McGraw-Hill Ryerson

Pre-Calculus 11

Chapter 2
Trigonometry

#4. $\dfrac{2\sqrt{48}-\sqrt{24}}{2\sqrt{3}-\sqrt{6}}$

$\dfrac{8\sqrt{3}-2\sqrt{6}}{2\sqrt{3}-\sqrt{6}}\left(\dfrac{2\sqrt{3}+\sqrt{6}}{2\sqrt{3}+\sqrt{6}}\right)$

$\dfrac{16(3)+8\sqrt{18}-4\sqrt{18}-2(6)}{4(3)-6}$

$\dfrac{36+4\sqrt{18}}{6}=\dfrac{3\cancel{6}+12\sqrt{2}}{6}\begin{matrix}\div 6\\ \div 6\end{matrix}$

$=\dfrac{6+2\sqrt{2}}{1}$

5. a) $\dfrac{2a(4b^5) - 5b(2a^5) + 2ab^2(3a^4b^2)}{12a^6b^5}$

$\dfrac{8ab^5 - 10a^5b + 6a^5b^4}{12a^6b^5}$ $\div 2ab$ $\div 2ab$

$\dfrac{4b^4 - 5a^4 + 3a^4b^3}{6a^5b^4}$

$3x^2+8x-3 \quad -9$

$(\frac{3x}{3}+\frac{9}{3})(3x-1)$

$(x+3)(3x-1)$

6. $\left(\frac{x+3}{x^2-5x+6}+\frac{6}{x^2-7x+12}\right)\times\frac{x^2-5x+4}{x^2-64}\div\frac{5x^2-5x}{x^2-10x+16}$

$\left(\frac{x+3}{(x-3)(x-2)}+\frac{6}{(x-4)(x-3)}\right)$

$\frac{(x+3)(x-4)+6(x-2)}{(x-3)(x-2)(x-4)}$

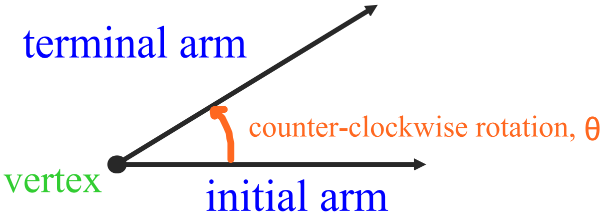
$x\neq 3, 2, 4, \pm 8, 0, 1$

$x^2+5x-24$

$\frac{\cancel{(x+8)}\cancel{(x-3)}}{\cancel{(x-3)}\cancel{(x-2)}\cancel{(x-4)}}\cdot\frac{\cancel{(x-4)}\cancel{(x-1)}}{\cancel{(x-8)}\cancel{(x+8)}}\cdot\frac{\cancel{(x-8)}\cancel{(x-2)}}{5x\cancel{(x-1)}}$

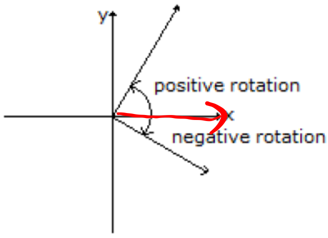
$\frac{1}{5x}$

Rotation Angles

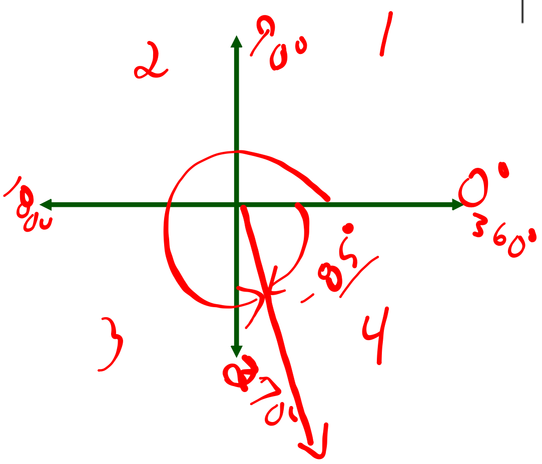


- **standard position** - when the initial arm is on the **positive** *x*-axis and the vertex is at the origin.

