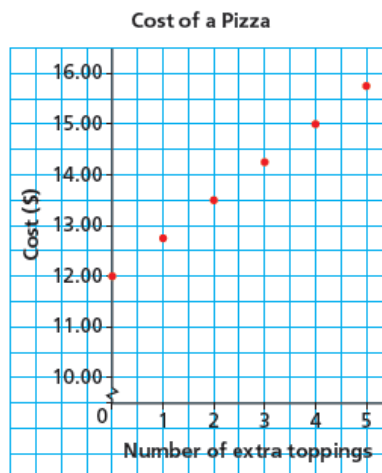# Linear

Relations

5.6 Properties of Linear Relations

LESSON FOCUS Identify and represent linear relations in different ways.
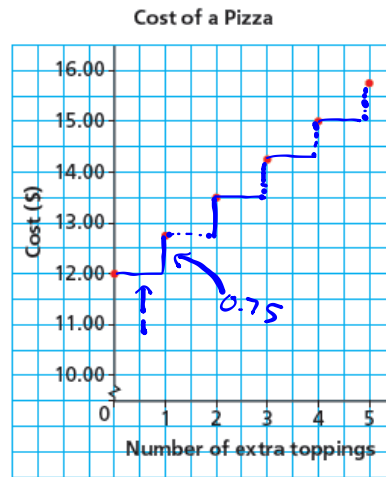
Make Connections

The table of values and graph show the cost of a pizza with up to 5 extra toppings.

Number of Extra Toppings	Cost ($)
0	12.00
1	12.75
2	13.50
3	14.25
4	15.00
5	15.75

Cost of a Pizza

Number of Extra Toppings	Cost ($)
0	12.00
1	12.75
2	13.50
3	14.25
4	15.00
5	15.75

(handwritten annotations: +1 between each topping, 0.75 increase between each cost)

Cost of a Pizza

(graph: Cost ($) on y-axis from 10.00 to 16.00, Number of extra toppings on x-axis from 0 to 5, showing a step pattern with 0.75 increases)

What patterns do you see in the table? *topping up by 1 / cost 0.75 increase/topping*

Write a rule for the pattern that relates the cost of a pizza to the number of its toppings. *$C = 0.75t + 12$*

How are the patterns in the table shown in the graph?

How can you tell from the table that the graph represents a linear relation?
For every one topping there is 0.75 increase in cost.

5.6 Properties of Linear Relations

EXERCISE...

Which table of values represents a linear relation? Justify your answer.

a) The relation between the number of bacteria in a culture, n, and time, t minutes.

t	n
0	1
20	2
40	4
60	8
80	16
100	32

(handwritten: +20 between each t; +1, +2, +4, +8, +16 between each n) **Non linear**

b) The relation between the amount of goods and services tax charged, T dollars, and the amount of the purchase, A dollars

A	T
60	3
120	6
180	9
240	12
300	15

(handwritten: +60 between each A; +3 between each T) **linear**

Here is another example of a linear relation...

The cost for a car rental is $60, plus $20 for every 100 km driven.
The independent variable is the distance driven and the dependent variable
is the cost.
We can identify that this is a linear relation in different ways.

- a table of values

X (independent)   y (dependent)

Independent variable →	Distance (km)	Cost ($)	← Dependent variable
+100	0	60	+20
+100	100	80	+20
+100	200	100	+20
+100	300	120	+20
+100	400	140	+20

For a linear relation, a constant change in the independent variable results in a constant change in the dependent variable.
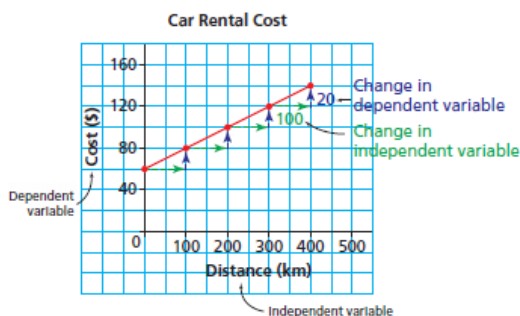
5.6 Properties of Linear Relations

How to identify a linear relation...

- a set of ordered pairs

+100   +100   +100   +100

{(0, 60), (100, 80), (200, 100), (300, 120), (400, 140)}

+20   +20   +20   +20

← ?
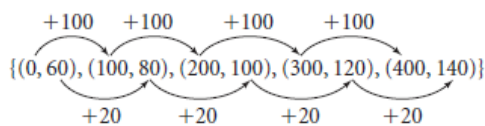
Why is it important that the ordered pairs are listed so their first elements are in numerical order?

