September 16, 2019

UNIT 1: ROOTS AND POWERS

SECTION 4.4:
FRACTIONAL EXPONENTS AND RADICALS

K. Sears

NUMBERS, RELATIONS AND FUNCTIONS 10

#2

WHAT'S THE POINT OF TODAY'S LESSON?

We will continue working on the NRF 10 Specific Curriculum Outcome (SCO) "Algebra and Numbers 3" OR "AN3" which states:

"Demonstrate an understanding of powers with integral and rational exponents."

#3

What does THAT mean???

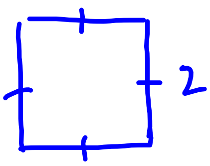
SCO AN3 means that we will:

- **apply the 6 exponent laws you learned in grade 9:**
  - $a^0 = 1$
  - $(a^m)(a^n) = a^{m+n}$
  - $a^m \div a^n = a^{m-n}$
  - $(a^m)^n = a^{mn}$
  - $(ab)^m = a^m b^m$
  - $(a \div b)^n = a^n \div b^n$
- **use patterns to explain $a^{-n} = \dfrac{1}{a^n}$ and $a^{\frac{1}{n}} = \sqrt[n]{a}$**
- **apply all exponent laws to evaluate a variety of expressions**
- **express powers with rational exponents as radicals and vice versa**
- **identify and correct errors in work that involves powers**

#4

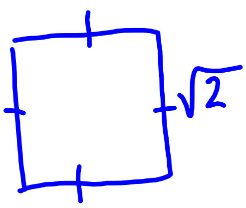
ANY QUESTIONS
(page 221, #1, #3, #4, #6a, #7b, #8, #9 & #11)

8. a)



$P = \overset{side}{s} + \overset{side}{s} + \overset{side}{s} + \overset{side}{s}$
$= 4(2)$
$= 8$

b)



$P = \sqrt{2} + \sqrt{2} + \sqrt{2} + \sqrt{2}$
$= 4\sqrt{2}$

Feb 10-8:24 AM

EXPONENT LAWS (separate sheet):

1. Zero Exponent Law: $a^0 = 1$ $-13^0 = -1$
2. Product of Powers: $(a^m)(a^n) = a^{m+n}$
3. Quotient of Powers: $a^m \div a^n = a^{m-n}$
4. Power of a Power: $(a^m)^n = a^{mn}$
5. Power of a Product: $(ab)^m = a^m b^m$
6. Power of a Quotient: $(a \div b)^n = a^n \div b^n$

Feb 10-10:00 AM

7. MULTIPLICATION PROPERTY OF RADICALS:

$$\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$$

EX.: $\sqrt{24}$ (Factors: 1, 2, 3, 4, 6, 8, 12, 24)

$$= \sqrt{4 \cdot 6}$$
$$= \sqrt{4} \cdot \sqrt{6}$$
$$= 2 \cdot \sqrt{6}$$
$$= 2\sqrt{6}$$ (MIXED RADICAL)

EX.: $\sqrt[3]{24}$ (ENTIRE RADICAL)

$$= \sqrt[3]{8 \quad 3}$$
$$= \sqrt[3]{8} \quad \sqrt[3]{3}$$
$$= 2 \quad \sqrt[3]{3}$$
$$= 2\sqrt[3]{3}$$

Feb 10-10:00 AM

LET'S COPY AND COMPLETE THESE TABLES TOGETHER.

$\sqrt[3]{x}$

x	$x^{\frac{1}{2}}$ $(x^{0.5})$
1	$1^{\frac{1}{2}} = 1$
4	$4^{\frac{1}{2}} = 2$
9	$9^{\frac{1}{2}} = 3$
16	$16^{\frac{1}{2}} = 4$
25	$25^{\frac{1}{2}} = 5$

x	$x^{\frac{1}{3}}$ $[x^{(1\div 3)}]$
1	$\sqrt[3]{1} = 1$
8	$8^{\frac{1}{3}} = 2$
27	$27^{\frac{1}{3}} = 3$
64	$64^{\frac{1}{3}} = 4$
125	$125^{\frac{1}{3}} = 5$

Feb 10-10:00 AM

WHAT DO YOU THINK THE EXPONENT $\frac{1}{2}$ MEANS?

WHAT DO YOU THINK THE EXPONENT $\frac{1}{3}$ MEANS?

WHAT DO YOU THINK $a^{\frac{1}{4}}$ AND $a^{\frac{1}{5}}$ MEAN? (TEST YOUR PREDICTIONS WITH A CALCULATOR.)

WHAT DO YOU THINK $a^{\frac{1}{n}}$ MEANS?