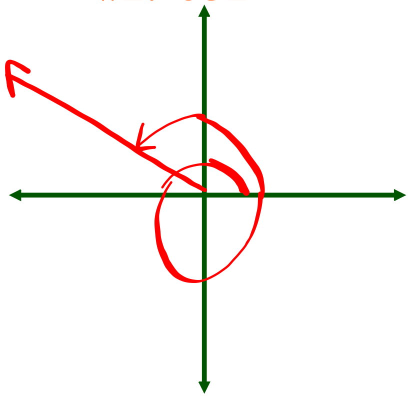
ex: positive rotation - counter clockwise (ccw)



#1: 275°

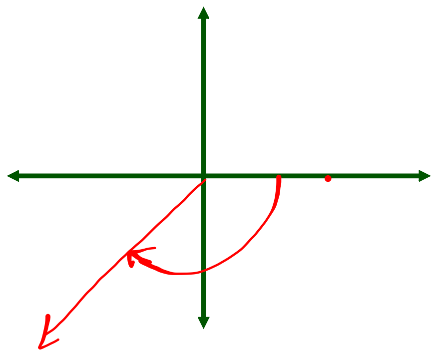


#2: 532°



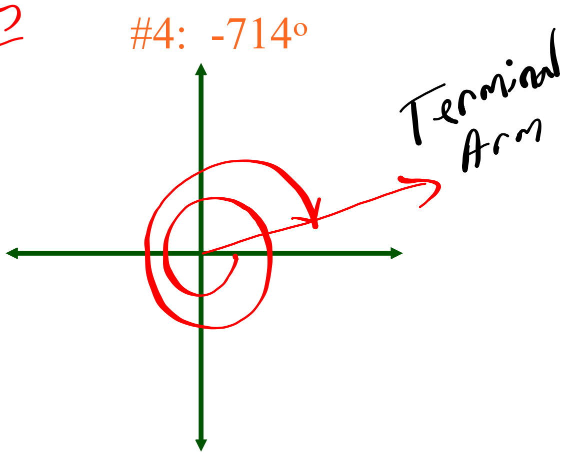
ex: negative rotation - clockwise (cw)

#3: -140°



-31335°

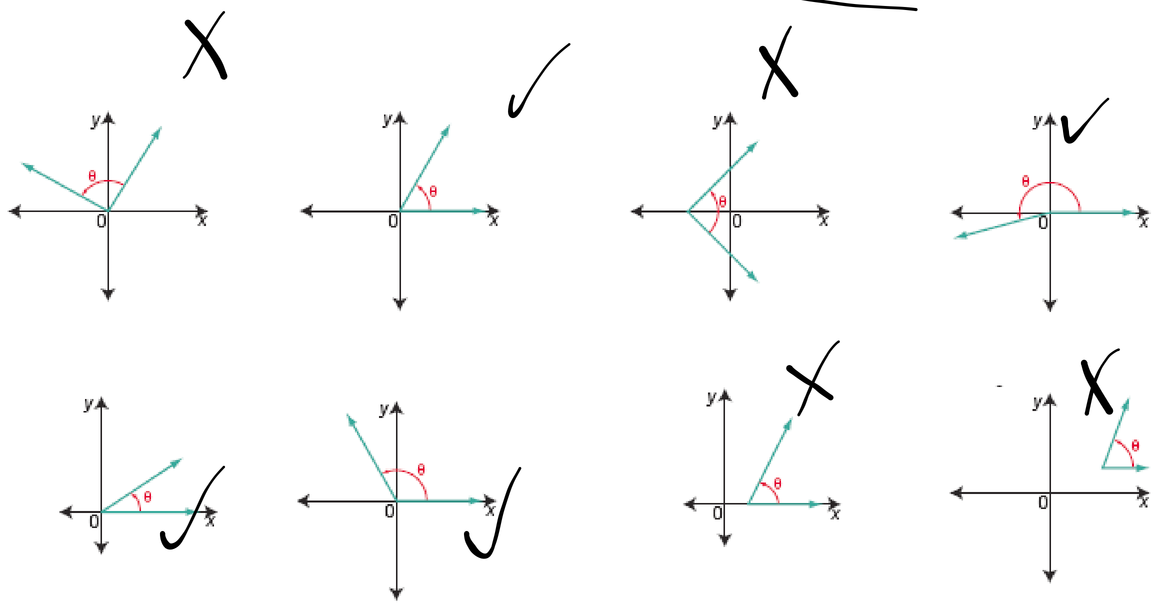
#4: -714°



Chapter 2

Angles in Standard Position

Circle the angles that are in standard position.



