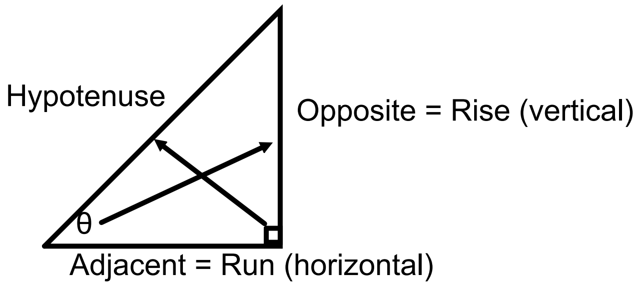
Section 6.2

Slope and Angle of Elevation

Use your prior knowledge (Trigonometry Unit) to now make the connections between the idea of slope as a fraction that is rise over run, to the Tangent relationship that is a fraction that is opposite over adjacent. Both are based on the arms that form the right angle in a triangle.

$$\text{Slope} = \frac{\text{Rise}}{\text{Run}} = \frac{\text{Opposite}}{\text{Adjacent}} = \text{Tangent}\,\theta \text{ (angle of elevation)}$$

We now see the Rise as the Opposite side and the Run as the Adjacent side.



Example:

Relationship between Slope and Angle of Elevation

Yanick and Emily plan to build a wheelchair ramp for their grandmother. They learn that outdoor ramps must have a slope close to 1:12 but not greater. This ensures that the person in the wheelchair can safely travel up and down the ramp.

a) To start, they measure the space they have to build the ramp.

- The distance from the doorway straight down to the ground is 2 feet.
- The walkway along the ground is 40 feet.

Do they have enough space in order for the ramp to be safe?

New Ramp

$$\text{Slope} = \frac{\text{Rise}}{\text{Run}} = \frac{2}{40} = 0.05$$

Maximum Safe Slope

1:12

$$\text{Slope} = \frac{1}{12} = 0.083...$$

0.05 is less than 0.083...

Therefore they have enough space.

b) Once the ramp is built, what angle will the ramp make with the ground?

$$\text{tangent } \theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$\text{tangent } \theta = \frac{2}{40}$$

$$\text{tangent } \theta = 0.05$$

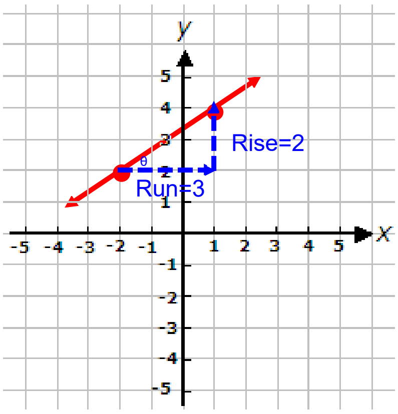
$$\theta = \tan^{-1} 0.05$$

$$\theta = 2.862...$$

The ramp makes an angle of 3° with the ground.

Angle of Elevation from a Graph

Find the angle of elevation in the graph below.



***Remember that Angle of Elevation is the Tangent function.

$\frac{\text{Rise = Opposite}}{\text{Run = Adjacent}}$

$\text{tangent } \theta = \frac{\text{rise}}{\text{run}}$

$\tan \theta = \frac{2}{3}$

$\tan \theta = 0.6667$ (remember to round to 4 decimal places)

$\theta = \tan^{-1} 0.6667$

$\theta = 34°$

Slope as a Percent Grade

Grade: the slope of a road or railway; usually expressed as a percent

Now let's call on prior knowledge about percent.

Percent can be represented as a fraction out of 100.

Once we have a fraction, we now have a Rise and a Run.

Example: $18\% = \frac{18}{100} = \frac{\text{Rise}}{\text{Run}}$ Now we have slope

$\frac{18}{100} = \frac{9}{50}$ as a reduced fraction

The grade of the road above is 18%. It means that the road drops 9 units for every 50 units horizontally.

On roads we often see signs like the one above. These signs are placed as warnings of upcoming steep hills.

