September, 2019

UNIT 1: ROOTS AND POWERS

SECTION 3.1:
FACTORS AND MULTIPLES OF WHOLE NUMBERS

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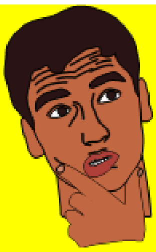
NUMBERS, RELATIONS AND FUNCTIONS 10

WHAT'S THE POINT OF TODAY'S LESSON?

We will begin working on the NRF 10 Specific Curriculum Outcome (SCO) "Algebra and Numbers 1" OR "AN1" which states:

"Demonstrate an understanding of factors of whole numbers by determining the prime factors, greatest common factor, least common multiple, square root and cube root."



What does THAT mean???

SCO AN1 means that we will:

* find the prime factors of whole numbers like 8 (2^3 or $2 \cdot 2 \cdot 2$)
* determine the greatest common factor (GCF) of numbers like 28 and 49 (GCF = 7)
* determine the least common multiple (LCM) of numbers like 6 and 9 (LCM = 18)
* determine if a given whole number is a perfect square, like 25 ($5 \cdot 5$) or a perfect cube, like 8 ($2 \cdot 2 \cdot 2$)
* determine the square root of perfect squares, like 36 ($\sqrt{36} = 6$), and the cube root of perfect cubes, like 64 ($\sqrt[3]{64} = 4$)

WARM UP:

Use prime factorization to determine the

a) GCF of 856, 1200 and 1368

b) LCM of 28, 42 and 63

WARM UP:

10×12
12×10

a) GCF of 856, 1200 and 1368:

GCF = Greatest Common Factor

Method I

856	1200	1368
1x 856	1x 1200	1x 1368
2x 428	2x 600	2x 684
4x 214	3x 400	3x 456
8x 107	4x 300	4x 342
	5x 240	6x 228
	6x 200	8x 171
⋮	8x 150	9x 152
continued	10x 120	

Method II

856
4×214
$2 \times 2 \times 2 \times 107$
$2^3 \times 107$

1200
4×300
$2 \times 2 \times 2 \times 150$
$2 \times 2 \times 2 \times 2 \times 75$
$2 \times 2 \times 2 \times 2 \times 25 \times 3$
$2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 3$
$2^4 \times 3 \times 5^2$

1368
8×171
$2 \times 4 \times 3 \times 57$
$2 \times 2 \times 2 \times 3 \times 57$
$2^3 \times 3 \times 57$

$GCF = 2^3$
$= 8$

WARM UP:

b) LCM of 28, 42 and 63:

LCM = Least Common Multiple

28 **42** **63**

Method I

28 56 84 112 140 168 196 224 252

42 84 126 168 210 252

63 126 189 252

Method II

28
4×7
$2 \times 2 \times 7$
$2^2 \times 7$

42
6×7
$2 \times 3 \times 7$

63
7×9
$7 \times 3 \times 3$
$3^2 \times 7$

$LCM = 2^2 \times 3^2 \times 7$
$= 252$

Choose higher power for each number

WARM UP:

a) What is the GCF of 340 and 380?

b) Reduce the fraction $\frac{340}{380}$ to its simplest form.

c) What is the LCM of 340 and 380?

d) What is $\frac{15}{340} + \frac{10}{380}$?

340
10×34
$2 \times 5 \times 2 \times 17$
$2^2 \times 5 \times 17$

380
10×38
$2 \times 5 \times 2 \times 19$
$2^2 \times 5 \times 19$

$GCF = 2^2 \times 5$
$= 4 \times 5$
$= 20$

$LCM = 2^2 \times 5 \times 17 \times 19$
$= 6460$

$340 = 2 \times 2 \times 5 \times 17$
$380 = 2 \times 2 \times 5 \times 19$

a) GCF

340 → $34 \cdot 10$ → $2 \cdot 17 \cdot 2 \cdot 5$

380 → $38 \cdot 10$ → $2 \cdot 19 \cdot 2 \cdot 5$

$$GCF = 2 \cdot 2 \cdot 5$$
$$= 4 \cdot 5$$
$$= 20$$

b)

$$\frac{340}{380} \begin{matrix} \div 20 \\ \div 20 \end{matrix}$$
$$= \frac{17}{19}$$

c) LCM

340 → $34 \cdot 10$ → $2 \cdot 17 \cdot 2 \cdot 5$

380 → $38 \cdot 10$ → $2 \cdot 19 \cdot 2 \cdot 5$

$2^2 \cdot 5 \cdot 17$ $\quad$ $2^2 \cdot 5 \cdot 19$

$$LCM = 2^2 \cdot 5 \cdot 17 \cdot 19$$
$$= 4 \cdot 5 \cdot 17 \cdot 19$$
$$= 20 \cdot 17 \cdot 19$$
$$= 340 \cdot 19$$
$$= 6460$$

d)

$$\frac{15 \times 19}{340 \times 19} + \frac{10 \times 17}{380 \times 17}$$
$$= \frac{285}{6460} + \frac{170}{6460}$$
$$= \frac{455 \div 5}{6460 \div 5}$$
$$= \frac{91}{1292}$$

