February 10, 2020

UNIT 1: ROOTS AND POWERS

SECTION 4.4: FRACTIONAL EXPONENTS AND RADICALS

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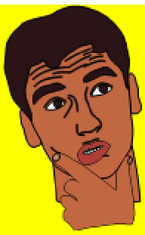
NUMBERS, RELATIONS AND FUNCTIONS 10

WHAT'S THE POINT OF TODAY'S LESSON?

We will continue working on the NRF 10 Specific Curriculum Outcome (SCO) "Algebra and Numbers 3" OR "AN3" which states:

"Demonstrate an understanding of powers with integral and rational exponents."



What does THAT mean???

SCO AN3 means that we will:

* apply the 6 exponent laws you learned in grade 9:
  $a^0 = 1$
  $(a^m)(a^n) = a^{m+n}$
  $a^m \div a^n = a^{m-n}$
  $(a^m)^n = a^{mn}$
  $(ab)^m = a^m b^m$
  $(a \div b)^n = a^n \div b^n$
* use patterns to explain $a^{-n} = \dfrac{1}{a^n}$ and $a^{\frac{1}{n}} = \sqrt[n]{a}$
* apply all exponent laws to evaluate a variety of expressions
* express powers with rational exponents as radicals and vice versa
* identify and correct errors in work that involves powers

EXPONENT LAWS (separate sheet):

1. Zero Exponent Law: $a^0 = 1$
2. Product of Powers: $(a^m)(a^n) = a^{m+n}$
3. Quotient of Powers: $a^m \div a^n = a^{m-n}$
4. Power of a Power: $(a^m)^n = a^{mn}$
5. Power of a Product: $(ab)^m = a^m b^m$
6. Power of a Quotient: $(a \div b)^n = a^n \div b^n$

$\dfrac{a^n}{b^n}$

7. MULTIPLICATION PROPERTY OF RADICALS:

$$\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$$

EX.: $\sqrt{24}$ (Factors: 1, 2, 3, 4, 6, 8, 12, 24)

$= \sqrt{4 \cdot 6}$

$= \sqrt{4} \cdot \sqrt{6}$

$= 2 \cdot \sqrt{6}$

$= 2\sqrt{6}$ (MIXED RADICAL)

EX.: $\sqrt[3]{24}$ (ENTIRE RADICAL)

$= \sqrt[3]{8 \cdot 3}$

$= \sqrt[3]{8} \cdot \sqrt[3]{3}$

$= 2 \cdot \sqrt[3]{3}$

$= 2\sqrt[3]{3}$

LET'S COPY AND COMPLETE THESE TABLES TOGETHER.

x	$x^{\frac{1}{2}}$ $(x^{0.5})$ $\sqrt{x}$
1	$1^{\frac{1}{2}} = 1$
4	$4^{\frac{1}{2}} = 2$
9	$9^{\frac{1}{2}} = 3$
16	$16^{\frac{1}{2}} = 4$
25	$25^{\frac{1}{2}} = 5$

x	$x^{\frac{1}{3}}$ $[x^{(1 \div 3)}]$
1	$1^{\frac{1}{3}} = 1$
8	$8^{\frac{1}{3}} = 2$
27	$27^{\frac{1}{3}} = 3$
64	$64^{\frac{1}{3}} = 4$
125	$125^{\frac{1}{3}} = 5$

WHAT DO YOU THINK THE EXPONENT $\frac{1}{2}$ MEANS?

WHAT DO YOU THINK THE EXPONENT $\frac{1}{3}$ MEANS?

WHAT DO YOU THINK $a^{\frac{1}{4}}$ AND $a^{\frac{1}{5}}$ MEAN? (TEST YOUR PREDICTIONS WITH A CALCULATOR.)

WHAT DO YOU THINK $a^{\frac{1}{n}}$ MEANS?

In grade 9, you learned that for powers with integral bases and whole number exponents:

$a^m \cdot a^n = a^{m+n}$

WE CAN EXTEND THIS LAW TO POWERS WITH FRACTIONAL EXPONENTS WITH A NUMERATOR OF 1:

$$5^{\frac{1}{2}} \cdot 5^{\frac{1}{2}} = 5^{\frac{1}{2}+\frac{1}{2}} = 5^1 = 5$$

$$\sqrt{5} \cdot \sqrt{5} = \sqrt{25} = 5$$

$5^{\frac{1}{2}}$ and $\sqrt{5}$ are equivalent expressions; that is, $5^{\frac{1}{2}} = \sqrt{5}$.