Check answer

Definition of Coterminal Angles

Coterminal Angles are angles drawn in standard position that share a terminal side.

For any angle θ, an angle coterminal with θ can be obtained by using the formula $\theta + k \cdot (360^\circ)$, where k is any integer.

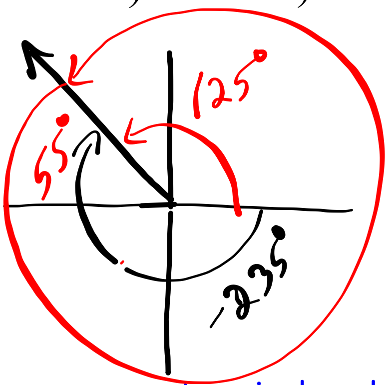
State both a positive and negative angle that would be coterminal with each of the following...

a) 583^o b) -235^o c) -810^o

$+720^\circ$

$-235^\circ + 360^\circ = 125^\circ$

$- 360^\circ = -595^\circ$



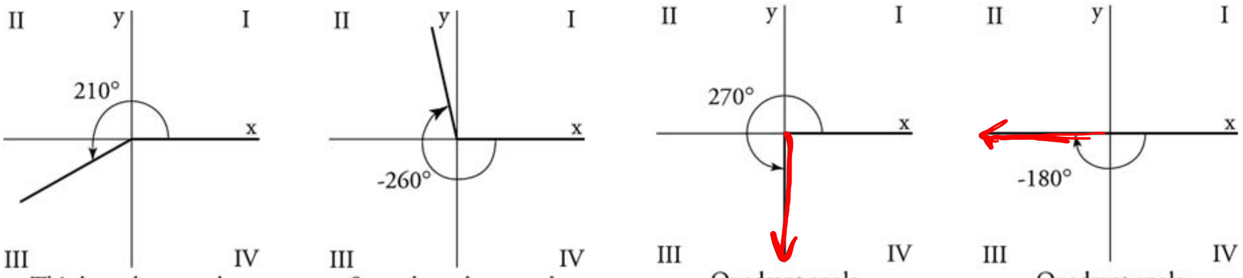
a) $583^\circ \Rightarrow -137^\circ, 223^\circ, 943^\circ$

c) $-810^\circ \Rightarrow -450^\circ, 270^\circ$

How many co-terminal angles exist for any rotation angle?

∞

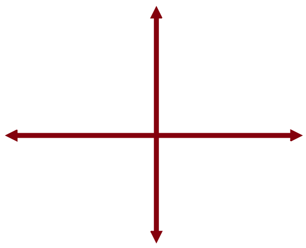
Angles that are in standard position are said to be **quadrantal** if their terminal side coincides with a coordinate axis. Angles in standard position that are not quadrantal fall in one of the four quadrants, as shown below...



Third quadrant angle (a) Second quadrant angle (b) Quadrant angle (e) Quadrant angle (f)

- Quadrantal angle: terminal arm lies on a quadrant boundary (axis)

examples...



Within which quadrant would the terminal arm for each of the following rotation angles be found?

94° 2

500° 2

−100° 3

180° Quadrantal

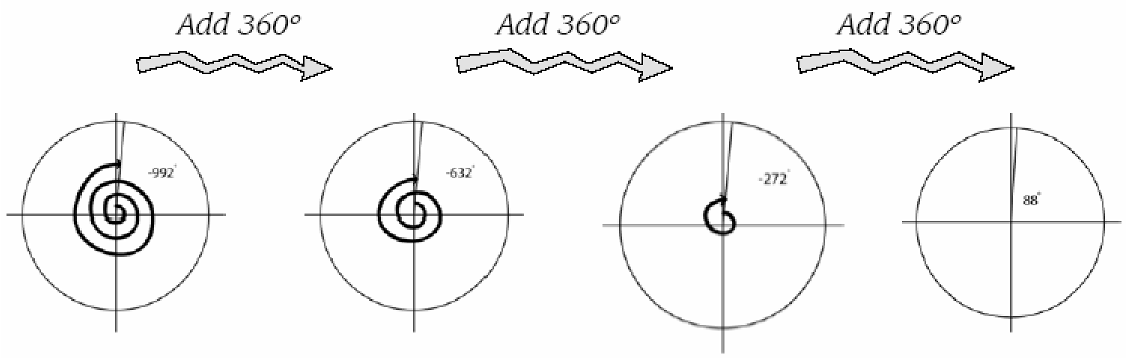
−300° 1

- Principal angle: is the **smallest positive** angle that describes the position of the terminal arm.