Apr 28-2:48 PM

Exercises for practice

Pages 278 - 279 Questions: 1 - 6

Pages 282 - 283 Questions: 1 - 8

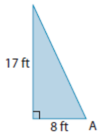
Answers

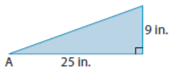
Pages 395 - 396

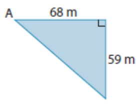
Check Your Understanding

Try It

1. Determine the tangent ratio of ∠A in each triangle.

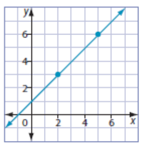
a)


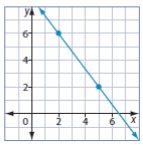
b)


c)


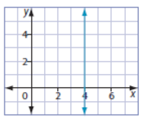
2. For each triangle in #1, determine the measure of ∠A to the nearest degree.

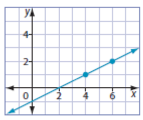
3. For each graph, what is the measure of the angle of elevation to the nearest degree?

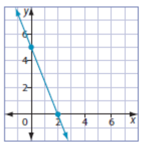
a)


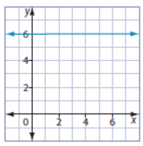
b)


Remember a vertical line has a slope stated as undefined

c)


d)


e)


f)


Remember a horizontal line has a slope of 0

Apr 28-2:48 PM

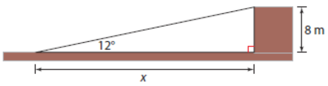
F.Y.I.

Some spaces do not allow a wheelchair ramp to be built with a slope of 1:12. The ramp might need to be built winding from side to side to meet the safety rule. Sometimes permission is given for a ramp to have a maximum slope of 1:8.

Apply It

4. What angle does a ramp with a slope of 1:8 make with the ground? Express your answer to the nearest degree.

5. a) An underground parking ramp has been designed to rise at an angle of 12°. The parking lot is 8 m below ground. What is the run of the ramp to the nearest tenth of a metre?

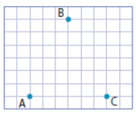


b) Would this make a good wheelchair ramp? Explain.

6. a) Indoor ramps have a steepness of 1 cm of rise for every 9 cm of run. For an indoor ramp to rise 90 cm, how far along the ground should it run?

b) What is the angle of elevation of the ramp? Express your answer to the nearest degree.

7. a) What type of triangle is formed by the points A, B, and C on the grid? How do you know?



b) Determine the slope of line segments AB, BC, and AC.

c) What can you conclude about the slopes of the two equal sides in this triangle?

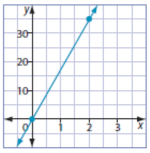
d) Without using a protractor, determine the measure of ∠A. Express your answer to the nearest degree.

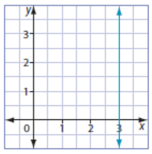
e) What can you conclude about the measure of ∠C? Explain.

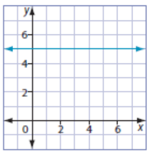
Check Your Understanding

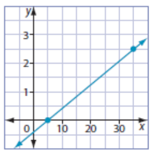
Try It

1. Determine the slope of each line.

a) 

b) 

c) 

d) 

2. a) Suppose each line in #1 represents a road. Which one would be the easiest to travel?
   b) Which one would be impossible to travel? Why?

3. a) Explain the meaning of the sign shown.
   b) Write the slope of the road as a fraction.

4. The table shows details of the grades of various roads. Copy and complete the table. Express slope as a decimal to the nearest hundredth.

Road Name	Rise	Run	Slope as a Fraction	Slope as a Decimal	Percent Grade
Rarely Driven Route	1500	5000			
Snail Pace Strip	9	42			
Pothole Path				0.05	
Maniac Motorway			$\frac{17}{90}$		
Hurry-Up Highway			$\frac{3}{50}$		
Traffic Jam Thoroughfare					1%
Reckless Ramp				0.5	
Boggy Boulevard					6.25%

F.Y.I.
Warning signs are usually placed where the grade of the road is 6% or greater.

5. Determine the angle of elevation of a road with each grade. Express your answers to the nearest degree.
   a) 16% b) 2%
   c) 6% d) 20%

Apply It

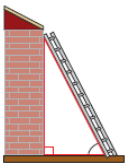
6. Refer to the table you completed in #4. Which of the roads require a warning sign?

