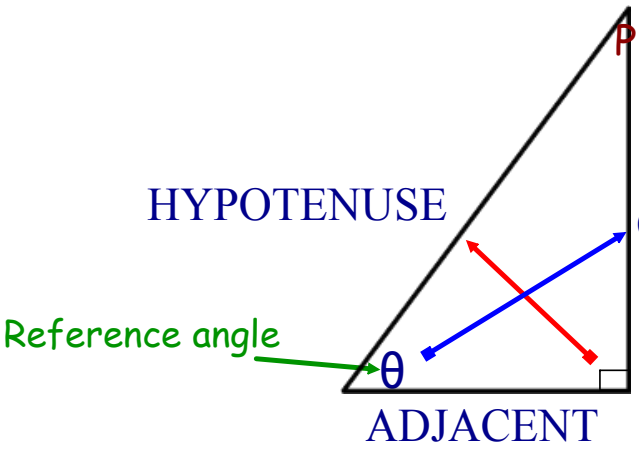
Trigonometric Ratios

*** Must have calculator in DEGREE mode ***



Primary Trigonometric Ratios

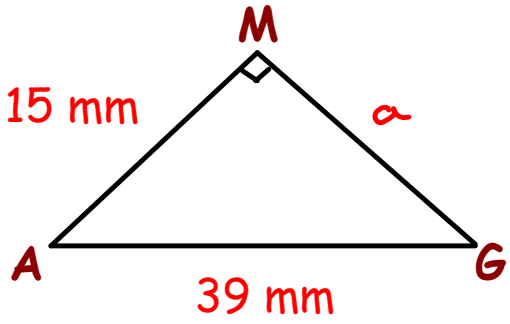
$$\sin\theta = \frac{opp}{hyp}$$

$$\cos\theta = \frac{adj}{hyp}$$

$$\tan\theta = \frac{opp}{adj}$$

Memory Aid: "SOH CAH TOA"

Solve the triangle (find ALL sides and angles)

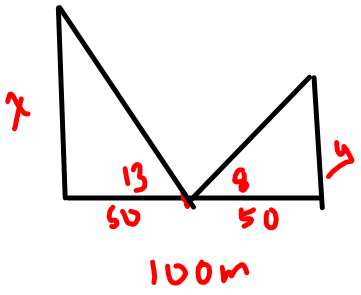


$a^2 = c^2 - b^2$
$a^2 = 39^2 - 15^2$
$a^2 = 1296$
$\sqrt{a^2} = \sqrt{1296}$
$a = 36$ mm

$\cos A = \frac{15}{39}$ $\left(\frac{opp}{hyp}\right)$
$\cos A = 0.3846\ldots$
$A = 67°$

$G = 180 - 90 - 67$
$= 23°$

Two trees are 100m apart. From a point on midway between them, the angles of elevation to their tops are 8° and 13°. How much taller is one tree than the other?



$\tan 13 = \frac{x}{50}$ $\tan 8 = \frac{y}{50}$

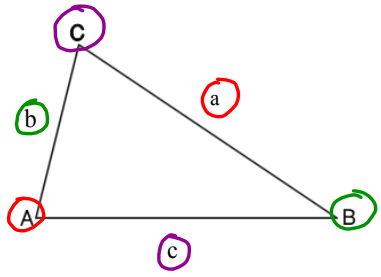
$x = 50(\tan 13)$ $y = 50(\tan 8)$

$x = 11.54$m $y = 7.03$

Difference $11.54 - 7.03$
4.51 or 4.5

** **Used when the triangle does not contain 90°** angle (Oblique Triangle)

** In order to use you must be given 1)an angle and an opposite side



Lower case letters "a,b,c" represent side lengths
Upper case letters "A,B,C" represent angle measures

Law of Sines

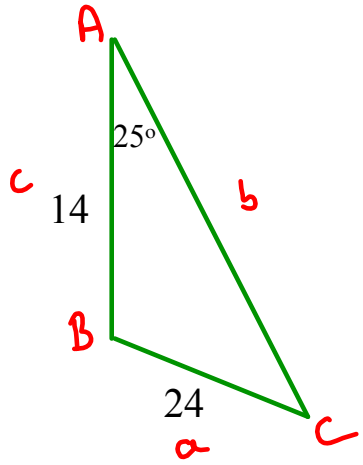
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

"when looking for a side"

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

"when looking for an angle"

Solve the triangle.