Boundary??? $0 \leq \text{principal angle} \leq 360°$

Example Given the co-terminal angle -992°, find the principal angle.

We need to "unwind" our way back to between 0° and 360° by making revolutions of 360°.



The principal angle is 88°

Examples... a) -260° b) 680°

a) PA = 100°

b) PA = 320°

$41\,387° \div 360 = 114.963...$

-114

$0.963...$

$\times 360$

$347°$

PA = 347°

What about a strategy for much larger angles??

Find the principal angle for the following angles:

(1) 134 723°

(2) -34 211°

(3) 5 345 781°

PA = 141°

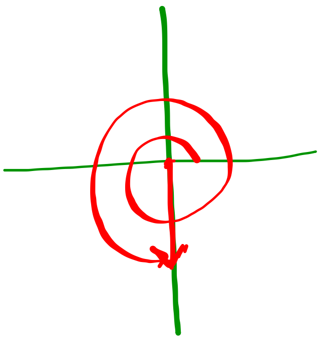
(4) -278 153°

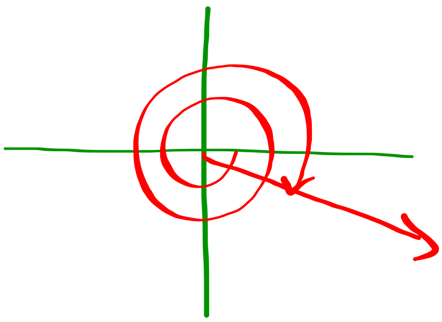
PA = 127°

Warm Up

1. Sketch each of the following:

(a) 630° (b) -740°





2. Determine the principal angle for each of the following:

(a) 13 679° (b) -376 895°

$= 359°$

$= 25°$

$-376\,895 \div 360$

$= -1046.9305..$

$+1046$

$= -0.9305..$

$\times 360$

$-335°$

$+360$

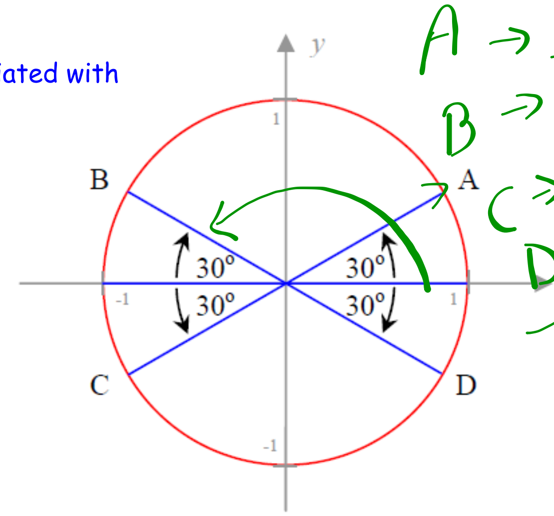
25

Reference Angles

Definition of Reference Angle

When an angle is drawn in standard position, its reference angle is the positive acute angle measured from x-axis to the angle's terminal side.

What rotation angles would be associated with a 30° reference angle?



$A \to 30°$
$B \to 150°$
$C \to 210°$
$D \to 330°$

Determining rotation angle from a known reference angle ...

Let's develop a rule for each of the quadrants...

Reference Angle = θ

II: $180 - \theta$

I: θ

$180 + \theta$

$360 - \theta$

Ref ∡ = 50°

Q1 → 50° ← RA
Q2 → 130°
Q3 → 230°
Q4 → 310

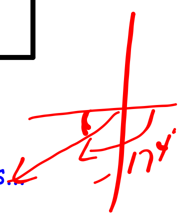
Ref ∡ 72°

Q3 → 180 + 72° = 252

Complete the chart shown below...