SIMILARLY...

$$5^{\frac{1}{3}} \cdot 5^{\frac{1}{3}} \cdot 5^{\frac{1}{3}} = 5^{\frac{1}{3}+\frac{1}{3}+\frac{1}{3}}$$
$$= 5^1$$
$$= 5$$

$$\sqrt[3]{5} \cdot \sqrt[3]{5} \cdot \sqrt[3]{5}$$
$$= \sqrt[3]{125}$$
$$= 5$$

SO...

$$5^{\frac{1}{3}} = \sqrt[3]{5}$$

8. POWERS WITH RATIONAL EXPONENTS WITH A NUMERATOR OF 1:

$$x^{\frac{1}{n}} = \sqrt[n]{x}$$

EX.:

$$8^{\frac{1}{3}}$$

$$= \sqrt[3]{8}$$

$$= 2$$

EXAMPLE:

Evaluate each power without using a calculator.

a) $27^{\frac{1}{3}}$ $= 3$

b) $0.49^{\frac{1}{2}}$ 0.7

c) $(-64)^{\frac{1}{3}}$ -4

d) $\left(\frac{4}{9}\right)^{\frac{1}{2}}$ $\frac{2}{3}$

YOU TRY!

Evaluate each power without using a calculator.

a) $1000^{\frac{1}{3}} = 10$

b) $0.25^{\frac{1}{2}} = 0.5$

c) $(-8)^{\frac{1}{3}} = -2$

d) $\left(\frac{16}{81}\right)^{\frac{1}{4}} = \frac{2}{3}$

NOTE: Because a fraction can be written as a terminating or repeating decimal, we can interpret powers with decimal exponents.

EX.:

$$32^{0.2}$$
$$=32^{\frac{2}{10}}$$
$$=32^{\frac{1}{5}}$$
$$=\sqrt[5]{32}$$
$$=\mathbf{2}$$

YOU TRY!

Evaluate $100^{0.5}$.

$$100^{\frac{1}{2}}$$
$$=\sqrt{100}$$
$$=10$$

To give meaning to a power such as $8^{\frac{2}{3}}$ **,**

we use the exponent law $(a^m)^n = a^{mn}$**.**

EX.:

$$
\begin{aligned}
&8^{\frac{2}{3}}\\
&= 8^{\frac{1}{3}\cdot 2}\\
&= \left(8^{\frac{1}{3}}\right)^2\\
&= \left(\sqrt[3]{8}\right)^2\\
&= \mathbf{2^2}\\
&= \mathbf{4}
\end{aligned}
$$

EX.:

$$
\begin{aligned}
&8^{\frac{2}{3}}\\
&= 8^{2\cdot\frac{1}{3}}\\
&= \left(8^2\right)^{\frac{1}{3}}\\
&= \sqrt[3]{8^2}\\
&= \sqrt[3]{64}\\
&= \mathbf{4}
\end{aligned}
$$

9. POWERS WITH RATIONAL EXPONENTS:

EXPONENT

$$x^{\frac{m}{n}} = \left(x^{\frac{1}{n}}\right)^m = \left(\sqrt[n]{x}\right)^m$$

INDEX

AND

EXPONENT

$$x^{\frac{m}{n}} = \left(x^m\right)^{\frac{1}{n}} = \sqrt[n]{x^m}$$

INDEX

EX.: **Evaluate** $16^{\frac{3}{2}}$.

$16^{\frac{3}{2}}$ 3 (EXPONENT) 2 (INDEX)

$$
\begin{aligned}
&= \left(\sqrt[2]{16}\right)^3\\
&= \mathbf{4^3}\\
&= \mathbf{64}
\end{aligned}
$$

$\overset{7}{\cancel{56}}\left(\frac{7}{8}\right) = 7(7)$

$= 49$

EXAMPLE:

a) Write $40^{\frac{2}{3}}$ in radical form in 2 ways.

$\left(\sqrt[3]{40}\right)^2$ $\sqrt[3]{40^2}$

b) Write $\sqrt{3^5}$ and $\left(\sqrt[3]{25}\right)^2$ in exponent form.