$\frac{\sin C}{c} = \frac{\sin A}{a}$

$\frac{\sin C}{14} = \frac{\sin 25}{24}$

$\sin C = \frac{14(\sin 25)}{24}$

$\sin C = 0.2465$

$\angle C = 14$

$\angle B = 180 - 25 - 14$

$= 141$

$\frac{b}{\sin B} = \frac{a}{\sin A}$

$\frac{b}{\sin 141} = \frac{24}{\sin 25}$

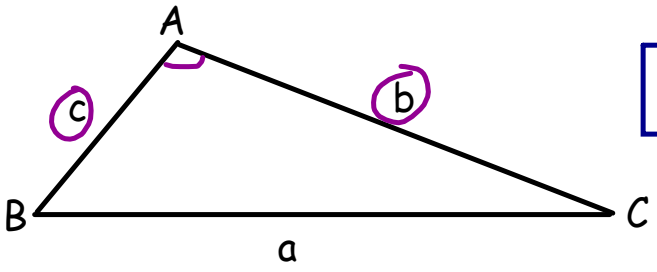
$b = \frac{24(\sin 141)}{\sin 25}$

$b = 35.7$

Law of Cosines

Finding an unknown side...

- 2 sides and a contained angle (SAS)



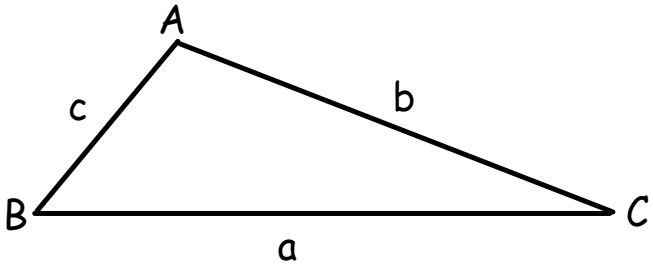
$$a^2 = b^2 + c^2 - 2bc\cos A$$

$b^2 = a^2 + c^2 - 2ac\cos B$

$c^2 = a^2 + b^2 - 2ab\cos C$

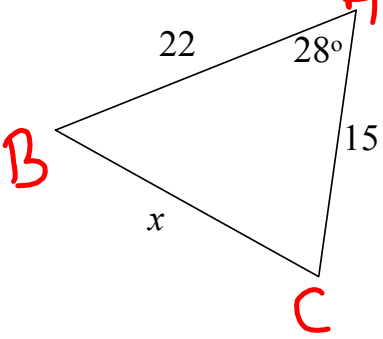
Finding an unknown angle...

- 3 known sides (SSS)



$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

EXAMPLE: Finding an unknown side.



$a^2 = b^2 + c^2 - 2bc\cos A$

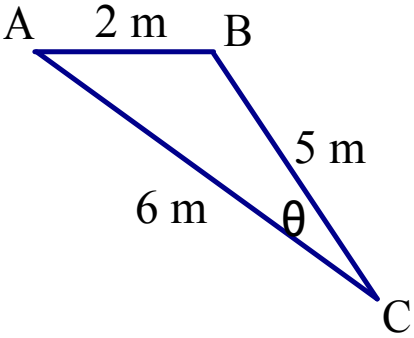
$a^2 = 15^2 + 22^2 - 2(15)(22)\cos 28$

$a^2 = 709 - 582.7454\ldots$

$a^2 = 126.25$

$a = 11.2$

EXAMPLE: Finding an unknown angle.



$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$

$\cos C = \frac{5^2 + 6^2 - 2^2}{2(5)(6)}$

$\cos C = \frac{57}{60}$

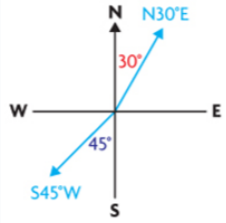
$\cos C = 0.95$

$C = 18°$

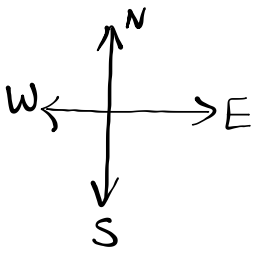
NOTE:

Communication | *Tip*

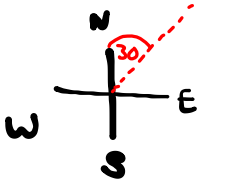
Directions are often stated in terms of north and south on a compass. For example, N30°E means travelling in a direction 30° east of north. S45°W means travelling in a direction 45° west of south.