7. Jason wants to know the grade of his new driveway. He measures that the driveway rises 5′ over 50′ along the ground. What is the grade of his driveway?

8. What is the grade of a wheelchair ramp that rises $1\frac{1}{2}$ ft over a horizontal distance of 18 ft? Express your answer to the nearest percent.

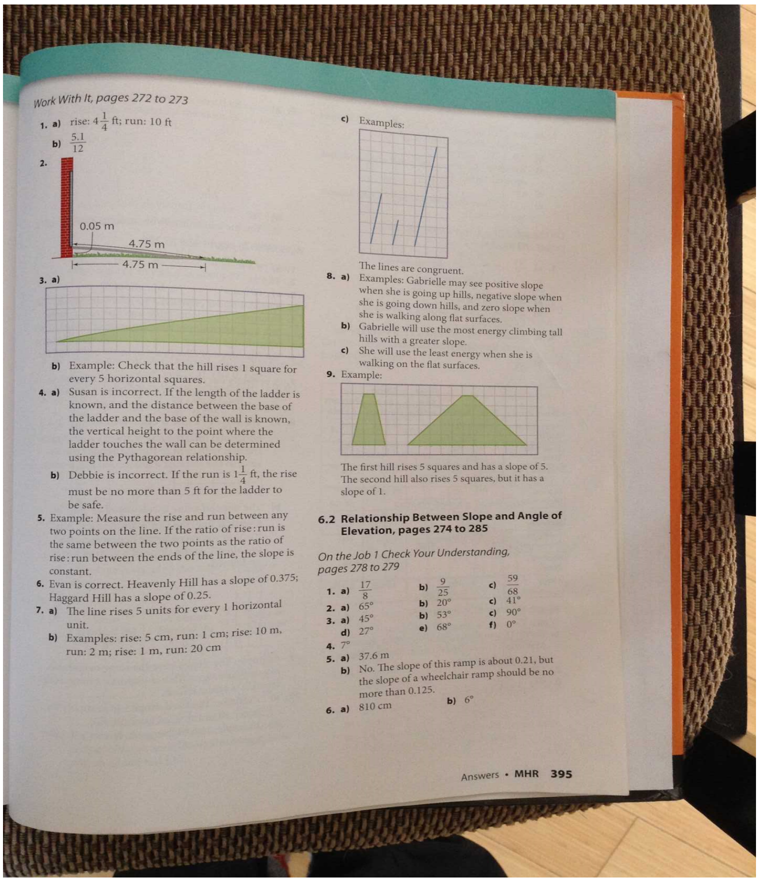
F.Y.I.
One third of all reports of people falling from a height involve ladders. Many of these injuries are caused by incorrect use of the ladder.

9. Tyler is helping his mother paint the eavestrough on their house. He leans the ladder on the bricks 4 m above the ground. The bottom of the ladder is 2 m away from the house.



   a) Make a sketch similar to the one shown. Include the given dimensions.
   b) Safety guidelines state that the maximum slope of a ladder should be 4. What is the angle of elevation of a ladder placed at this slope? Round your answer to the nearest degree.
   c) Does Tyler have the ladder positioned safely? Explain.

Answers



Work With It, pages 272 to 273

1. a) rise: $4\frac{1}{4}$ ft; run: 10 ft
b) $\frac{5.1}{12}$

2.

0.05 m
4.75 m
4.75 m

3. a)

b) Example: Check that the hill rises 1 square for every 5 horizontal squares.

4. a) Susan is incorrect. If the length of the ladder is known, and the distance between the base of the ladder and the base of the wall is known, the vertical height to the point where the ladder touches the wall can be determined using the Pythagorean relationship.
b) Debbie is incorrect. If the run is $1\frac{1}{4}$ ft, the rise must be no more than 5 ft for the ladder to be safe.

5. Example: Measure the rise and run between any two points on the line. If the ratio of rise : run is the same between the two points as the ratio of rise : run between the ends of the line, the slope is constant.

6. Evan is correct. Heavenly Hill has a slope of 0.375; Haggard Hill has a slope of 0.25.

7. a) The line rises 5 units for every 1 horizontal unit.
b) Examples: rise: 5 cm, run: 1 cm; rise: 10 m, run: 2 m; rise: 1 m, run: 20 cm
c) Examples:

The lines are congruent.

