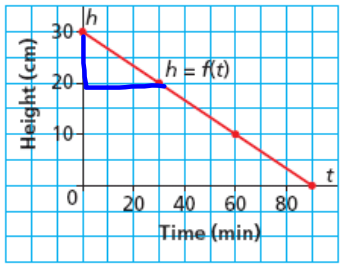
6.5 Slope-Point Form of the Equation for a Linear Function

LESSON FOCUS Relate the graph of a linear function to its equation in slope-point form.

Make Connections

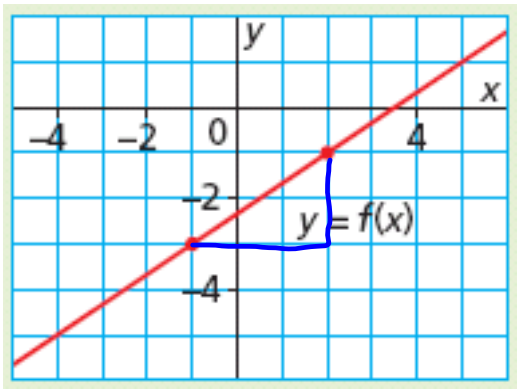
This graph shows the height of a candle as it burns.
How would you write an equation to describe this line?
Suppose you could not identify the h-intercept.
How could you write an equation for the line?



$y = mx + b$

$m = \frac{\text{rise}}{\text{run}} = \frac{-2}{3}$ $b = 30$ $y = -\frac{2}{3}x + 30$

How about this one using y = mx + b?

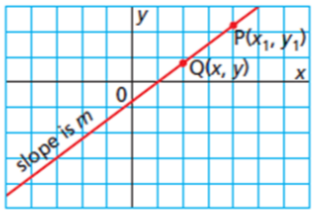


$m = \frac{\text{rise}}{\text{run}} = \frac{-2}{-3} = \frac{2}{3}$

$b = ?$

We can use this strategy to develop a formula for the slope-point form for the equation of a line.

This line has slope m and passes through the point $P(x_1, y_1)$.
Another point on the line is $Q(x, y)$.



The slope, m, of the line is:

$$m = \frac{\text{rise}}{\text{run}}$$

$$m = \frac{y - y_1}{x - x_1}$$

Multiply each side by $(x - x_1)$.

$$m(x - x_1) = (x - x_1)\left(\frac{y - y_1}{x - x_1}\right)$$

Simplify.

$$m(x - x_1) = y - y_1$$

$$y - y_1 = m(x - x_1)$$

Slope-Point Form of the Equation of a Linear Function

The equation of a line that passes through $P(x_1, y_1)$ and has slope m is:

$$y - y_1 = m(x - x_1)$$

6.5 Slope-Point Form of the Equation for a Linear Function

Slope - Point Formula...

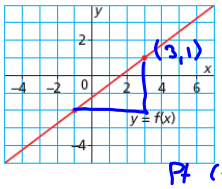
$$y - y_1 = m(x - x_1)$$

YOU NEED... 1) slope & 2) a point on the line

Example 2 Writing an Equation Using a Point on the Line and Its Slope

a) Write an equation in slope-point form for this line.

b) Write the equation in part a in slope-intercept form. What is the y-intercept of this line?



Pt. (3,1) Pt (-1,-2)

a) $m = \frac{\text{rise}}{\text{run}} = \frac{-3}{-4} = \frac{3}{4}$

$y - y_1 = m(x - x_1)$ $y - 1 = \frac{3}{4}(x-3)$

$y + 2 = \frac{3}{4}(x+1)$

b) $y - 1 = \frac{3}{4}(x-3)$

$4(y-1) = 3(x-3)$

$4y - 4 = 3x - 9$

$4y = 3x - 9 + 4$

$\frac{4y}{4} = \frac{3x}{4} - \frac{5}{4}$

$y = \frac{3}{4}x - \frac{5}{4}$

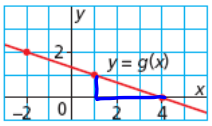
$b = -\frac{5}{4}$ $(0, -\frac{5}{4})$

6.5 Slope-Point Form of the Equation for a Linear Function

YOUR TURN...

2. a) Write an equation in slope-point form for this line.

b) Write the equation in part a in slope-intercept form. What is the y-intercept of this line?



Pt. (1,1)

a) $m = \frac{\text{rise}}{\text{run}} = \frac{-1}{3}$

$y - y_1 = m(x - x_1)$ $y - 1 = \frac{-1}{3}(x-1)$

b) $y - 1 = \frac{-1}{3}(x-1)$

$3(y-1) = -1(x-1)$

$3y - 3 = -x + 1$

$3y = -x + 1 + 3$

$\frac{3y}{3} = -\frac{x}{3} + \frac{4}{3}$

$y = -\frac{1}{3}x + \frac{4}{3}$

$y\text{-int} = \frac{4}{3}$ $(0, \frac{4}{3})$

$b = \frac{4}{3}$

EXAMPLE #3:

Determine the equation of the line that passes through (-1, 4) & (3, -12).

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-(-12)}{-1-3} = \frac{16}{-4} = -4$

$y - y_1 = m(x - x_1)$

$y - 4 = -4(x+1)$

$y - 4 = -4x - 4$

$4x + y - 4 + 4 = 0$

$4x + y = 0$

$y + 12 = -4(x-3)$

$y + 12 = -4x + 12$

$4x + y + 12 - 12 = 0$

$4x + y = 0$

Example 4 Writing an Equation of a Line That Is Parallel or Perpendicular to a Given Line

Write an equation for the line that passes through R(1, −1) and is:

a) parallel to the line $y = \frac{2}{3}x - 5$

b) perpendicular to the line $y = \frac{2}{3}x - 5$

a) $m = \frac{2}{3}$

$y - y_1 = m(x - x_1)$

$y + 1 = \frac{2}{3}(x-1)$

$3(y+1) = 2(x-1)$

$3y + 3 = 2x - 2$

$2x - 3y - 3 - 2 = 0$

$2x - 3y - 5 = 0$

b) $m = \frac{2}{3}$ $\perp m = -\frac{3}{2}$

$y - y_1 = m(x - x_1)$

$y + 1 = -\frac{3}{2}(x-1)$

$2(y+1) = -3(x-1)$

$2y + 2 = -3x + 3$

$3x + 2y + 2 - 3 = 0$

$3x + 2y - 1 = 0$

CHECK YOUR UNDERSTANDING

6.5 Slope-Point Form of the Equation for a Linear Function

Practice problems...

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9. $m = \frac{\Delta y}{\Delta x} = -\frac{4}{3}$

$y - 4 = -\frac{4}{3}(x+2)$

$3(y-4) = -4(x+2)$

$3y - 12 = -4x - 8$

$3y = -4x - 8 + 12$

$\frac{3y}{3} = -\frac{4}{3}x + \frac{4}{3}$

$y = -\frac{4}{3}x + \frac{4}{3}$

y-int $= \frac{4}{3}$ $(0, \frac{4}{3})$

x int, let $y = 0$

$0 = -\frac{4}{3}x + \frac{4}{3}$ (×3)

$0 = -4x + 4$

$-4x + 4 = 0$

$-4x = -4$

$\frac{-4x}{-4} = \frac{-4}{-4}$

$x = 1$

x-int $= 1$

$(1, 0)$