Curriculum Outcomes:

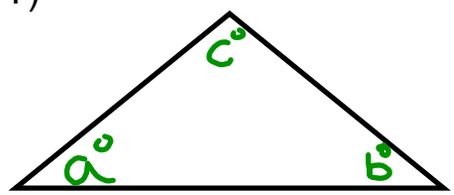
(SS1) Solve problems and justify the solution strategy using circle properties, including: the perpendicular from the centre of a circle to a chord bisects the chord; the measure of the central angle is equal to twice the measure of the inscribed angle subtended by the same arc; the inscribed angles subtended by the same arc are congruent; a tangent to a circle is perpendicular to the radius at the point of tangency.

Student Friendly:
How we can use the tangent properties to solve for unknown lengths. (Tangent properties go hand and hand with Pythagorean theorem)

Sum of Angles in a Triangle Theorem (SATT)

-Angles in a triangle add up to 180°

$a° + b° + c° = 180°$

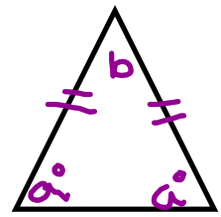


Isosceles Triangle Theorem (ITT)

-Base angles in an isosceles triangles are equal

$b = 180 - a - a$

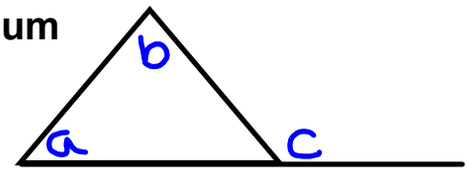
$a = \frac{180 - b}{2}$



Exterior Angle Theorem (EAT)

-Exterior angle of a triangle is equal to the sum of the opposite interior angles.

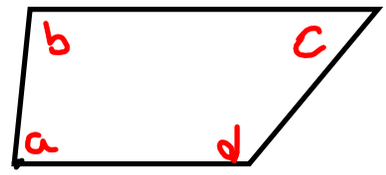
$c = a + b$



Quadrilateral Angle Theorem (QuadT)

-Angles in a quadrilateral add up to 360°

$a + b + c + d = 360°$



Supplementary Angle Theorem (SAT)

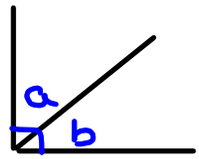
-Angles on a straight line add up to 180°

$a + b = 180°$

Complementary Angle Theorem (CAT)

-Angles add up to 90°

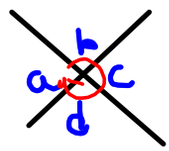
$a + b = 90°$



Cyclic Angle theorem (CyAT)

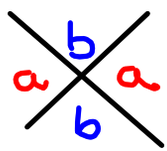
-Angles in a circle add up to 360°

$a + b + c + d = 360°$



Opposite Angle theorem (OAT)

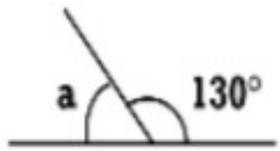
-Opposite angles are equal

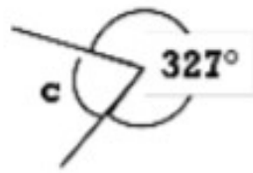


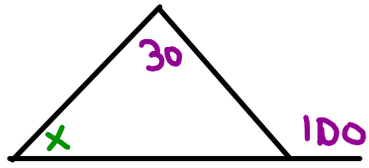
a = 50° (SAT)

b = 52° (SAT)

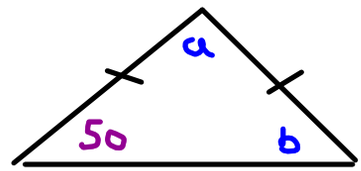
c = 33° (CyAT)





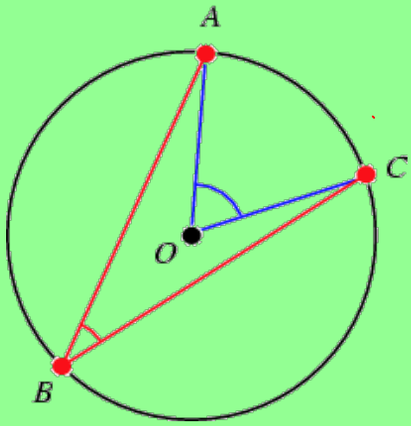


x = 70° (EAT)



b = 50° (Itt)

a = 80° (SATT)
(Itt)

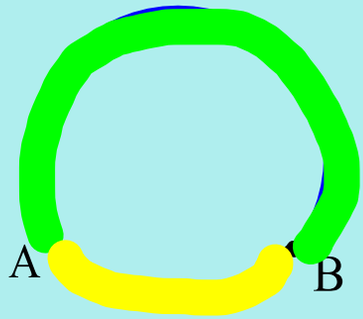


Section 8.3

Properties of Angles in Circles

- The longer arc AB is the major arc .

