

February 13, 2020

UNIT 1: ROOTS AND POWERS

**SECTION 4.6:
APPLYING THE
EXPONENT LAWS**



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NUMBERS, RELATIONS AND FUNCTIONS 10

WHAT'S THE POINT OF TODAY'S LESSON?

We will continue working on the NRF 10 Specific Curriculum Outcome (SCO) "Algebra and Numbers 3" OR "AN3" which states:

"Demonstrate an understanding of powers with integral and rational exponents."



What does THAT mean???

SCO AN3 means that we will:

- * apply the 6 exponent laws you learned in grade 9:

$$a^0 = 1$$

$$(a^m)(a^n) = a^{m+n}$$

$$a^m \div a^n = a^{m-n}$$

$$(a^m)^n = a^{mn}$$

$$(ab)^m = a^m b^m$$

$$(a \div b)^n = a^n \div b^n$$

- * use patterns to explain $a^{-n} = \frac{1}{a^n}$ and $a^{\frac{1}{n}} = \sqrt[n]{a}$

- * apply all exponent laws to evaluate a variety of expressions
- * express powers with rational exponents as radicals and vice versa
- * identify and correct errors in work that involves powers



EXPONENT LAWS (separate sheet):

1. Zero Exponent Law: $a^0 = 1$
2. Product of Powers: $(a^m)(a^n) = a^{m+n}$
3. Quotient of Powers: $a^m \div a^n = a^{m-n}$
4. Power of a Power: $(a^m)^n = a^{mn}$
5. Power of a Product: $(ab)^m = a^m b^m$
6. Power of a Quotient: $(a \div b)^n = a^n \div b^n$

7. MULTIPLICATION PROPERTY OF RADICALS:

$$\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$$

$$\begin{aligned} \text{EX.:} \quad & \sqrt{24} \quad (\text{Factors: } 1, 2, 3, 4, 6, 8, 12, 24) \\ & = \sqrt{4 \cdot 6} \\ & = \sqrt{4} \cdot \sqrt{6} \\ & = 2 \cdot \sqrt{6} \\ & = 2\sqrt{6} \quad (\text{MIXED RADICAL}) \end{aligned}$$

$$\begin{aligned} \text{EX.:} \quad & \sqrt[3]{24} \quad (\text{ENTIRE RADICAL}) \\ & = \sqrt[3]{8 \cdot 3} \\ & = \sqrt[3]{8} \cdot \sqrt[3]{3} \\ & = 2 \sqrt[3]{3} \\ & = 2\sqrt[3]{3} \end{aligned}$$

8. POWERS WITH RATIONAL EXPONENTS WITH A NUMERATOR OF 1:

$$x^{\frac{1}{n}} = \sqrt[n]{x}$$

$$\begin{aligned} \text{EX.:} \quad & 8^{\frac{1}{3}} \\ & = \sqrt[3]{8} \\ & = 2 \end{aligned}$$

9. POWERS WITH RATIONAL EXPONENTS:

$$\begin{array}{ccc}
 \text{EXPONENT} & & \text{EXPONENT} \\
 \swarrow & & \swarrow \\
 x^{\frac{m}{n}} & = & \left(x^{\frac{1}{n}}\right)^m \\
 \uparrow & & \uparrow \\
 \text{INDEX} & & \text{INDEX} \\
 & = & \left(\sqrt[n]{x}\right)^m
 \end{array}
 \quad \text{AND} \quad
 \begin{array}{ccc}
 \text{EXPONENT} & & \text{EXPONENT} \\
 \swarrow & & \swarrow \\
 x^{\frac{m}{n}} & = & \left(x^m\right)^{\frac{1}{n}} \\
 \uparrow & & \uparrow \\
 \text{INDEX} & & \text{INDEX} \\
 & = & \sqrt[n]{x^m}
 \end{array}$$

EX.: Evaluate $16^{\frac{3}{2}}$.

$$\begin{array}{ccc}
 \begin{array}{l}
 \frac{3}{2} \text{ (EXPONENT)} \\
 16^{\frac{3}{2}} \text{ (INDEX)}
 \end{array} & \text{OR} & \begin{array}{l}
 \frac{3}{2} \text{ (EXP.)} \\
 16^{\frac{3}{2}} \text{ (INDEX)}
 \end{array} \\
 = \left(\sqrt[2]{16}\right)^3 & & = \sqrt[2]{16^3} \\
 = 4^3 & & = \sqrt{4096} \\
 = 64 & & = 64
 \end{array}$$

10. POWERS WITH NEGATIVE EXPONENTS:

$$x^{-n} = \frac{1}{x^n} \quad \text{AND} \quad \frac{1}{x^{-n}} = x^n$$

EX.: 4^{-2}
 $= \frac{1}{4^2}$
 $= \frac{1}{16}$

EX.: $\frac{1}{5^{-2}}$
 $= 5^2$
 $= 25$

Basically, remember to take the reciprocal of the ENTIRE base and change the negative exponent to a positive exponent.

