

SEPTEMBER 19, 2019

UNIT 1: ROOTS AND POWERS

**SECTION 4.6:
APPLYING THE
EXPONENT LAWS**



K. Sears
NUMBERS, RELATIONS AND FUNCTIONS 10

#2

WHAT'S THE POINT OF TODAY'S LESSON?

We will continue working on the NRF 10 Specific Curriculum Outcome (SCO) "Algebra and Numbers 3" OR "AN3" which states:

"Demonstrate an understanding of powers with integral and rational exponents."

#3



What does THAT mean???

SCO AN3 means that we will:

- * apply the 6 exponent laws you learned in grade 9:

$$a^0 = 1$$

$$(a^m)(a^n) = a^{m+n}$$

$$a^m \div a^n = a^{m-n}$$

$$(a^m)^n = a^{mn}$$

$$(ab)^m = a^m b^m$$

$$(a \div b)^n = a^n \div b^n$$

- * use patterns to explain $a^{-n} = \frac{1}{a^n}$ and $a^{\frac{1}{n}} = \sqrt[n]{a}$

- * apply all exponent laws to evaluate

a variety of expressions

- * express powers with rational exponents as

radicals and vice versa

- * identify and correct errors in work that involves powers



#4

EXPONENT LAWS (separate sheet):

1. Zero Exponent Law: $a^0 = 1$

2. Product of Powers: $(a^m)(a^n) = a^{m+n}$

3. Quotient of Powers: $a^m \div a^n = a^{m-n}$

4. Power of a Power: $(a^m)^n = a^{mn}$

5. Power of a Product: $(ab)^m = a^m b^m$

6. Power of a Quotient: $(a \div b)^n = a^n \div b^n$

7. MULTIPLICATION PROPERTY OF RADICALS:

$$\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$$

EX.: $\sqrt{24}$ (Factors: 1, 2, 3, 4, 6, 8, 12, 24)

$$= \sqrt{4 \cdot 6}$$

$$= \sqrt{4} \cdot \sqrt{6}$$

$$= 2 \cdot \sqrt{6}$$

$$= 2\sqrt{6} \text{ (MIXED RADICAL)}$$

EX.: $\sqrt[3]{24}$ (ENTIRE RADICAL)

$$= \sqrt[3]{8 \cdot 3}$$

$$= \sqrt[3]{8} \cdot \sqrt[3]{3}$$

$$= 2 \cdot \sqrt[3]{3}$$

$$= 2\sqrt[3]{3}$$

Feb 10-10:00 AM

8. POWERS WITH RATIONAL EXPONENTS WITH A NUMERATOR OF 1:

$$x^{\frac{1}{n}} = \sqrt[n]{x}$$

EX.: $8^{\frac{1}{3}}$

$$= \sqrt[3]{8}$$

$$= 2$$

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9. POWERS WITH RATIONAL EXPONENTS:

$$\begin{array}{ccc}
 \text{EXPONENT} & & \text{EXPONENT} \\
 \swarrow & & \swarrow \\
 x^{\frac{m}{n}} & = & \left(x^{\frac{1}{n}}\right)^m \\
 \uparrow & & \uparrow \\
 \text{INDEX} & & \text{INDEX} \\
 & = & \left(\sqrt[n]{x}\right)^m
 \end{array}
 \quad \text{AND} \quad
 \begin{array}{ccc}
 \text{EXPONENT} & & \text{EXPONENT} \\
 \swarrow & & \swarrow \\
 x^{\frac{m}{n}} & = & \left(x^m\right)^{\frac{1}{n}} \\
 \uparrow & & \uparrow \\
 \text{INDEX} & & \text{INDEX} \\
 & = & \sqrt[n]{x^m}
 \end{array}$$

EX.: Evaluate $16^{\frac{3}{2}}$.