- The shorter arc AB is the minor arc.

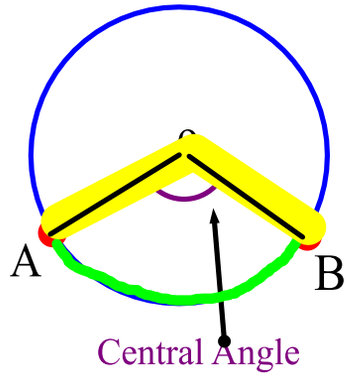


$\overset{\frown}{AB}$

Central Angle:

The angle formed by joining the endpoints of a arc to the centre of a circle
(Made with 2 radii)

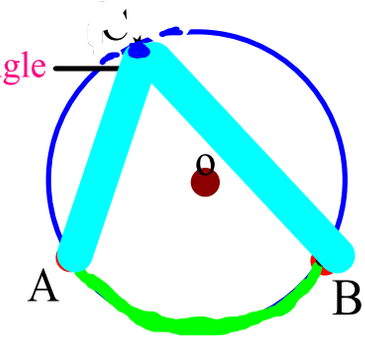
$\angle AOB$



Inscribed Angle:

The angle formed by joining the endpoints of a arc to a point on the circle
(Made with two chords)

$\angle ACB$

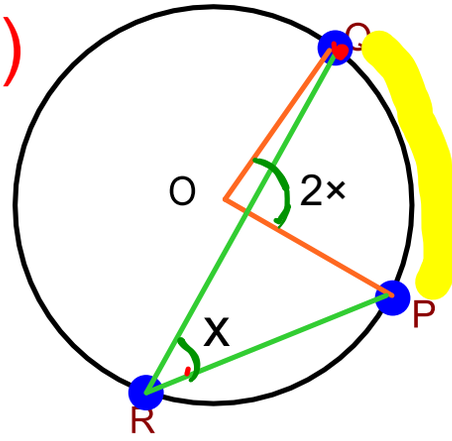


Inscribed and central angles are SUBTENDED by the MINOR arc

come from the same 'smaller arc'

Central Angle & Inscribed Angle Property

Property 1: (Ins/Cent <, ⌒)



In a circle, the measure of a central angle coming from an arc is TWICE the measure of an inscribe angle coming from the same arc.

$\angle POQ = 2\angle PRQ$

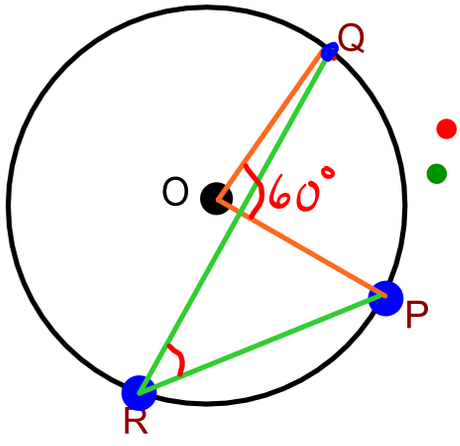
Central angle is twice the inscribed angle

or

$\angle PRQ = \frac{1}{2}\angle POQ$

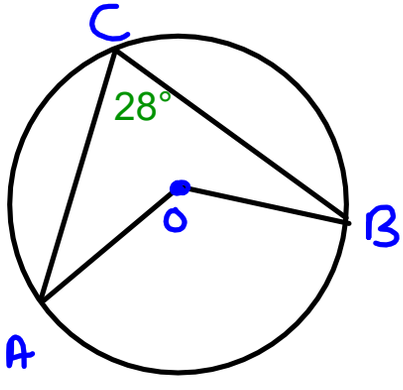
Inscribed angle is half the center angle

Example:



$\angle QOP = 60°$ (central)

$\angle QRP = 30°$ (ins/cent>, $\overset{\frown}{QP}$)