- a graph

Car Rental Cost

Change in dependent variable
Change in independent variable

← ○

The graph of a linear relation is a straight line.

We can use each representation to calculate the rate of change.

The rate of change can be expressed as a fraction:

$$\frac{\text{change in dependent variable}}{\text{change in independent variable}} = \frac{\$20}{100 \text{ km}}$$

$$= \$0.20/\text{km}$$

5.6 Properties of Linear Relations

Rate of Change = change in dependent variable

(ROC)   change in independent variable
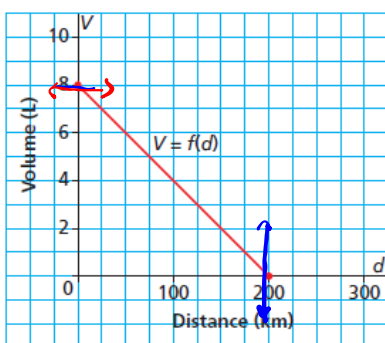
$$ROC = \frac{\Delta y}{\Delta x}$$

EXERCISE...

This graph shows the fuel consumption of a scooter with a full
tank of gas at the beginning of a journey.

Volume of Gas in a Scooter

$y = 0$ ⟵⟶

$x = 200$ ↓

what scooter
starts with
for gas.
↓

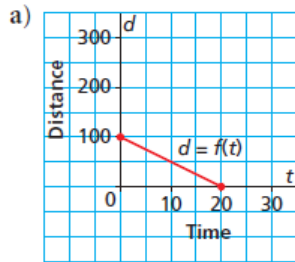y-int: $(0, 8)$
x-int: $(200, 0)$
- max kms + its
out of gas.

a) Write the coordinates of the points where the graph intersects
the axes. Determine the vertical and horizontal intercepts.
Describe what the points of intersection represent.

b) What are the domain and range of this function?

Domain: $\{x \mid 0 \le x \le 200, x \in R\}$

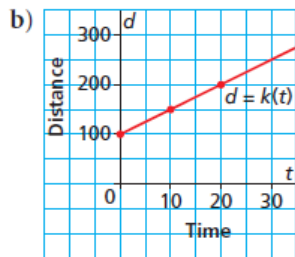Range: $\{y \mid 0 \le y \le 8, y \in R\}$

ONE MORE...

Which graph has a rate of
change of −5 and a vertical
intercept of 100? Justify your
answer.

a)

Distance — d — 300, 200, 100

Time — t — 10, 20, 30

$d = f(t)$

$$RoC = \frac{\Delta y}{\Delta x}$$

$$= \frac{-100}{20}$$

$$= -5$$

b)

Distance — d — 300, 200, 100

Time — t — 10, 20, 30

$d = k(t)$

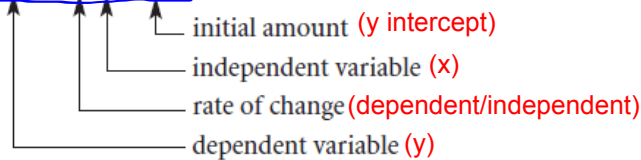$$RoC = \frac{\Delta y}{\Delta x}$$

$$= \frac{-100}{-20}$$

$$= 5$$

The rate of change is $0.20/km; that is, for each additional 1 km driven, the
rental cost increases by 20¢. The rate of change is constant for a linear relation.

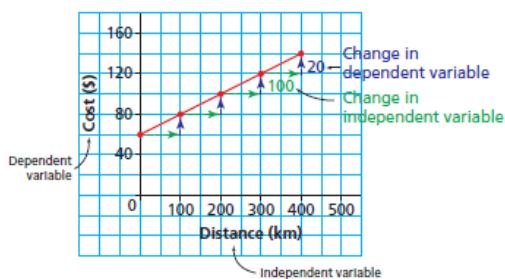We can determine the rate of change from the equation that represents
the linear function.

Let the cost be C dollars and the distance driven be d kilometres.