$$\begin{array}{ccc}
 & \begin{array}{c} 3(\text{EXPONENT}) \\ 16^{\frac{3}{2}} \end{array} & \begin{array}{c} 3(\text{EXP.}) \\ 16^{\frac{3}{2}} \end{array} \\
 & \begin{array}{c} (\text{INDEX}) \\ \text{OR} \\ \end{array} & \begin{array}{c} (\text{INDEX}) \\ \end{array} \\
 = & \left(\sqrt[2]{16}\right)^3 & = \sqrt[2]{16^3} \\
 = & 4^3 & = \sqrt{4096} \\
 = & 64 & = 64
 \end{array}$$

Feb 10-10:00 AM

10. POWERS WITH NEGATIVE EXPONENTS:

$$x^{-n} = \frac{1}{x^n} \quad \text{AND} \quad \frac{1}{x^{-n}} = x^n$$

EX.:

$$\begin{aligned}
 & 4^{-2} \\
 & = \frac{1}{4^2} \\
 & = \frac{1}{16}
 \end{aligned}$$

EX.:

$$\begin{aligned}
 & \frac{1}{5^{-2}} \\
 & = 5^2 \\
 & = 25
 \end{aligned}$$

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Basically, remember to take the reciprocal of the ENTIRE base and change the negative exponent to a positive exponent.

EX.: $\left(-\frac{3}{4}\right)^{-3}$

Feb 21-3:28 PM

LAST MINUTE QUESTIONS???
(page 236, #1 to #8)

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QUIZ TIME!
(Sections 4.4 and 4.5 - 10 to 15 min.)

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4.6 - APPLYING EXPONENT LAWS:

Let's build gradually on what we knew in grade 9...

For example:

$$\begin{aligned} & (2^2)(2^6) \\ = & 2^8 \\ = & 256 \end{aligned}$$

Mar 9-8:13 AM

APPLYING EXPONENT LAWS:**Let's build gradually on what we knew in grade 9...****For example:**

$$\begin{aligned}
 & (2^2)(2^{-6}) \\
 = & 2^{-4} \\
 = & \frac{1}{2^4} \\
 = & \frac{1}{16}
 \end{aligned}$$

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APPLYING EXPONENT LAWS:**Let's build gradually on what we knew in grade 9...****For example:**

$$\begin{aligned}
 & \left[(2^2)(2^{-6}) \right]^{\frac{-3}{2}} \\
 = & (2^{-4})^{\frac{-3}{2}} \\
 = & \left(\frac{1}{2^4} \right)^{\frac{-3}{2}} \\
 = & \left(\frac{1}{16} \right)^{\frac{-3}{2}} \\
 = & 16^{\frac{3}{2}} \\
 = & (\sqrt{16})^3 \\
 = & 4^3 \\
 = & 64
 \end{aligned}$$

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EXAMPLE:

a) $0.3^{-3} \cdot 0.3^5$

b) $\left[\left(-\frac{3}{2} \right)^{-4} \right]^2 \cdot \left[\left(-\frac{3}{2} \right)^2 \right]^3$

c) $\frac{(1.4^3)(1.4^4)}{1.4^{-2}}$

d) $\left(\frac{7^{\frac{2}{3}}}{7^{\frac{1}{3}} \cdot 7^{\frac{5}{3}}} \right)^6$

Mar 8-5:50 PM

SOLUTIONS:

$$\begin{aligned} \text{a) } 0.3^{-3} \cdot 0.3^5 &= 0.3^{(-3) + 5} \\ &= 0.3^2 \end{aligned}$$

$$\begin{aligned} \text{b) } \left[\left(-\frac{3}{2} \right)^{-4} \right]^2 \cdot \left[\left(-\frac{3}{2} \right)^2 \right]^3 &= \left(-\frac{3}{2} \right)^{-8} \cdot \left(-\frac{3}{2} \right)^6 \\ &= \left(-\frac{3}{2} \right)^{-2} \\ &= \left(-\frac{2}{3} \right)^2 \end{aligned}$$