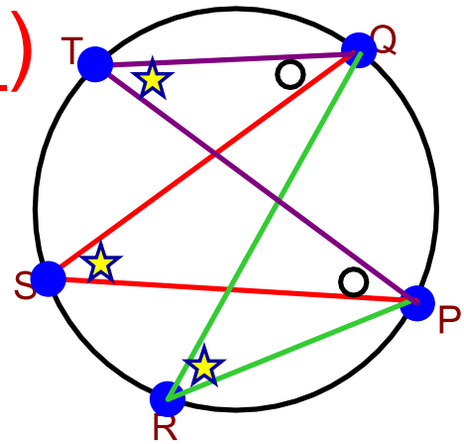
$\angle ACB = 28°$ (ins>)

$\angle AOB = 56°$ (ins/cent>, $\overset{\frown}{AB}$)

Inscribed Angle Property

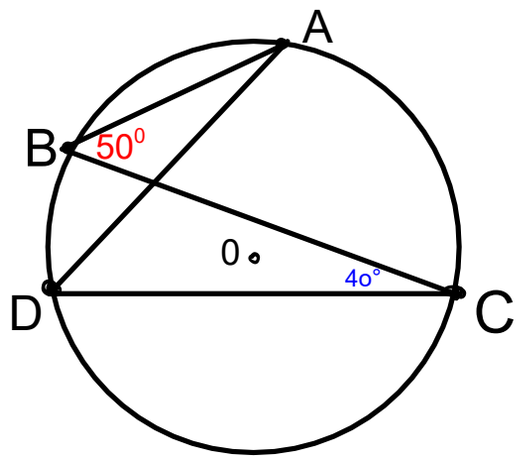
Property 2: (Ins<, ⌒)

In a circle, all inscribed angles coming from the same arc are equal.



$\angle PTQ = \angle PSQ = \angle PRQ$

Example:



$\angle ABC = 50°$ (ins)

$\angle ADC = 50°$ (ins >, $\overset{\frown}{AC}$)

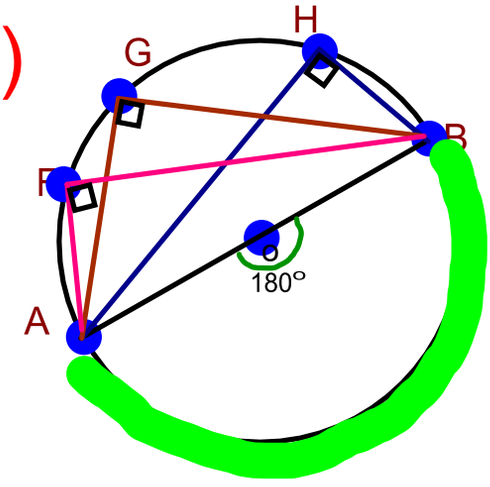
$\angle BCD = 40°$ (ins)

$\angle BAD = 40°$ (ins >, $\overset{\frown}{BD}$)

Angles is a Semicircle Property

Property 3: (Inc <, diam)

All inscribed angles subtended by a semicircle (diameter)are right angles



Makes sense

Inscribed angles are always half the centre

Center Angle = 180° (Straight Line)

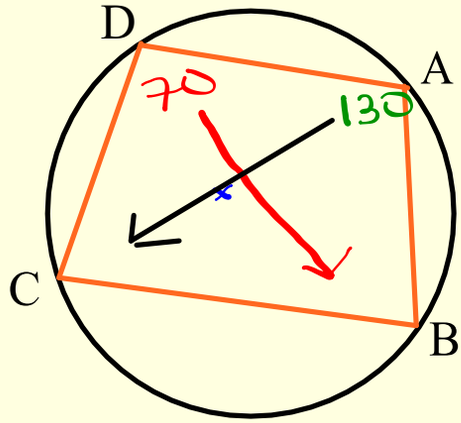
Inscribed angle is half the Central Angle

Inscribed = (1/2) central
=(1/2) 180°
= 90°

Cyclic Quadrilateral Angle Properties:

Property 4: (Cyclic Quad)

__The opposite angles of an inscribed **quadrilateral** are supplementary.
(add up to 180°)



∠ABC = 110° (CyQuad)

∠BCD = 50° (CyQuad)

ABCD is an inscribed quadrilateral.
< A and < C are opposite
therefore, < A + < C = 180°

< B and < D are opposite
therefore, < B + < D = 180°

(SATT) (ITT) (SAT) (CAT) (OAT) (CyAT)	< ___ = 90° (Tang P) <___= <___= 90° (Chord P) ___ = ____(Chord P) ___ = ____=___(Radii)	< ___= __° (ins/cent >, $\overset{\frown}{\_\_}$) < ___= __° (ins >, $\overset{\frown}{\_\_}$) < ___= __° (ins >, diam) < ___= __° (CyQuad)