442$

[5]

$$\begin{aligned} 86\,093\,442 &= 2(3)^{n-1} \\ 43\,046\,721 &= (3)^{n-1} \end{aligned}$$

$$3^n = 3^{n-1}$$

$$\begin{aligned} 3^n &= n-1 \\ 16 &= n-1 \\ \therefore & \end{aligned}$$

$$S = \frac{2((3)^{16} - 1)}{3 - 1}$$

$$S = 129\,140\,162$$

4. (a) Expand the following and express as a polynomial in simplest form:

$$(-2a^4 + 4b^3)^5$$

1 1 2 1
1 3 3 1
1 4 6 4 5 [5]

$$\begin{aligned}
 &= (-2a^4)^5 + 5(-2a^4)^4(4b^3) + 10(-2a^4)^3(4b^3)^2 + 10(-2a^4)^2(4b^3)^3 + 5(-2a^4)(4b^3)^4 + (4b^3)^5 \\
 &= -32a^{20} + 320a^{16}b^3 - 1280a^{12}b^6 + 2560a^8b^9 - 2560a^4b^{12} + 1024b^{15}
 \end{aligned}$$

- (b) Given that the binomial expression $(2x^7 - 3y^9)^{13}$ is expanded, determine the numerical coefficient of the term that would have the variable part $x^{21}y^{90}$. [3]

$$\begin{aligned}
 &\binom{13}{3} (2x^7)^3 (-3y^9)^{10} \\
 &286 (8x^{21})(-3)^{10} (y^{90}) \\
 &= \underline{\underline{1351\ 041\ 112}}
 \end{aligned}$$

5. Solve the following equation: $6x^3 - 15 = 7x(2-x)$

[4]

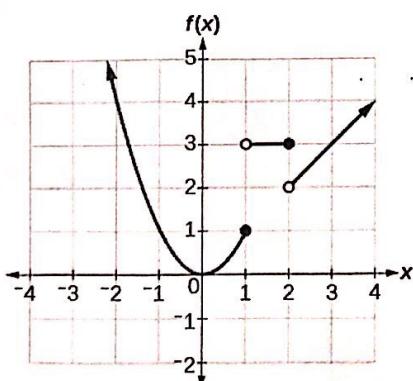
$$\begin{aligned}
 &x=1 \quad 6x^3 - 15 = 14x - 7x^2 \\
 &6x^3 + 7x^2 - 14x - 15 = 0 \\
 &-6 + 7 + 14 - 15 = 0
 \end{aligned}$$

$$\begin{aligned}
 &(x+1) \quad 1 \cancel{1} \overset{6}{\cancel{6}} \overset{7}{\cancel{7}} \overset{-14}{\cancel{-14}} \overset{-15}{\cancel{-15}} \\
 &\quad \cancel{6} \quad \cancel{1} \quad \cancel{-15} \quad 0 \\
 &(x+1)(6x^2 + x - 15) = 0 \\
 &(6x+10)(6x-9) = 0 \\
 &(x+1)(3x+5)(2x-3) = 0
 \end{aligned}
 \qquad \qquad \qquad x = -1, -\frac{5}{3}, \frac{3}{2}$$

6. Use the graph provided to fill in the blanks below.

Use does not exist (DNE) where appropriate.

[5]



- (a) $\lim_{x \rightarrow 1^+} f(x) = \underline{\underline{3}}$
 (b) $\lim_{x \rightarrow 2^-} f(x) = \underline{\underline{3}}$
 (c) $\lim_{x \rightarrow -2} f(x) = \underline{\underline{4}}$
 (d) $\lim_{x \rightarrow 1} f(x) = \underline{\underline{DNE}}$
 (e) $f(2) = \underline{\underline{3}}$

7. (a) Examine the following function for any point(s) of discontinuity. Clearly show all work and provide appropriate mathematical proof that supports your conclusions. [4]

$$f(x) = \begin{cases} (x+2)^2 - 2 & , \text{ if } x < -1 \\ -2-x & , \text{ if } -1 \leq x < 2 \\ 0 & , \text{ if } x = 2 \\ -(x-3)^2 - 3 & , \text{ if } x > 2 \end{cases}$$

$$F(-1) = \frac{x=-1}{-2-(-1)} = -1$$

$$\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^+} f(x) = -1$$

$$= (1)^2 - 2 = -1$$

continuous

$$\frac{x=2}{F(2)=0}$$

$$\lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^+} f(x)$$

$$= -2-2 = -4$$

$$= -(-1)^2 - 3 = -1$$

$$= -4$$

$$\lim_{x \rightarrow 2} f(x) \neq f(2)$$

- ∵ discontinuous

- (b) Provide a sketch of $f(x)$. [3]

$$y = (x+2)^2 - 2$$

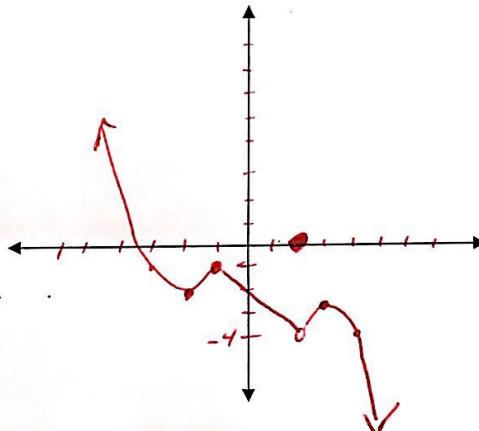
$$V(-2, -2)$$

$$\frac{x}{-1} / \frac{y}{-1}$$

$$y = -2-x$$

$$\frac{x}{-1} / \frac{y}{-1}$$

$$\frac{x}{2} / \frac{y}{-4}$$



8. If $x-2, -3x+1$ and $7x+1$ represent three consecutive terms of a geometric sequence of INTEGERS, determine the value of the eleventh term in this sequence. [5]

$$\frac{-3x+1}{x-2} = \frac{7x+1}{-3x+1}$$

$$9x^2 - 6x + 1 = 7x^2 + 14x + x - 2$$

$$2x^2 + 17x + 3 = 0$$

$$(2x+1)(2x+3) = 0$$

$$(x+3)(2x+1) = 0$$

$$x = -3 \text{ or } x = -\frac{1}{2}$$

$$\downarrow$$

$$-5, 1^0, -20$$

$$a = -5$$

$$r = -2$$

$$t_{11} = (-5)(-2)^{10}$$

$$t_{11} = -5120$$