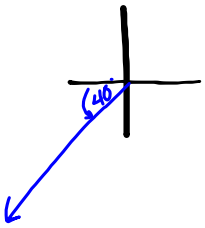
Bearings



N30°E

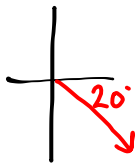


W40°S



S15°W

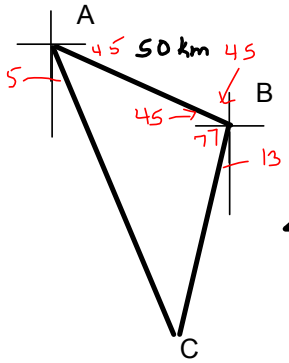
E20°S



Example:

Exercise 10.13

3. Two ships are 50 km apart. Ship A sights a distress flare at S5E whereas ship B sights the same flare at S13W. If ship A is N45W of ship B, find the distance each ship is from the distressed ship.



In triangle

$\angle A = 40°$

$\angle B = 122°$

$c = 50$

$\angle C = 180 - 40 - 122$

$= 18°$

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

$$a = \frac{50(\sin 40)}{\sin 18°}$$

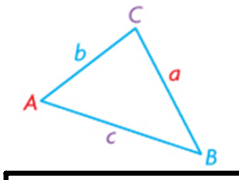
$$a = 104 \text{ km}$$

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$

$$b = \frac{50(\sin 122)}{\sin 18}$$

$b =$

Trigonometry Summary AND 'The AMBIGUOUS Case'...



sine law

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

cosine law

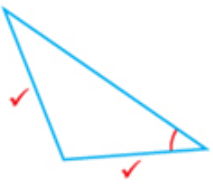
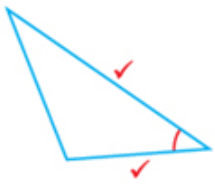
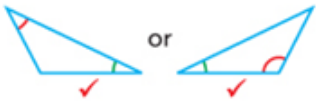
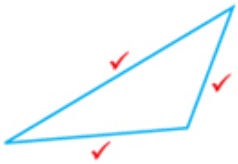
$$a^2 = b^2 + c^2 - 2bc\cos A$$

oblique triangle

A triangle that does not contain a 90° angle.

Need to Know

- The sine law and cosine law are used with obtuse triangles in the same way that they are used with acute triangles.

Use the sine law when you know ...	Use the cosine law when you know ...
- the lengths of two sides and the measure of the angle that is opposite a known side 	- the lengths of two sides and the measure of the contained angle 
- the measures of two angles and the length of any side 	- the lengths of all three sides 

Ambiguous Case

- Be careful when using the sine law to determine the measure of an angle. The inverse sine of a ratio always gives an acute angle, but the supplementary angle has the same ratio. You must decide whether the acute angle, θ, or the obtuse angle, $180° - \theta$, is the correct angle for your triangle.

Ambiguous Case

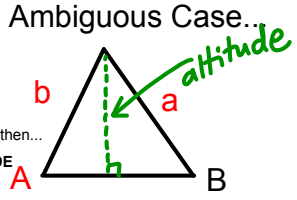
Criteria for the Ambiguous Case..

- Must be given SSA
- Given angle is acute
- a < b

*** If ALL 3 criteria are met, then...

CALCULATE THE ALTITUDE

alt = b sin A



CASE 1: a < altitude; there is NO SOLUTION

CASE 2: a = altitude; there is ONE SOLUTION [Right Triangle]

CASE 3: a >altitude; this is the 'AMBIGUOUS CASE'...TWO SOLUTIONS

1) Acute Triangle (angle, θ, is found with Law of Sines)
2) Obtuse Triangle (angle is 180° - θ)