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<p>c) $\frac{(1.4^3)(1.4^4)}{1.4^{-2}}$</p> <p>$= \frac{1.4^{3+4}}{1.4^{-2}}$</p> <p>$= \frac{1.4^7}{1.4^{-2}}$</p> <p>$= 1.4^{7-(-2)}$</p> <p>$= 1.4^9$</p>	<p>d) $\left(\frac{7^{\frac{2}{3}}}{\frac{1}{7^3} \cdot \frac{5}{7^3}}\right)^6$</p> <p>$= \left(\frac{7^{\frac{2}{3}}}{\frac{5}{7^6}}\right)^6$</p> <p>$= \left(7^{\frac{2}{3}-\frac{6}{3}}\right)^6$</p> <p>$= \left(7^{-\frac{4}{3}}\right)^6$</p> <p>$= 7^{-\frac{24}{3}}$</p> <p>$= 7^{-8}$</p> <p>$= \frac{1}{7^8} \left(\frac{1}{7}\right)^8$</p>
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YOU TRY!

<p>a) $0.8^{-5} = \left(\frac{8}{10}\right)^{-5}$</p> <p>$= \left(\frac{10}{8}\right)^5$</p> <p>$= \left(\frac{5}{4}\right)^5$</p> <p>$= \frac{3125}{1024}$</p>	<p>a) $0.8^2 \cdot 0.8^{-7} = 0.8^{-5}$</p> <p>b) $\left[\left(-\frac{4}{5}\right)^2\right]^{-3} \div \left[\left(-\frac{4}{5}\right)^4\right]^{-5}$</p> <p>c) $\frac{(1.5^{-3})^{-5}}{1.5^5}$</p> <p>d) $\frac{9^{\frac{5}{4}} \cdot 9^{-\frac{1}{4}}}{9^{\frac{3}{4}}}$</p>	<p>b) $\left(-\frac{4}{5}\right)^{-6} \div \left(-\frac{4}{5}\right)^{-20}$</p> <p>$\left(-\frac{4}{5}\right)^{-6-(-20)}$</p> <p>$\left(-\frac{4}{5}\right)^{14}$</p> <p>d) $9^{\frac{5}{4} + \frac{-1}{4} - \frac{3}{4}}$</p> <p>$= 9^{\frac{1}{4}}$</p> <p>$\approx 1.7$</p>
<p>c) $\frac{(1.5)^{15}}{1.5^5} = 1.5^{10}$</p>		

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[Answers: a) $\frac{1}{0.8^5}$ b) $\left(-\frac{4}{5}\right)^{14}$
c) 1.5^{10} d) $9^{\frac{1}{4}}$]

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EXAMPLE:

a) $(x^3y^2)(x^2y^{-4}) = x^5y^{-2}$
 $= \frac{x^5}{y^2}$

b) $\frac{10a^5b^3}{2a^2b^{-2}}$
 $= 5a^3b^{3-(-2)}$
 $= 5a^3b^5$

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SOLUTIONS:

$$\begin{aligned}\text{a) } (x^3y^2)(x^2y^{-4}) &= x^3 \cdot y^2 \cdot x^2 \cdot y^{-4} \\ &= x^3 \cdot x^2 \cdot y^2 \cdot y^{-4} \\ &= x^{3+2} \cdot y^{2+(-4)} \\ &\checkmark = x^5 \cdot y^{-2} \\ &= x^5 \cdot \frac{1}{y^2} \\ &\checkmark = \frac{x^5}{y^2}\end{aligned}$$

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$$\begin{aligned}\text{b) } \frac{10a^5b^3}{2a^2b^{-2}} &= \frac{10}{2} \cdot \frac{a^5}{a^2} \cdot \frac{b^3}{b^{-2}} \\ &= 5 \cdot a^{5-2} \cdot b^{3-(-2)} \\ &= 5 \cdot a^3 \cdot b^5 \\ &= 5a^3b^5\end{aligned}$$

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HOW I WOULD ACTUALLY SHOW MY WORK:

a) $(x^3y^2)(x^2y^{-4})$

b) $\frac{10a^5b^3}{2a^2b^{-2}}$

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YOU TRY!