GCF & LCM

GCF - Greatest Common Factor

LCM - Least Common Multiple

VOCABULARY:

prime number: a whole number with exactly two factors, itself and 1.
EX.: 2, 3, 5, 7, 11, 13, 17, 19, 23, and 29 are the first 10 prime numbers.

composite number: a number with three or more factors.
EX.: 8 is a composite number; it has four factors (1, 2, 4 and 8).

prime factor: a prime number that is a factor of a number that is a prime number.
EX.: The prime factors of 30 are 2, 3, and 5.

prime factorization: writing a number as a product of its prime factors.
EX.: The prime factorization of 20 is $2 \cdot 2 \cdot 5$ or $2^2 \cdot 5$.

USING FACTOR TREES FOR PRIME FACTORIZATION:

• means multiply

3300

$33 \cdot 100$

$3 \cdot 11 \cdot 4 \cdot 25$

$3 \cdot 11 \cdot 2 \cdot 2 \cdot 5 \cdot 5$

$$P.F. = 2 \cdot 2 \cdot 3 \cdot 5 \cdot 5 \cdot 11$$

OR

$$2^2 \cdot 3 \cdot 5^2 \cdot 11$$

Warm Up

Find the GCF and LCM of the folllowing numbers using prime factorization (factor tree).

180, 240, 340

GCF

180	240	340
1 x 180	1 x 240	1x 340
2x 90	2 x 120	2x 170
3x 60	3x 80	4x85
4x 45	4x 60	5x 68
5x 36	5x 48	10 x 34
6 x 30	6x 40	17 x 20
9x 20	8x 30	
10 x 18	10 x 24	
12x15	12 x 20	
	15x 16	

GCF = 20

180

5×36

$5 \times 6 \times 6$

$5 \times 3 \times 2 \quad 3 \times 2$

$2^2 \times 3^2 \times 5$

240

24×10

$12 \times 2 \times 5 \times 2$

$2 \times 6 \times 2 \times 5 \quad 2$

$2 \times 2 \times 3 \times 2 \times 5 \times 2$

$2^4 \times 3 \times 5$

340

34×10

$17 \times 2 \times 2 \times 5$

$2^2 \times 5 \times 17$

$GCF = 5 \times 2 \times 2$

$= 20$

$LCM = 2^4 \times 3^2 \times 5 \times 17$

$= 16 \times 9 \times 5 \times 17$

$= 12240$

LCM

180 360 540 720 900 1080 1260 1440

240 480 720 960 1200 1440....

340 680 1020 1360 1700...

USING FACTOR TREES FOR PRIME FACTORIZATION:

YOU TRY! :)

2646

2×1323

$2 \times 3 \times 441$

$2 \times 3 \times 3 \times 147$

$2 \quad 3 \times 3 \times 7 \times 21$

$2 \times 3 \times 3 \times 7 \times 3 \times 7$

$2 \times 3^3 \times 7^2$

DETERMINING THE GREATEST COMMON FACTOR:

EX.: **Determine the GCF of 12 and 15.**

Method I

12: 1, 2, 3, 4, 6, 12

15: 1, 3, 5, 15

GCF = 3

Method II

12: 3×4; $3 \times 2 \times 2$

15: 3×5

DETERMINING THE GREATEST COMMON FACTOR:

EX.: **You try! Determine the GCF of 18 and 24.**

18: 1, 2, 3, 6, 9, 18

24: 1, 2, 3, 4, 6, 8, 12, 24

GCF = 6

18
3 x 6
3 x 3 x 2

24
4 x 6
2 x 2   2 x 3

GCF = 3 x 2
= 6

18 = 2 x 3 x 3
24 = 2 x 2 x 2 x 3

DETERMINING THE GREATEST COMMON FACTOR FOR LARGER NUMBERS: PRIME FACTORIZATION

EX.: **Determine the GCF of 138 and 198.**

138
2 x 69
2 x 3 x 23

198
2 x 99
2 x 3 x 33
2 x 3 x 3 x 11

138 = 2 x 3 x 23
198 = 2 x 3 x 3 x 11

GCF = 2 x 3
= 6

Once you have the prime factorization of the numbers, multiply all the prime factors they have in common to determine their GCF.