8. a) Examples: Gabrielle may see positive slope when she is going up hills, negative slope when she is going down hills, and zero slope when she is walking along flat surfaces.
b) Gabrielle will use the most energy climbing tall hills with a greater slope.
c) She will use the least energy when she is walking on the flat surfaces.

9. Example:

The first hill rises 5 squares and has a slope of 5. The second hill also rises 5 squares, but it has a slope of 1.

6.2 Relationship Between Slope and Angle of Elevation, pages 274 to 285

On the Job 1 Check Your Understanding, pages 278 to 279

1. a) $\frac{17}{8}$ **b)** $\frac{9}{25}$ **c)** $\frac{59}{68}$
2. a) 65° **b)** 20° **c)** 41°
3. a) 45° **b)** 53° **c)** 90°
d) 27° **e)** 68° **f)** 0°
4. 7°
5. a) 37.6 m
b) No. The slope of this ramp is about 0.21, but the slope of a wheelchair ramp should be no more than 0.125.
6. a) 810 cm **b)** 6°

Answers • MHR 395

7. a) isosceles; the points A and C are each 3 squares horizontally from point B.
   b) AB: 2, BC: −2, AC: 0
   c) Example: The slopes have equal magnitude but opposite signs.
   d) 63°
   e) ∠C will have the same measure as ∠A, because tan A = tan C.

On the Job 2 Check Your Understanding, pages 282 to 283

1. a) $\frac{35}{2}$ b) undefined
   c) 0 d) $\frac{1}{12}$
2. a) the line in part c)
   b) the line in part b) because it would be a vertical road
3. a) Example: The road drops 12 m vertically for every 100 m travelled horizontally.
   b) $\frac{3}{25}$
4.

Road Name	Rise	Run	Slope as a Fraction	Slope as a Decimal	Percent Grade
Rarely Driven Route	1500	5000	$\frac{3}{10}$	0.3	30%
Snail Pace Strip	9	42	$\frac{3}{14}$	0.21	21%
Pothole Path	1	20	$\frac{1}{20}$	0.05	5%
Maniac Motorway	17	90	$\frac{17}{90}$	0.19	19%
Hurry-Up Highway	3	50	$\frac{3}{50}$	0.06	6%
Traffic Jam Thoroughfare	1	100	$\frac{1}{100}$	0.01	1%
Reckless Ramp	1	2	$\frac{1}{2}$	0.5	50%
Boggy Boulevard	1	16	$\frac{1}{16}$	0.0625	6.25%

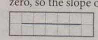
5. a) 9° b) 1°
   c) 3° d) 11°
6. Rarely Driven Route, Snail Pace Strip, Maniac Motorway, Hurry-Up Highway, Reckless Ramp, and Boggy Boulevard
7. 10%
8. 8%

9. a)

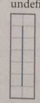
   b) 76°
   c) Yes, the ladder is safe because the slope is only 2.

Work With It, pages 284 to 285

1. a) 4.3 m b) 4.7 m
2. 48.6 m
3. a) 60″ or 5′ b) about 6°
4. 89 432 ft
5. Examples: zero slope: table top, horizon, counter top; undefined slope: table leg, side of house, hinged edge of door
6. Yes. The slope of Chad's set of stairs is 0.8, and the guideline slope is 0.8.
7. A perfectly flat road has a 0% grade. Examples: This road will require more gas than a downward slope, but less gas than an upward slope.
8. a) The slope of a line is calculated as $\frac{\text{rise}}{\text{run}}$. In a horizontal line, the rise is always zero. Any fraction with zero in the numerator equals zero, so the slope of a horizontal line is zero.



   b) The slope of a line is calculated as $\frac{\text{rise}}{\text{run}}$. In a vertical line, the run is always zero. Any fraction with zero in the denominator is undefined, so the slope of a vertical line is undefined.



9. Example: The slopes have the same magnitude, but opposite signs. This is important because the triangle is symmetrical.
10. a) Example: The road drops 1 m vertically for every 10 m travelled horizontally.
    b) Example: In a higher gear, drivers will gain speed too quickly.