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In grade 9, you learned that for powers with integral bases and whole number exponents:

$a^m \cdot a^n = a^{m+n}$ $\quad 2^3 \cdot 2^4 = 2^7$

WE CAN EXTEND THIS LAW TO POWERS WITH FRACTIONAL EXPONENTS WITH A NUMERATOR OF 1:

$$5^{\frac{1}{2}} \cdot 5^{\frac{1}{2}} = 5^{\frac{1}{2}+\frac{1}{2}}$$
$$= 5^{\frac{2}{2}}$$
$$= 5$$

$$\sqrt{5} \cdot \sqrt{5} = \sqrt{25}$$
$$= 5$$

Feb 19-4:10 PM

$5^{\frac{1}{2}}$ and $\sqrt{5}$ are equivalent expressions; that is, $5^{\frac{1}{2}} = \sqrt{5}$.

$$x^{\frac{1}{n}} = \sqrt[n]{x}$$

$$9^{\frac{1}{2}} = \sqrt{9}$$

Feb 19-4:17 PM

SIMILARLY...

$$5^{\frac{1}{3}} \cdot 5^{\frac{1}{3}} \cdot 5^{\frac{1}{3}} \qquad \sqrt[3]{5} \cdot \sqrt[3]{5} \cdot \sqrt[3]{5}$$

$$5^{\frac{1}{3}+\frac{1}{3}+\frac{1}{3}} = 5^{\frac{3}{3}}$$

$$= 5^{1}$$

$$= 5$$

Feb 19-4:18 PM

SO...

$$5^{\frac{1}{3}} = \sqrt[3]{5}$$

Feb 19-4:18 PM

8. POWERS WITH RATIONAL EXPONENTS WITH A NUMERATOR OF 1:

$$x^{\frac{1}{n}} = \sqrt[n]{x}$$

EX.:

$$8^{\frac{1}{3}}$$
$$= \sqrt[3]{8}$$
$$= 2$$

Feb 10-10:00 AM

EXAMPLE:

Evaluate each power without using a calculator.

a) $27^{\frac{1}{3}}$ $=3$

b) $0.49^{\frac{1}{2}}$ 0.7

c) $(-64)^{\frac{1}{3}}$ -4

d) $\left(\frac{4}{9}\right)^{\frac{1}{2}}$ $\frac{2}{3}$

Feb 20-9:06 AM

YOU TRY!

Evaluate each power without using a calculator.

a) $1000^{\frac{1}{3}} = 10$ b) $0.25^{\frac{1}{2}} = 0.5$

c) $(-8)^{\frac{1}{3}} = -2$ d) $\left(\frac{16}{81}\right)^{\frac{1}{4}} = \frac{2}{3}$

Feb 20-9:08 AM

NOTE: Because a fraction can be written as a terminating or repeating decimal, we can interpret powers with decimal exponents.

$32^{\frac{2}{10}}$

$32^{\frac{1}{5}}$

EX.:

$$32^{0.2}$$
$$=32^{\frac{2}{10}}$$
$$=32^{\frac{1}{5}}$$
$$=\sqrt[5]{32}$$
$$= \mathbf{2}$$

Feb 20-12:22 PM

YOU TRY!

Evaluate $100^{0.5}$.

$= 100^{\frac{1}{2}}$

$= \sqrt{100}$

$= 10$

Feb 21-3:43 PM

To give meaning to a power such as $8^{\frac{2}{3}}$,

we use the exponent law $(a^m)^n = a^{mn}$.

EX.:

$$8^{\frac{2}{3}}$$
$$= 8^{\frac{1}{3}\cdot 2}$$
$$= \left(8^{\frac{1}{3}}\right)^2$$
$$= \left(\sqrt[3]{8}\right)^2$$
$$= 2^2$$
$$= 4$$

EX.:

$$8^{\frac{2}{3}}$$
$$= 8^{2\cdot\frac{1}{3}}$$
$$= \left(8^2\right)^{\frac{1}{3}}$$
$$= \sqrt[3]{8^2}$$
$$= \sqrt[3]{64}$$
$$= 4$$

Feb 20-12:38 PM

9. POWERS WITH RATIONAL EXPONENTS:

EXPONENT

$$x^{\frac{m}{n}} = \left(x^{\frac{1}{n}}\right)^m = \left(\sqrt[n]{x}\right)^m$$

INDEX

AND

EXPONENT

$$x^{\frac{m}{n}} = \left(x^m\right)^{\frac{1}{n}} = \sqrt[n]{x^m}$$

INDEX

EX.: **Evaluate** $16^{\frac{3}{2}}$.

$16^{\frac{3}{2}}$ 3 (EXPONENT), 2 (INDEX)

$= \left(\sqrt[2]{16}\right)^3$

$= 4^3$

$= 64$

OR

$16^{\frac{3}{2}}$ 3 (EXP.), 2 (INDEX)

$= \sqrt[2]{16^3}$

$= \sqrt{4096}$

$= 64$

Feb 10-10:00 AM

EXAMPLE: $\left(40^{\frac{1}{3}}\right)^2$ $\left(\sqrt[3]{40}\right)^2$ or $\left(40^2\right)^{\frac{1}{3}}$

a) Write $40^{\frac{2}{3}}$ in radical form in 2 ways. $\sqrt[3]{40^2}$

b) Write $\sqrt{3^5}$ and $\left(\sqrt[3]{25}\right)^2$ in exponent form.