DETERMINING THE GREATEST COMMON FACTOR FOR LARGER NUMBERS: PRIME FACTORIZATION

tree

EX.: You try! Determine the GCF of 126 and 144.

126
2 x 63
2 x 3 x 21
2 x 3 x 3 x 7

144
2 x 72
2 x 2 x 36
2 x 2 x 2 x 18
2 x 2 x 2 x 2 x 9
2 x 2 x 2 x 2 x 3 x 3

GCF = 2 x 3 x 3
= 18

DETERMINING THE LOWEST COMMON MULTIPLE:

EX.: Determine the LCM of 8 and 12.

8: 8, 16, 24, 32, 40

12: 12, 24, 36, 48

LCM = 24

DETERMINING THE LOWEST COMMON MULTIPLE:

EX.: **You try! Determine the LCM of 6 and 10.**

6: 6, 12, 18, 24, 30, 36

10: 10, 20, 30, 40, 50

LCM = 30

DETERMINING THE LOWEST COMMON MULTIPLE FOR LARGER NUMBERS OR GROUPS OF NUMBERS: PRIME FACTORIZATION

EX.: **Determine the LCM of 18, 20 and 30.**

18
3 x 6
3 x 2 x 3
2×3^2

20
4 x 5
2 x 2 x 5
$2^2 \times 5$

30
5 x 6
5 x 2 x 3

$LCM = 2^2 \times 3^2 \times 5$
$= 4 \times 9 \times 5$
$= 180$

Once you have the prime factorization of the numbers, multiply the greatest power of each of the prime factors to determine the LCM.

DETERMINING THE LOWEST COMMON MULTIPLE FOR LARGER NUMBERS OR GROUPS OF NUMBERS: PRIME FACTORIZATION

EX.: You try! Determine the GCF + LCM of 28, 42 and 63.

28 → 4 × 7 → 2 × 2 × 7

$2^2 \times 7$

42 → 6 × 7 → 2 × 3 × 7

63 → 3 × 21 → 3 × 3 × 7

$3^2 \times 7$

2 × 2 × 7
2 × 3 × 7
3 × 3 × 7

GCF = 7

$LCM = 2^2 \times 3^2 \times 7$
$= 4 \times 9 \times 7$
$= 252$

SOLVING PROBLEMS THAT INVOLVE GCF AND LCM:

EX.: What is the side length of the SMALLEST square that could be tiled with rectangles that measure 16 cm by 40 cm? (Assume the tiles cannot be cut.) Sketch the square and the rectangular tiles.

LCM

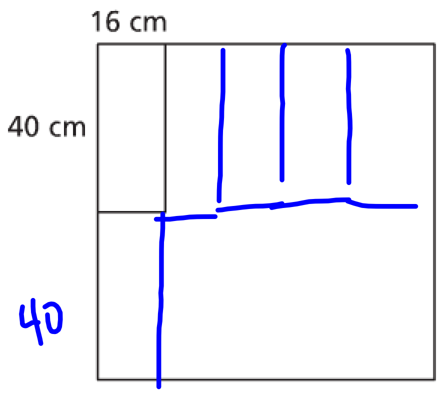
16 → 4 × 4 → 2 × 2 × 2 × 2

2^4

40 → 4 × 10 → 2 × 2 × 2 × 5

$2^3 \times 5$

$LCM = 2^4 \times 5$
$= 80$ ✓



SOLVING PROBLEMS THAT INVOLVE GCF AND LCM:

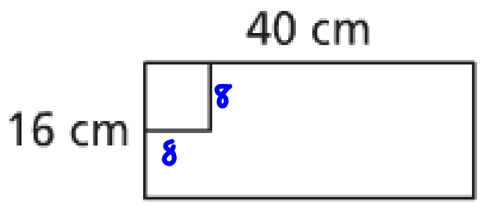
EX.: **What is the side length of the LARGEST square that could be used to tile a rectangle that measures 16 cm by 40 cm? (Assume the tiles cannot be cut.) Sketch the rectangle and the square tiles.**

GCF

16 → 4×4 → $2 \times 2 \times 2 \times 2$

40 → 5×8 → $5 \times 2 \times 4$ → $5 \times 2 \times 2 \times 2$

$GCF = 2 \times 2 \times 2$
$= 8$



CONCEPT REINFORCMENT:

"FOUNDATIONS AND PRE-CALCULUS - MATHEMATICS 10" (<u>FPCM 10</u>)

a, d

pages 140 / 141: #3 <u>TO</u> #13, #15 <u>TO</u> #19a and #20

16 a, c, e