a) m^6n^1

b) $\frac{3x^3y^{-3-2}}{7}$

$\frac{3x^3y^{-5}}{7}$

$\frac{3x^3}{7y^5}$

a) $m^4n^{-2} \cdot m^2n^3$

b) $\frac{6x^4y^{-3}}{14xy^2}$

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$$[\text{Answers: a) } m^6 n \quad \text{b) } \frac{3x^3}{7y^5}]$$

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EXAMPLE: $3^{\frac{1}{3}}$

$$\text{a) } (8a^3b^6)^{\frac{1}{3}} = 2a^1b^2$$

$$\begin{aligned} \text{c) } \frac{4a^{-2}b^{\frac{2}{3}}}{2a^2b^{\frac{1}{3}}} &= 2a^{-2-2}b^{\frac{2}{3}-\frac{1}{3}} \\ &= 2a^{-4}b^{\frac{1}{3}} \\ &= \frac{2b^{\frac{1}{3}}}{a^4} \end{aligned}$$

$$\begin{aligned} \text{b) } (x^{\frac{3}{2}}y^2)(x^{\frac{1}{2}}y^{-1}) &= x^2y \\ \text{d) } \left(\frac{x^2y}{25a^5b^{-\frac{1}{2}}} \right)^{\frac{1}{2}} &= \left(\frac{x^2y}{25a^5b^{-\frac{1}{2}}} \right)^{\frac{1}{2}} \\ &= (4a^{-4}b^{\frac{1}{2}})^{\frac{1}{2}} \\ &= 2a^{-2}b^{\frac{1}{4}} \\ &= \frac{2b^{\frac{1}{4}}}{a^2} \end{aligned}$$

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SOLUTIONS:

$$\begin{aligned}\text{a) } (8a^3b^6)^{\frac{1}{3}} &= 8^{\frac{1}{3}} \cdot a^{3\left(\frac{1}{3}\right)} \cdot b^{6\left(\frac{1}{3}\right)} \\ &= (2^3)^{\frac{1}{3}} \cdot a^1 \cdot b^2 \\ &= 2ab^2\end{aligned}$$

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$$\begin{aligned}\text{b) } (x^{\frac{3}{2}}y^2)(x^{\frac{1}{2}}y^{-1}) &= x^{\frac{3}{2}} \cdot x^{\frac{1}{2}} \cdot y^2 \cdot y^{-1} \\ &= x^{\frac{3}{2} + \frac{1}{2}} \cdot y^{2 + (-1)} \\ &= x^2y\end{aligned}$$

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$$\begin{aligned}
 \text{c) } \frac{4a^{-2}b^{\frac{2}{3}}}{2a^2b^{\frac{1}{3}}} &= \frac{4}{2} \cdot \frac{a^{-2}}{a^2} \cdot \frac{b^{\frac{2}{3}}}{b^{\frac{1}{3}}} \\
 &= 2 \cdot a^{(-2) - 2} \cdot b^{\frac{2}{3} - \frac{1}{3}} \\
 &= 2 \cdot a^{-4} \cdot b^{\frac{1}{3}} \\
 &= \frac{2b^{\frac{1}{3}}}{a^4}
 \end{aligned}$$

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$$\begin{aligned}
 \text{d) } \left(\frac{100a}{25a^5b^{-\frac{1}{2}}} \right)^{\frac{1}{2}} &= \left(\frac{100}{25} \cdot \frac{a^1}{a^5} \cdot \frac{1}{b^{-\frac{1}{2}}} \right)^{\frac{1}{2}} \\
 &= \left(4 \cdot a^{1-5} \cdot b^{\frac{1}{2}} \right)^{\frac{1}{2}} \\
 &= \left(4 \cdot a^{-4} \cdot b^{\frac{1}{2}} \right)^{\frac{1}{2}} \\
 &= 4^{\frac{1}{2}} \cdot a^{(-4)\left(\frac{1}{2}\right)} \cdot b^{\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)} \\
 &= 2 \cdot a^{-2} \cdot b^{\frac{1}{4}} \\
 &= \frac{2b^{\frac{1}{4}}}{a^2}
 \end{aligned}$$