An equation for this linear function is:

$$C = 0.20d + 60$$

— initial amount (y intercept)

— independent variable (x)

— rate of change (dependent/independent)

— dependent variable (y)

Car Rental Cost

Cost ($) — 160, 120, 80, 40

Distance (km) — 100, 200, 300, 400, 500

Change in dependent variable
Change in independent variable
Dependent variable
Independent variable

5.6 Properties of Linear Relations

- Graphing Relations

I. Using a table of values:

Using a Table of Values to Graph a Linear Relation Worksheet

Find the value of "y" in the following table(s) of values.

a) $y = x + 2$ b) $y = 2x + 1$ c) $y = 3x - 1$ d) $y = 2x$

x	y
0	2
1	3
2	4
3	5
4	6

x	y
3	7
4	9
5	11
6	13
20	41

x	y
8	23
9	26
10	29
11	32
25	74

x	y
2	4
3	6
4	8
5	10
100	200

e) $y = \dfrac{1}{2}x + 2$

x	y
0	2
2	3
4	4
6	5
8	6

f) $y = \dfrac{1}{3}x - 3$

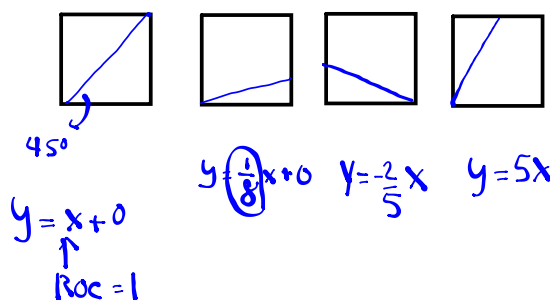| x	y
3	-2
6	-1
9	0
12	1

EXAMPLE: Create a table of values and graph...

- Graph $y = 2x + 3$ $y = -\dfrac{2}{3}x + 1$

x	y
-2	-1
-1	1
0	3
1	5
2	7

x	y
-6	5
-3	3
0	1
3	-1
6	-3

45°

$y = x + 0$
↑
Roc = 1

$y = \dfrac{1}{8}x + 0$ $y = -\dfrac{2}{5}x$ $y = 5x$

$y = \dfrac{-}{\underset{Roc}{↑}}x + \dfrac{-}{\underset{y\,int}{↑}}$

$y = -x + 0$
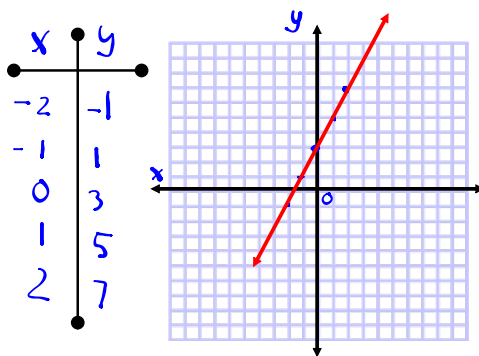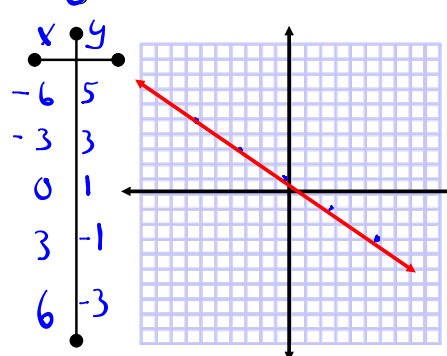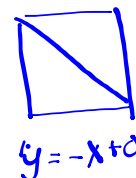
YOUR TURN...

Graph the equation: $y = -3x + 5$

x	y
-2	11
-1	8
0	5
1	2
2	-1
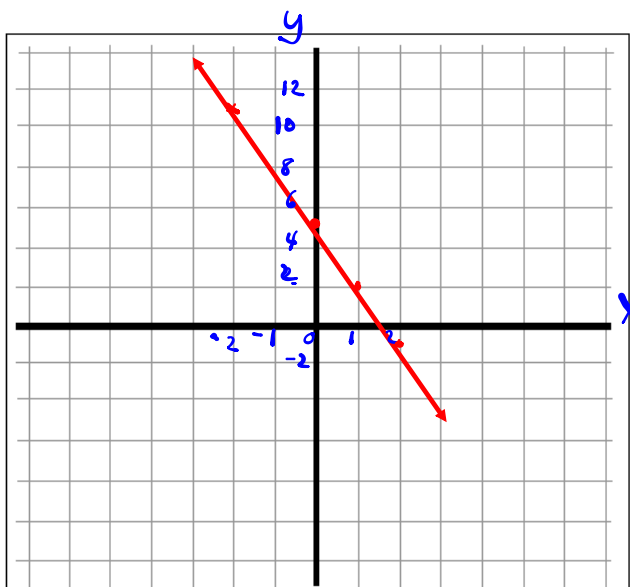
PRACTICE PROBLEMS...

p. 308: #3 - 8, 12, 14, 16