EX.: $\left(-\frac{3}{4}\right)^{-3}$

LAST MINUTE QUESTIONS???
(page 236, #1 to #8)

QUIZ TIME!
(Sections 4.4 and 4.5 - 10 to 15 min.)

4.6 - APPLYING EXPONENT LAWS:

Let's build gradually on what we knew in grade 9...

For example:

$$\begin{aligned} & (2^2)(2^6) \\ = & 2^8 \\ = & 256 \end{aligned}$$

APPLYING EXPONENT LAWS:

Let's build gradually on what we knew in grade 9...

For example:

$$\begin{aligned}
 & (2^2)(2^{-6}) \\
 = & 2^{-4} \\
 = & \frac{1}{2^4} \\
 = & \frac{1}{16}
 \end{aligned}$$

APPLYING EXPONENT LAWS:

Let's build gradually on what we knew in grade 9...

For example:

$$\begin{aligned}
 & \left[(2^2)(2^{-6}) \right]^{\frac{-3}{2}} \\
 = & (2^{-4})^{\frac{-3}{2}} && 2^{-4\left(\frac{3}{2}\right)} \\
 = & \left(\frac{1}{2^4} \right)^{\frac{-3}{2}} && 2^6 \\
 & && 64 \\
 = & \left(\frac{1}{16} \right)^{\frac{-3}{2}} \\
 = & 16^{\frac{3}{2}} \\
 = & (\sqrt{16})^3 \\
 = & 4^3 \\
 = & 64
 \end{aligned}$$

EXAMPLE:

a) $0.3^{-3} \cdot 0.3^5$

b) $\left[\left(-\frac{3}{2} \right)^{-4} \right]^2 \cdot \left[\left(-\frac{3}{2} \right)^2 \right]^3$

c) $\frac{(1.4^3)(1.4^4)}{1.4^{-2}}$

d) $\left(\frac{7^{\frac{2}{3}}}{7^{\frac{1}{3}} \cdot 7^{\frac{5}{3}}} \right)^6$

SOLUTIONS:

$$\begin{aligned} \text{a) } 0.3^{-3} \cdot 0.3^5 &= 0.3^{(-3) + 5} \\ &= 0.3^2 \end{aligned}$$

$$\begin{aligned} \text{b) } \left[\left(-\frac{3}{2} \right)^{-4} \right]^2 \cdot \left[\left(-\frac{3}{2} \right)^2 \right]^3 &= \left(-\frac{3}{2} \right)^{-8} \cdot \left(-\frac{3}{2} \right)^6 \\ &= \left(-\frac{3}{2} \right)^{-2} \\ &= \left(-\frac{2}{3} \right)^2 \end{aligned}$$

$$\begin{aligned}
 \text{c) } & \frac{(1.4^3)(1.4^4)(1.4)^2}{1.4^{-2}} \\
 &= \frac{1.4^{3+4}}{1.4^{-2}} \\
 &= \frac{1.4^7}{1.4^{-2}} \\
 &= 1.4^{7 - (-2)} \\
 &= 1.4^9
 \end{aligned}$$

$$\begin{aligned}
 \text{d) } & \left(\frac{7^{\frac{2}{3}}}{7^{\frac{1}{3}} \cdot 7^{\frac{5}{3}}} \right)^6 \\
 &= \left(\frac{7^{\frac{2}{3}}}{7^{\frac{6}{3}}} \right)^6 \\
 &= \left(7^{\frac{2}{3} - \frac{6}{3}} \right)^6 \\
 &= \left(7^{-\frac{4}{3}} \right)^6 \\
 &= 7^{-\frac{24}{3}} \\
 &= 7^{-8} \\
 &= \frac{1}{7^8}
 \end{aligned}$$

YOU TRY!