$3^{\frac{5}{2}}$ $25^{\frac{2}{3}}$

SOLUTION:

a) $40^{\frac{2}{3}} = \left(\sqrt[3]{40}\right)^2$ or $\sqrt[3]{40^2}$

b) $\sqrt{3^5} = 3^{\frac{5}{2}}$ AND $\left(\sqrt[3]{25}\right)^2 = 25^{\frac{2}{3}}$

YOU TRY!

a) Write $26^{\frac{2}{5}}$ in radical form in 2 ways. $(\sqrt[5]{26})^2$ $\sqrt[5]{26^2}$ 3.68

26^(2/5)

b) Write $\sqrt{6^5}$ and $(\sqrt[4]{19})^3$ in exponent form.

$6^{\frac{5}{2}} \doteq 88.2$ $19^{\frac{3}{4}} \doteq 9.1$

SOLUTION:

a) $(\sqrt[5]{26})^2$ or $\sqrt[5]{26^2}$

b) $6^{\frac{5}{2}}$, $19^{\frac{3}{4}}$

EXAMPLE:

Evaluate.

a) $0.04^{\frac{3}{2}} = 0.008$ **b)** $27^{\frac{4}{3}} = (3)^4 = 81$

c) $(-32)^{0.4}$ **d)** $1.8^{1.4} \doteq 2.3$

$= (-32)^{\frac{4}{10}}$

$= (-32)^{\frac{2}{5}}$

$= (-2)^2$

$= 4$

SOLUTION:

a) $0.04^{\frac{3}{2}} = \left(0.04^{\frac{1}{2}}\right)^3$

$= \left(\sqrt{0.04}\right)^3$

$= 0.2^3$

$= 0.008$

b) $27^{\frac{4}{3}} = \left(27^{\frac{1}{3}}\right)^4$

$= \left(\sqrt[3]{27}\right)^4$

$= 3^4$

$= 81$

c) The exponent $0.4 = \frac{4}{10}$ or $\frac{2}{5}$

So, $(-32)^{0.4} = (-32)^{\frac{2}{5}}$

$= \left[(-32)^{\frac{1}{5}}\right]^2$

$= \left(\sqrt[5]{-32}\right)^2$

$= (-2)^2$

$= 4$

d) $1.8^{1.4}$

Use a calculator.

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1.8^1.4
          2.277096874
```

$1.8^{1.4} = 2.2770\ldots$

YOU TRY!

Evaluate.

a) $0.01^{\frac{3}{2}}$ $= (0.1)^3$ $= 0.001$ **b)** $(-27)^{\frac{4}{3}}$ $= (-3)^4$ $= 81$

c) $81^{\frac{3}{4}}$ $(3)^3$ $= 27$ **d)** $0.75^{1.2}$ $\doteq 0.708\ldots$ $\doteq 0.7$

SOLUTION:

a) 0.001 b) 81 c) 27 d) 0.7080…

EXAMPLE:

Biologists use the formula $b = 0.01m^{\frac{2}{3}}$ to estimate the brain mass, b kilograms, of a mammal with body mass m kilograms. Estimate the brain mass of each animal.

a) a husky with a body mass of 27 kg

b) a polar bear with a body mass of 200 kg

SOLUTION:

Use the formula $b = 0.01m^{\frac{2}{3}}$.

a) Substitute: $m = 27$

$b = 0.01(27)^{\frac{2}{3}}$

$b = 0.01\left(\sqrt[3]{27}\right)^2$

$b = 0.01(3)^2$

$b = 0.01(9)$

$b = 0.09$

Use the order of operations. Evaluate the power first.

The brain mass of the husky is approximately 0.09 kg.

b) Substitute: $m = 200$

$b = 0.01(200)^{\frac{2}{3}}$

Use a calculator.

```
0.01(200)^(2/3)
        0.341995189
```

The brain mass of the polar bear is approximately 0.34 kg.

YOU TRY!

Use the formula $b = 0.01m^{\frac{2}{3}}$ to estimate the brain mass of each animal.

a) a moose with a body mass of 512 kg $= 0.64$

b) a cat with a body mass of 5 kg $\doteq 0.029$

SOLUTION:

a) approximately 0.64 kg

b) approximately 0.03 kg

CONCEPT REINFORCEMENT:

***FPCM 10*:**

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