February 11, 2020

UNIT 1: ROOTS AND POWERS

SECTION 4.5: NEGATIVE EXPONENTS AND RECIPROCALS

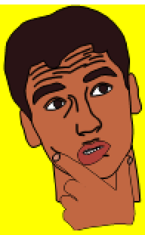
K. Sears

NUMBERS, RELATIONS AND FUNCTIONS 10

WHAT'S THE POINT OF TODAY'S LESSON?

We will continue working on the NRF 10 Specific Curriculum Outcome (SCO) "Algebra and Numbers 3" OR "AN3" which states:

"Demonstrate an understanding of powers with integral and rational exponents."



What does THAT mean???

SCO AN3 means that we will:

* apply the 6 exponent laws you learned in grade 9:
  - $a^0 = 1$
  - $(a^m)(a^n) = a^{m+n}$
  - $a^m \div a^n = a^{m-n}$
  - $(a^m)^n = a^{mn}$
  - $(ab)^m = a^m b^m$
  - $(a \div b)^n = a^n \div b^n$
* use patterns to explain $a^{-n} = \frac{1}{a^n}$ and $a^{\frac{1}{n}} = \sqrt[n]{a}$
* apply all exponent laws to evaluate a variety of expressions
* express powers with rational exponents as radicals and vice versa
* identify and correct errors in work that involves powers

WARM-UP:

Write the power below as a radical then evaluate.

$$\left(\frac{64}{125}\right)^{\frac{2}{3}}$$

$$\left(\sqrt[3]{\frac{64}{125}}\right)^2$$

$$\left(\frac{4}{5}\right)^2$$

$$\frac{16}{25}$$

WHITE BOARD WARM-UP (Day 2):

First, write the power below with a positive exponent. At this point, write the power as a radical then evaluate.

$$\left(\frac{64}{125}\right)^{\frac{-2}{3}}$$

$$\left(\frac{125}{64}\right)^{\frac{2}{3}}$$

$$\left(\frac{5}{4}\right)^{2}$$

$$\frac{25}{16}$$

HOMEWORK QUESTIONS???
(pages 227 / 228, #7, #8, #10, #11 and #15 TO #19)

EXPONENT LAWS (separate sheet):

1. Zero Exponent Law: $a^0 = 1$
2. Product of Powers: $(a^m)(a^n) = a^{m+n}$
3. Quotient of Powers: $a^m \div a^n = a^{m-n}$
4. Power of a Power: $(a^m)^n = a^{mn}$
5. Power of a Product: $(ab)^m = a^m b^m$
6. Power of a Quotient: $(a \div b)^n = a^n \div b^n$

7. MULTIPLICATION PROPERTY OF RADICALS:

$$\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$$

EX.: $\sqrt{24}$ (Factors: 1, 2, 3, 4, 6, 8, 12, 24)

$= \sqrt{4 \cdot 6}$

$= \sqrt{4} \cdot \sqrt{6}$

$= 2 \cdot \sqrt{6}$

$= 2\sqrt{6}$ (MIXED RADICAL)

EX.: $\sqrt[3]{24}$ (ENTIRE RADICAL)

$= \sqrt[3]{8 \cdot 3}$

$= \sqrt[3]{8} \cdot \sqrt[3]{3}$

$= 2 \sqrt[3]{3}$

$= 2\sqrt[3]{3}$

8. POWERS WITH RATIONAL EXPONENTS WITH A NUMERATOR OF 1:

$$x^{\frac{1}{n}} = \sqrt[n]{x}$$

EX.:

$$8^{\frac{1}{3}}$$
$$= \sqrt[3]{8}$$
$$= 2$$

9. POWERS WITH RATIONAL EXPONENTS:

EXPONENT

$$x^{\frac{m}{n}} = \left(x^{\frac{1}{n}}\right)^{m} = \left(\sqrt[n]{x}\right)^{m}$$

INDEX

AND

EXPONENT

$$x^{\frac{m}{n}} = \left(x^{m}\right)^{\frac{1}{n}} = \sqrt[n]{x^{m}}$$

INDEX

EX.: **Evaluate** $16^{\frac{3}{2}}$.

3 (EXPONENT)

2 (INDEX)

$$16^{\frac{3}{2}}$$
$$= \left(\sqrt[2]{16}\right)^{3}$$
$$= 4^{3}$$
$$= 64$$

OR

3 (EXP.)

2 (INDEX)

$$16^{\frac{3}{2}}$$
$$= \sqrt[2]{16^{3}}$$
$$= \sqrt{4096}$$
$$= 64$$

10. POWERS WITH NEGATIVE EXPONENTS:

$$x^{-n} = \frac{1}{x^n} \quad \left(\frac{1}{x}\right)^n \quad AND \quad \frac{1}{x^{-n}} = x^n$$

EX.: 4^{-2}

$= \frac{1}{4^2}$

$= \frac{1}{16}$

EX.: $\frac{1}{5^{-2}}$

$= 5^2$

$= 25$

VOCABULARY:

1. RECIPROCAL: Two numbers whose product is 1.

$5 \times \left(\frac{1}{5}\right) = \frac{5}{5}$

$= 1$

EX.: 2 and ½ are reciprocals.

We build on our understanding of powers to work with negative exponents.

For example:

$$5^{-2} \cdot 5^{2}$$
$$= 5^{-2+2}$$
$$= 5^{0}$$
$$= 1$$

This means that 5^{-2} and 5^{2} are RECIPROCALS! (Their product equals 1...)

If...

$$5^{-2} \cdot 5^{2} = 1$$

... then...

$$5^{-2} \cdot 25 = 1$$

... and this must actually mean...

$$\frac{1}{25} \cdot 25 = 1$$

... so...