$3^{\frac{5}{2}}$ $25^{\frac{2}{3}}$

SOLUTION:

a) $40^{\frac{2}{3}} = \left(\sqrt[3]{40}\right)^2$ or $\sqrt[3]{40^2}$

b) $\sqrt{3^5} = 3^{\frac{5}{2}}$ **AND** $\left(\sqrt[3]{25}\right)^2 = 25^{\frac{2}{3}}$

Feb 21-3:28 PM

YOU TRY!

Calculator
↳ 26 y^x (2/5)

a) Write $26^{\frac{2}{5}}$ in radical form in 2 ways. $\sqrt[5]{26^2}$ $(\sqrt[5]{26})^2$

b) Write $\sqrt{6^5}$ and $(\sqrt[4]{19})^3$ in exponent form. $6^{\frac{5}{2}}$ $19^{\frac{3}{4}}$

SOLUTION:

a) $(\sqrt[5]{26})^2$ or $\sqrt[5]{26^2}$

b) $6^{\frac{5}{2}}, 19^{\frac{3}{4}}$

Feb 21-3:28 PM

EXAMPLE:

Evaluate.

a) $0.04^{\frac{3}{2}} = 0.008$ **b)** $27^{\frac{4}{3}} = 81$

c) $(-32)^{0.4}$

$(-32)^{\frac{4}{10}}$

$(-32)^{\frac{2}{5}} = (-2)^2$

$= 4$

d) $1.8^{1.4}$

$1.8^{1\frac{4}{10}}$

$1.8^{1\frac{2}{5}}$

$1.8^{\frac{7}{5}} \doteq 2.277$

Feb 21-3:28 PM

SOLUTION:

a) $0.04^{\frac{3}{2}} = \left(0.04^{\frac{1}{2}}\right)^3$

$$= \left(\sqrt{0.04}\right)^3$$

$$= 0.2^3$$

$$= 0.008$$

Feb 21-3:50 PM

c) The exponent $0.4 = \frac{4}{10}$ or $\frac{2}{5}$

So, $(-32)^{0.4} = (-32)^{\frac{2}{5}}$

$$= \left[(-32)^{\frac{1}{5}}\right]^2$$

$$= \left(\sqrt[5]{-32}\right)^2$$

$$= (-2)^2$$

$$= 4$$

Feb 21-3:47 PM

d) $1.8^{1.4}$

Use a calculator.

```
1.8^1.4
              2.277096874
```

$1.8^{1.4} = 2.2770\ldots$

Feb 21-3:48 PM

YOU TRY!

Evaluate.

a) $0.01^{\frac{3}{2}} = (0.1)^3 = 0.001$ **b)** $(-27)^{\frac{4}{3}} = (-3)^4 = 81$

c) $81^{\frac{3}{4}} = 3^3 = 27$ **d)** $0.75^{1.2} \doteq 0.708$

SOLUTION:

a) 0.001 b) 81 c) 27 d) 0.7080…

Feb 21-3:28 PM

EXAMPLE:

Biologists use the formula $b = 0.01m^{\frac{2}{3}}$ to estimate the brain mass, b kilograms, of a mammal with body mass m kilograms. Estimate the brain mass of each animal.

a) a husky with a body mass of 27 kg

b) a polar bear with a body mass of 200 kg

$b = 0.01(27)^{\frac{2}{3}}$
$= 0.09$ kg

SOLUTION:

Use the formula $b = 0.01m^{\frac{2}{3}}$.

a) Substitute: $m = 27$

$b = 0.01(27)^{\frac{2}{3}}$

Use the order of operations. Evaluate the power first.

$b = 0.01(\sqrt[3]{27})^2$

$b = 0.01(3)^2$

$b = 0.01(9)$

$b = 0.09$

The brain mass of the husky is approximately 0.09 kg.

Feb 21-3:57 PM

b) Substitute: $m = 200$

$b = 0.01(200)^{\frac{2}{3}}$

Use a calculator.

```
0.01(200)^(2/3)
         0.341995189
```

$b = 0.01(200)\underbrace{\boxed{\wedge}}_{y^x}(2/3)$

$\doteq 0.34$ kg

The brain mass of the polar bear is approximately 0.34 kg.

Feb 21-4:00 PM

YOU TRY!

Use the formula $b = 0.01m^{\frac{2}{3}}$ to estimate the brain mass of each animal.

a) a moose with a body mass of 512 kg

$b = 0.01(512)\wedge(2/3)$

$= 0.64$ Kg

$\times 2.2$

1.408 lbs

b) a cat with a body mass of 5 kg

$b = 0.01(5)\wedge(2/3)$

$\doteq 0.03$ Kg

Feb 21-3:57 PM

SOLUTION:

a) approximately 0.64 kg

b) approximately 0.03 kg

Feb 21-4:05 PM

CONCEPT REINFORCEMENT:

FPCM 10:

Page 227: #3 to #16

Page 228: #17 to #21

page 221 1, 3, 4, 6a, 7b, 8, 9 & 11 ✓

Feb 10-8:24 AM