$$\begin{aligned}
 \text{a) } & (0.8)^{-5} \\
 & \frac{1}{0.8^5} \left\{ \begin{array}{l} \left(\frac{4}{5}\right)^{-5} \\ \left(\frac{5}{4}\right)^5 \end{array} \right.
 \end{aligned}$$

$$\begin{aligned}
 \text{c) } & \frac{(1.5)^{15}}{1.5^5} \\
 & 1.5^{10}
 \end{aligned}$$

$$\text{a) } 0.8^2 \cdot 0.8^{-7}$$

$$\text{b) } \left[\left(-\frac{4}{5} \right)^2 \right]^{-3} \div \left[\left(-\frac{4}{5} \right)^4 \right]^{-5}$$

$$\text{c) } \frac{(1.5^{-3})^{-5}}{1.5^5} \quad \left(-\frac{4}{5} \right)^{-6} \div \left(-\frac{4}{5} \right)^{-20}$$

$$\frac{9^{\frac{5}{4}} \cdot 9^{-\frac{1}{4}}}{9^{\frac{3}{4}}} \quad \left(-\frac{4}{5} \right)^{-6 - (-20)}$$

$$\text{d) } \frac{9^{\frac{5}{4}} \cdot 9^{-\frac{1}{4}} \cdot 9^{-\frac{3}{4}}}{9^{\frac{1}{4}}} \quad \left(-\frac{4}{5} \right)^4$$

$$\begin{aligned}
 & 9^{\frac{5}{4} - \frac{1}{4} - \frac{3}{4}} \\
 & 9^{\frac{1}{4}}
 \end{aligned}$$

$$[\text{Answers: a) } \frac{1}{0.8^5} \quad \text{b) } \left(-\frac{4}{5}\right)^{14}$$

$$\text{c) } 1.5^{10} \quad \text{d) } 9^{\frac{1}{4}}]$$

EXAMPLE:

$$\text{a) } (x^3 y^2)(x^2 y^{-4}) = x^3 \cdot x^2 \cdot y^2 \cdot y^{-4}$$

$$= x^5 y^{-2}$$

$$= \frac{x^5}{y^2}$$

$$\text{b) } \frac{10a^5 b^3}{2a^2 b^{-2}}$$

$$= \frac{10}{2} \frac{a^5}{a^2} \frac{b^3}{b^{-2}}$$

$$= 5 a^{5-2} b^{3-(-2)}$$

$$= 5a^3 b^5$$

SOLUTIONS:

$$\begin{aligned}\mathbf{a)} \quad (x^3y^2)(x^2y^{-4}) &= x^3 \cdot y^2 \cdot x^2 \cdot y^{-4} \\ &= x^3 \cdot x^2 \cdot y^2 \cdot y^{-4} \\ &= x^{3+2} \cdot y^{2+(-4)} \\ &= x^5 \cdot y^{-2} \\ &= x^5 \cdot \frac{1}{y^2} \\ &= \frac{x^5}{y^2}\end{aligned}$$

$$\begin{aligned}\mathbf{b)} \quad \frac{10a^5b^3}{2a^2b^{-2}} &= \frac{10}{2} \cdot \frac{a^5}{a^2} \cdot \frac{b^3}{b^{-2}} \\ &= 5 \cdot a^{5-2} \cdot b^{3-(-2)} \\ &= 5 \cdot a^3 \cdot b^5 \\ &= 5a^3b^5\end{aligned}$$

HOW I WOULD ACTUALLY SHOW MY WORK:

a) $(x^3y^2)(x^2y^{-4})$

$$x^3 \cdot x^2 \cdot y^2 \cdot y^{-4}$$

$$x^5 y^{-2}$$

$$\frac{x^5}{y^2}$$

b) $\frac{10a^5b^3}{2a^2b^{-2}}$

YOU TRY!

a) $m^4 \cdot m^2 \cdot n^{-2} \cdot n^3$
 $= m^6 n$

a) $m^4 n^{-2} \cdot m^2 n^3$

b) $\frac{6x^4y^{-3}}{14xy^2} = \frac{6}{14} \frac{x^4}{x} \frac{y^{-3}}{y^2}$
 $= \frac{3}{7} x^3 y^{-5}$

$$= \frac{3x^3}{7y^5}$$

$$[\text{Answers: a) } m^6 n \quad \text{b) } \frac{3x^3}{7y^5}]$$

EXAMPLE:

$$\text{a) } (8a^3b^6)^{\frac{1}{3}} = 8^{\frac{1}{3}}$$

$$\text{b) } (x^{\frac{3}{2}}y^2)(x^{\frac{1}{2}}y^{-1})$$

$$\text{c) } \frac{4a^{-2}b^{\frac{2}{3}}}{2a^2b^{\frac{1}{3}}}$$

$$\text{d) } \left(\frac{100a}{25a^5b^{-\frac{1}{2}}} \right)^{\frac{1}{2}}$$

SOLUTIONS:

$$\begin{aligned}\mathbf{a)} \quad (8a^3b^6)^{\frac{1}{3}} &= 8^{\frac{1}{3}} \cdot a^{3\left(\frac{1}{3}\right)} \cdot b^{6\left(\frac{1}{3}\right)} \\ &= (2^3)^{\frac{1}{3}} \cdot a^1 \cdot b^2 \\ &= 2ab^2\end{aligned}$$

$$\begin{aligned}\mathbf{b)} \quad (x^{\frac{3}{2}}y^2)(x^{\frac{1}{2}}y^{-1}) &= x^{\frac{3}{2}} \cdot x^{\frac{1}{2}} \cdot y^2 \cdot y^{-1} \\ &= x^{\frac{3}{2} + \frac{1}{2}} \cdot y^{2 + (-1)} \\ &= x^2y\end{aligned}$$

$$\begin{aligned}
 \text{c) } \frac{4a^{-2}b^{\frac{2}{3}}}{2a^2b^{\frac{1}{3}}} &= \frac{4}{2} \cdot \frac{a^{-2}}{a^2} \cdot \frac{b^{\frac{2}{3}}}{b^{\frac{1}{3}}} \\
 &= 2 \cdot a^{(-2) - 2} \cdot b^{\frac{2}{3} - \frac{1}{3}} \\
 &= 2 \cdot a^{-4} \cdot b^{\frac{1}{3}} \\
 &= \frac{2b^{\frac{1}{3}}}{a^4}
 \end{aligned}$$

$$\text{d) } \left(\frac{100a}{25a^5b^{-\frac{1}{2}}} \right)^{\frac{1}{2}} = \left(\frac{100}{25} \cdot \frac{a^1}{a^5} \cdot \frac{1}{b^{-\frac{1}{2}}} \right)^{\frac{1}{2}}$$

$$\begin{aligned}
 &= \left(4 \cdot a^{1-5} \cdot b^{\frac{1}{2}} \right)^{\frac{1}{2}} \\
 &= \left(4 \cdot a^{-4} \cdot b^{\frac{1}{2}} \right)^{\frac{1}{2}} \\
 &= 4^{\frac{1}{2}} \cdot a^{(-4)\left(\frac{1}{2}\right)} \cdot b^{\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)} \\
 &= 2 \cdot a^{-2} \cdot b^{\frac{1}{4}} \\
 &= \frac{2b^{\frac{1}{4}}}{a^2}
 \end{aligned}$$

HOW I WOULD ACTUALLY SHOW MY WORK:

a) $(8a^3b^6)^{\frac{1}{3}}$

b) $(x^{\frac{3}{2}}y^2)(x^{\frac{1}{2}}y^{-1})$

HOW I WOULD ACTUALLY SHOW MY WORK:

c) $\frac{4a^{-2}b^{\frac{2}{3}}}{2a^2b^{\frac{1}{3}}}$

d) $\left(\frac{100a}{25a^5b^{-\frac{1}{2}}}\right)^{\frac{1}{2}}$

YOU TRY!

a) $25^{3/2} a^{4(3/2)} b^{2(3/2)}$
 $125 a^6 b^3$

b) $x^3 \cdot x^{-1} \cdot y^{-3/2} \cdot y^{1/2}$
 $x^2 \cdot y^{-1}$
 $x^2 y^{-1}$
 $\frac{x^2}{y}$

a) $(25a^4b^2)^{3/2}$

b) $(x^3y^{-2})(x^{-1}y^{1/2})$

c) $\frac{12x^{-5}y^{5/2}}{3x^{1/2}y^{-1/2}} = 4x^{-5-1/2}y^{5/2+1/2}$
 $= 4x^{-11/2}y^3$

d) $\left(\frac{50x^2y^4}{2x^4y^7}\right)^{1/2} = \frac{4y^3}{x^{11/2}}$

$(25x^{-2}y^{-3})^{1/2}$
 $25^{1/2} x^{-2(1/2)} y^{-3(1/2)}$
 $5x^{-1}y^{-3/2}$
 $\frac{5}{xy^{3/2}}$

[Answers: a) $125a^6b^3$ b) $\frac{x^2}{y}$

c) $\frac{4y^3}{x^{11/2}}$ d) $\frac{5}{xy^{3/2}}$

CONCEPT REINFORCEMENT:

FPCM 10:

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Page 242: #7 to #11, #14 to #17 & #19

Page 243: #21 & #22

UNIT 1 TEST PREPARATION

FPCM 10:

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Page 198: Review Questions (3.1 / 3.2)

Page 201: Practice Test (#1 & #3)

Page 244: Study Guide

Page 245: Skills Summary

Pages 246 to 248: Review Questions

Page 249: Practice Test