5^{-2} must be equal to $\frac{1}{25}$ or $\frac{1}{5^2}$!!!

Another scenario based on exponent laws:

$$5^{-2} \cdot \frac{1}{5^{-2}}$$
$$= \frac{5^{-2}}{5^{-2}}$$
$$= 5^{-2-(-2)}$$
$$= 5^{-2+2}$$
$$= 5^{0}$$
$$= 1$$

This means that 5^{-2} and $\frac{1}{5^{-2}}$ are also RECIPROCALS!

(Their product also equals 1...)

"THE OLD FASHIONED WAY"... :)

The way we used to teach the negative exponent rule:

$2^3 = 8$

$2^2 = 4$

$2^1 = 2$

$2^0 = 1$

$2^{-1} = \frac{1}{2}$

$2^{-2} = \frac{1}{4}$

$2^{-3} = \frac{1}{8}$

EXAMPLE:

a) 3^{-2}

$\left(\frac{1}{3}\right)^2$

$\frac{1}{9}$

b) 0.3^{-4}

$\left(\frac{3}{10}\right)^{-4}$

$\left(\frac{10}{3}\right)^4$

$\frac{10000}{81}$

Basically, remember to take the reciprocal of the ENTIRE base and change the negative exponent to a positive exponent.

EX.: $\left(-\frac{3}{4}\right)^{-3}$ $=\left(-\frac{4}{3}\right)^3$

$-\frac{1}{2}$ $-\frac{1}{2}$ $\cancel{-\frac{1}{2}}$

$=\frac{-64}{27}$

YOU TRY!

Evaluate each power.

a) 7^{-2}

$\left(\frac{1}{7}\right)^2$

$\frac{1}{49}$

b) $\left(\frac{10}{3}\right)^{-3}$

$\left(\frac{3}{10}\right)^3$

$\frac{27}{1000}$

c) $(-1.5)^{-3}$

$\left(-\frac{3}{2}\right)^{-3}$

$\left(-\frac{2}{3}\right)^3$

$\frac{-8}{27}$

EXAMPLE:

Evaluate each power without using a calculator.

a) $8^{-\frac{2}{3}}$

$\left(\frac{1}{8}\right)^{\frac{2}{3}}$

$\left(\frac{1}{2}\right)^2$

$\frac{1}{4}$

b) $\left(\frac{9}{16}\right)^{-\frac{3}{2}}$

$\left(\frac{16}{9}\right)^{\frac{3}{2}}$

$\left(\frac{4}{3}\right)^3$

$\frac{64}{27}$

YOU TRY!

Evaluate each power without using a calculator.

a) $16^{-\frac{5}{4}}$

$\left(\frac{1}{16}\right)^{\frac{5}{4}}$

$\left(\frac{1}{2}\right)^{5}$

$\frac{1}{32}$

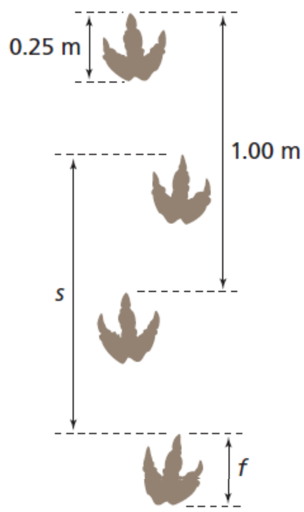
b) $\left(\frac{25}{36}\right)^{-\frac{1}{2}}$

$\left(\frac{36}{25}\right)^{\frac{1}{2}}$

$\frac{6}{5}$

EXAMPLE:

Paleontologists use measurements from fossilized dinosaur tracks and the formula $v = 0.155\,s^{\frac{5}{3}}f^{-\frac{7}{6}}$ to estimate the speed at which the dinosaur travelled. In the formula, v is the speed in metres per second, s is the distance between successive footprints of the same foot, and f is the foot length in metres.
Use the measurements in the diagram to estimate the speed of the dinosaur.



SOLUTION

Use the formula: $v = 0.155\,s^{\frac{5}{3}}f^{-\frac{7}{6}}$
Substitute: $s = 1$ and $f = 0.25$

$v = 0.155\,(1)^{\frac{5}{3}}(0.25)^{-\frac{7}{6}}$

$v = 0.155\,(0.25)^{-\frac{7}{6}}$

$v = 0.7811\ldots$

The dinosaur travelled at approximately 0.8 m/s.

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0.155(0.25)^(-7/6)
0.781151051
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YOU TRY!

Use the formula $v = 0.155\, s^{\frac{5}{3}} f^{-\frac{7}{6}}$ to estimate the speed of a dinosaur when $s = 1.5$ and $f = 0.3$.

$V = 0.155\,(1.5)^{\frac{5}{3}}\,(0.3)^{-\frac{7}{6}}$

Answer: approximately 1.2 m/s

CONCEPT REINFORCEMENT:

FPCM 10:

Page 233: #3 TO #14

Page 234: #15 TO #17ab and #18 TO #20

9. b) $(0.09)^{-\frac{1}{2}}$

$= \left(\frac{9}{100}\right)^{-\frac{1}{2}}$

$= \left(\frac{100}{9}\right)^{\frac{1}{2}}$

$= \frac{10}{3}$

e) $(-0.027)^{-\frac{2}{3}}$

$\left(-\frac{27}{1000}\right)^{-\frac{2}{3}}$

$\left(-\frac{1000}{27}\right)^{\frac{2}{3}}$

$\left(-\frac{10}{3}\right)^{2}$

$\frac{100}{9}$

QUIZ PREPARATION - SECTIONS 4.4 & 4.5:
(Fractional Exponents and Radicals; Negative Exponents and Reciprocals)

***FPCM 10*:**
Page 236: #1 to #8 (ALL!)