Reference Angle	Quadrant	Rotation Angle
24°	3	204°
48°	2	132°
75°	4	285°
80°	1	80°

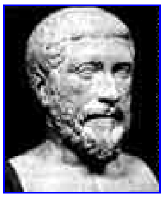
Determine the reference angle associated with each of the following rotation angles...



Rotation Angle	Reference Angle
325°	35°
-174°	6°
1240° (160)	20°

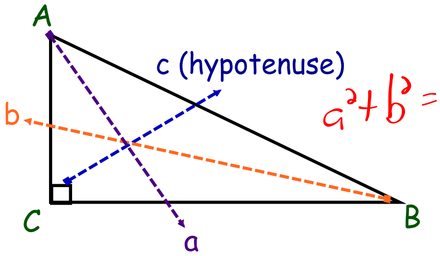
Practice problems...

#2, 3, 4, 5, 6, 7



Pythagorean Theorem

- is a fundamental relationship amongst the sides on a **RIGHT triangle**.



$a^2+b^2=c^2$

FORMULA???

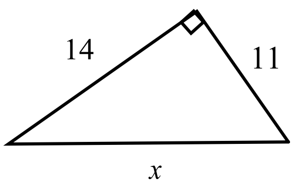
$$c^2 = a^2 + b^2$$

OPTIONS...

#1. Finding the unknown hypotenuse:

$$c^2 = a^2 + b^2$$

ex:



$x^2 = 14^2 + 11^2$

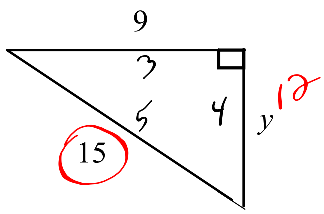
$x \doteq 17.8$

$x \approx 17.8$

#2. Finding an unknown side

$$a^2 = c^2 - b^2$$

ex:



$y^2 = 15^2 - 9^2$

$y = 12$

Pythagorean Triples...

#1

3, 4, 5

$9+16=25$

#2

5 - 12 - 13

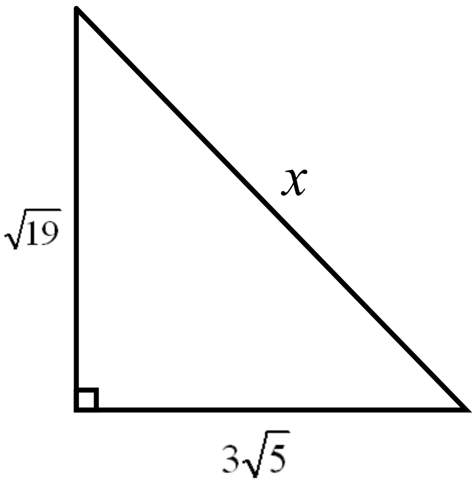
$25+144$

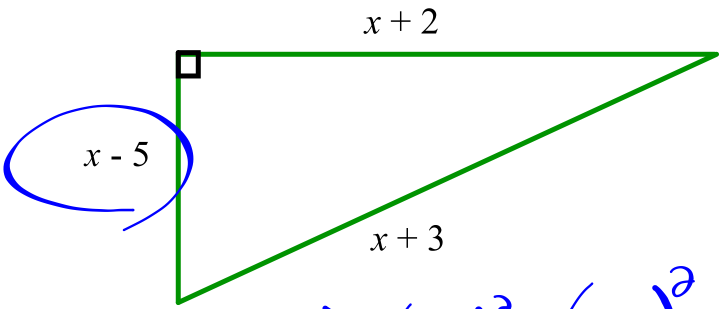
169

169

Check Up...