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HOW I WOULD ACTUALLY SHOW MY WORK:

a) $(8a^3b^6)^{\frac{1}{3}}$

b) $(x^{\frac{3}{2}}y^2)(x^{\frac{1}{2}}y^{-1})$

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HOW I WOULD ACTUALLY SHOW MY WORK:

c) $\frac{4a^{-2}b^{\frac{2}{3}}}{2a^2b^{\frac{1}{3}}}$

d) $\left(\frac{100a}{25a^5b^{-\frac{1}{2}}}\right)^{\frac{1}{2}}$

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YOU TRY!

a) $(25a^4b^2)^{\frac{3}{2}}$

b) $(x^3y^{-\frac{3}{2}})(x^{-1}y^{\frac{1}{2}})$

c) $\frac{12x^{-5}y^{\frac{5}{2}}}{3x^{\frac{1}{2}}y^{-\frac{1}{2}}}$

d) $\left(\frac{50x^2y^4}{2x^4y^7}\right)^{\frac{1}{2}}$

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a) $(25a^4b^2)^{\frac{3}{2}}$ $25^{\frac{3}{2}}(a^4)^{\frac{3}{2}}(b^2)^{\frac{3}{2}}$

$125 a^{\frac{12}{2}} b^{\frac{6}{2}}$

$125 a^6 b^3$

Sep 20-10:37 AM

$$\begin{aligned}
 \text{b) } (x^3 y^{-\frac{3}{2}})(x^{-1} y^{\frac{1}{2}}) &= x^{3-1} y^{-\frac{3}{2} + \frac{1}{2}} \\
 &= x^2 y^{-\frac{2}{2}} \\
 &= x^2 y^{-1} \\
 &= \frac{x^2}{y}
 \end{aligned}$$

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$$\begin{aligned}
 \text{c) } \frac{12x^{-5} y^{\frac{5}{2}}}{3x^{\frac{1}{2}} y^{-\frac{1}{2}}} &= 4 x^{-5 - \frac{1}{2}} y^{\frac{5}{2} - (-\frac{1}{2})} \\
 &= 4 x^{-5\frac{1}{2}} y^{\frac{6}{2}} \\
 &= 4 x^{-\frac{11}{2}} y^3 \\
 &= \frac{4 y^3}{x^{\frac{11}{2}}}
 \end{aligned}$$

Sep 20-10:36 AM

$$\begin{aligned}
 \text{d) } \left(\frac{50x^2y^4}{2x^4y^7} \right)^{\frac{1}{2}} &= \left(\frac{25x^2y^4}{x^4y^7} \right)^{\frac{1}{2}} \\
 &= \left(\frac{25}{x^2y^3} \right)^{\frac{1}{2}} \\
 &= \frac{\sqrt{25}}{\sqrt{x^2} \sqrt{y^3}} \quad (x^2)^{\frac{1}{2}} = x^{\frac{2}{2}} \\
 &= \frac{5}{xy^{\frac{3}{2}}} \quad = x
 \end{aligned}$$

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[Answers: a) $125a^6b^3$ b) $\frac{x^2}{y}$

c) $\frac{4y^3}{x^{\frac{11}{2}}}$ d) $\frac{5}{xy^{\frac{3}{2}}}$]

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CONCEPT REINFORCEMENT:

FPCM 10:

Page 241: #3 to #6

Page 242: #7 to #11, #14 to #17 & #19

Page 243: #21 & #22

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UNIT 1 TEST PREPARATION

FPCM 10:

Page 197: Skills Summary (3.1 / 3.2)

Page 198: Review Questions (3.1 / 3.2)

Page 201: Practice Test (#1 & #3)

Page 244: Study Guide

Page 245: Skills Summary

Pages 246 to 248: Review Questions

Page 249: Practice Test

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