Solve ΔABC,

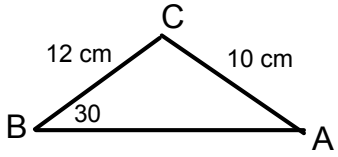
$\angle B = 30$, a= 12 cm, b = 10 cm

SSA alt = a sinB

$\angle B$ acute = 12 sin 30

b < a = 6 b > alt so 2 solutions

First Solution



$\angle A = 37$

$\angle C = 180 - 37 - 30$

$= 113$

c = 18.4 cm

$\frac{\sin A}{a} = \frac{\sin B}{b}$ $\frac{c}{\sin C} = \frac{b}{\sin B}$

$\sin A = \frac{12 \sin 30}{10}$ $c = \frac{10 \sin 113}{\sin 30}$

sin A = 0.6 c = 18.4 cm

A = 37

Second Solution

A = 180 -37

= 143

C = 180 - 143 - 30

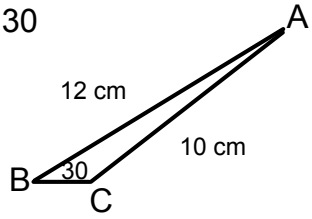
= 7

c = 2.4 cm

$\frac{c}{\sin C} = \frac{b}{\sin B}$

$c = \frac{10 \sin 7}{\sin 30}$

c = 2.4 cm



Word Problems / Applications

- When solving word problems, you have to draw a diagram with the given information, then decide if you need to use:

- Trig Ratios

- Law of Cosine

- Law of Sines (when using Law of Sines, remember to check for an ambiguous case)

Rounding Rules

Trig Ratios - round to 4 decimal places

Side - round to 1 decimal place

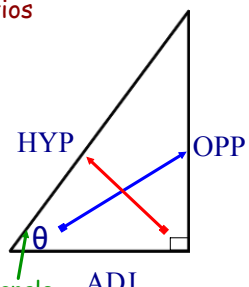
Angle - round to nearest whole number

REVIEW - Trigonometry

- Pythagorean Theorem & Primary Trig Ratios

REMEMBER: "SOH CAH TOA"

$\sin\theta = \frac{opp}{hyp}$ $\cos\theta = \frac{adj}{hyp}$ $\tan\theta = \frac{opp}{adj}$

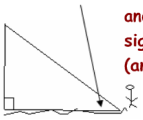


$c^2 = a^2 + b^2$

Reference angle

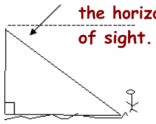
- Applications of Primary Trig

Angle of elevation - is the angle between the ground and the line of sight. (angle of inclination)



Always from the GROUND up

Angle of Depression - is the angle between the horizon and the line of sight.



Always outside the triangle

Also, note that the angle of elevation = angle of depression

- Law of Sines & Its Applications "Pair"

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

"when looking for a side" "when looking for an angle"

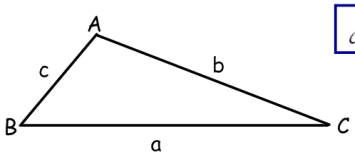
- Ambiguous Case??? (Law of Sines - finding an angle)

- given a side (a), the angle opposite (A) and another side (b)...

CASE #1: a > b -> only one solution
CASE #2: a = b -> only one solution
CASE #3: a < b ... Determine the altitude length (bsinA)
(i) a < altitude -> no solution
(ii) a = altitude -> one solution (right triangle)
(iii) a > altitude -> two solutions... (Ambiguous Case)
(1) acute angle
(2) obtuse angle (180° - acute)

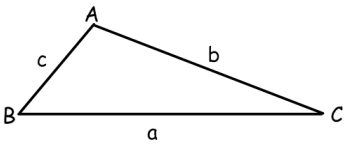
- Law of Cosines & Its Applications

Finding an unknown side...
- 2 sides and a contained angle (SAS)



$a^2 = b^2 + c^2 - 2bc\cos A$

Finding an unknown angle...
- 3 known sides (SSS)



$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$

- Bearings and Multi-step Word Problems
- "Solving" - find ALL angles & sides

**Review questions (these are optional, pick and choose the questions that you need/want to do) **

Lots of Practice from the Textbook!!!

Chapter Review - Frequently Asked Questions)

- Chapter 3 pg 128 and 153
- Chapter 4 pg 174 and 199

		Recommended
Practice Questions		
- Chapter 3	pg 129 # 1-9	#4,5,7,8
	pg 154 # 1-12	#4,5,8,9,(11,12 - Bearing)
-Chapter 4	pg 175 # 1-9	#4,5,7,8
	pg. 200 # 1-8	#5,6,7 - Ambiguous case #8 - Bearing
Practice Test		
Chapter 3	pg. 152 # 1-8	} your choice
Chapter 4	pg. 198 # 1-7	