Determine the measure of the variable in each of the following diagrams:

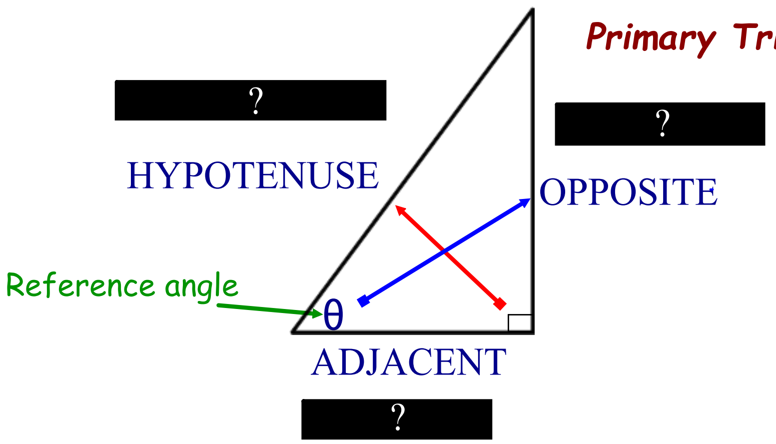




$$(\sqrt{19})^2 + (3\sqrt{5})^2 = x^2$$
$$19 + 9(5) = x^2$$
$$64 = x^2$$
$$8 = x$$

$$(x+2)^2 + (x-5)^2 = (x+3)^2$$
$$x^2 + 4x + 4 + x^2 - 10x + 25 = x^2 + 6x + 9$$
$$2x^2 - 6x + 29 = x^2 + 6x + 9$$
$$x^2 - 12x + 20 = 0$$
$$(x-10)(x-2) = 0$$
$$x = 10 \quad \cancel{x = 2}$$

Trigonometric Ratios



Primary Trigonometric Ratios

$$\sin\theta = \frac{opp}{hyp}$$

$$\cos\theta = \frac{adj}{hyp}$$

$$\tan\theta = \frac{opp}{adj}$$

Memory Aid: "SOH CAH TOA"

Reciprocal Trigonometric Ratios

$$\text{cosecant}\,\theta = \frac{hypotenuse}{opposite}$$

$$\text{secant}\,\theta = \frac{hypotenuse}{adjacent}$$

$$\text{cotangent}\,\theta = \frac{adjacent}{opposite}$$

Notice that these ratios are each the reciprocal of one of the primary trig ratios

Summary

Primary Ratios	Reciprocal Ratios
$\sin\theta = \frac{opp}{hyp}$	$\csc\theta = \frac{hyp}{opp}$
$\cos\theta = \frac{adj}{hyp}$	$\sec\theta = \frac{hyp}{adj}$
$\tan\theta = \frac{opp}{adj}$	$\cot\theta = \frac{adj}{opp}$

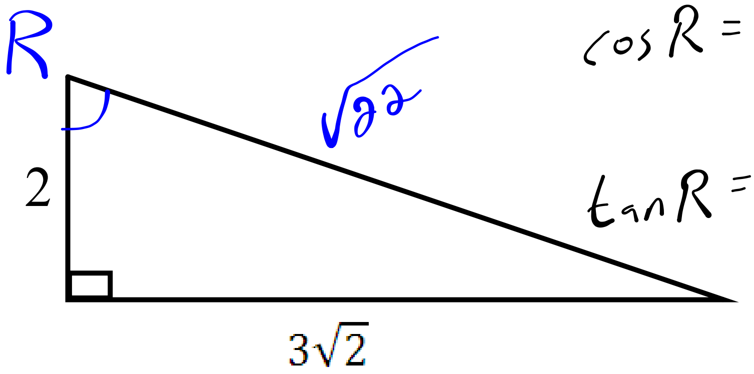
Check up...

State the six trigonometric ratios of angle R. Express your ratios as fractions in simplest form.

$$\sin R = \frac{3\sqrt{2}}{\sqrt{22}} \qquad \csc R = \frac{\sqrt{22}}{3\sqrt{2}}$$

$$\cos R = \frac{2}{\sqrt{22}} \qquad \sec R = \frac{\sqrt{22}}{2}$$

$$\tan R = \frac{3\sqrt{2}}{2} \qquad \cot R = \frac{2}{3\sqrt{2}}$$



Reciprocal ratios are not found on a calculator....we must learn how to use the reciprocal function on our calculator.

Reciprocal Functions

or

Inverse Trigonometric Functions
(Arc Trig Functions)

Trigonometric Functions

Evaluate each of the following:

$\sin 78^\circ =$ 0.9781

[redacted]

$\cos\theta = 0.6469$

$\theta =$ 50°

[redacted]

$\cot 118^\circ =$ ______

[redacted]

$\sec\theta = 3.2361$

$\theta =$ ______

[redacted]