UNIT 1: ROOTS AND POWERS

SECTION 3.1: FACTORS AND MULTIPLES OF WHOLE NUMBERS

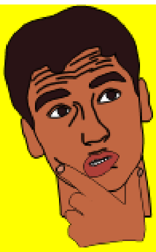
K. Sears

NUMBERS, RELATIONS AND FUNCTIONS 10

WHAT'S THE POINT OF TODAY'S LESSON?

We will begin working on the NRF 10 Specific Curriculum Outcome (SCO) "Algebra and Numbers 1" OR "AN1" which states:

"Demonstrate an understanding of factors of whole numbers by determining the prime factors, greatest common factor, least common multiple, square root and cube root."



What does THAT mean???

SCO AN1 means that we will:

* find the prime factors of whole numbers like 8 (2^3 or $2 \cdot 2 \cdot 2$)
* determine the greatest common factor (GCF) of numbers like 28 and 49 (GCF = 7)
* determine the least common multiple (LCM) of numbers like 6 and 9 (LCM = 18)
* determine if a given whole number is a perfect square, like 25 ($5 \cdot 5$) or a perfect cube, like 8 ($2 \cdot 2 \cdot 2$)
* determine the square root of perfect squares, like 36 ($\sqrt{36} = 6$), and the cube root of perfect cubes, like 64 ($\sqrt[3]{64} = 4$)

WARM UP:

Use prime factorization to determine the

a) GCF of 856, 1200 and 1368

b) LCM of 28, 42 and 63

WARM UP:

a) GCF of 856, 1200 and 1368:

856

2×428

$2 \times 4 \times 107$

$2 \times 2 \times 2 \times 107$

$2^3 \times 107$

1200

12×100

$2 \times 6 \times 10 \times 10$

$2 \times 2 \times 3 \times 2 \times 5 \times 2 \times 5$

$2^4 \times 3 \times 5^2$

1368

2×684

$2 \times 2 \times 342$

$2 \times 2 \times 2 \times 171$

$2 \times 2 \times 2 \times 3 \times 57$

$2 \quad 2 \quad 2 \times 3 \times 3 \times 19$

$2^3 \times 3^2 \times 19$

GCF (Greast Common Factor)

$GCF = 2^3$

$= 8$

$2 \times 2 \times 2 \times 107$

$2 \times 2 \times 2 \times 2 \times 5 \times 5$

$2 \times 2 \times 2 \times 3 \times 3 \times 19$

WARM UP:

b) LCM of 28, 42 and 63:

28

4×7

$2 \times 2 \times 7$

$2^2 \times 7$

42

6×7

$2 \times 3 \times 7$

63

9×7

$3 \times 3 \times 7$

$3^2 \times 7$

LCM (Least Common Multiple)

$LCM = 2^2 \times 3^2 \times 7$

$= 252$

WARM UP:

a) What is the <u>GCF</u> of 340 and 380?

b) Reduce the fraction $\frac{340}{380}$ to its simplest form. $\xrightarrow{\div 10} \frac{34}{38} \div 2 \; \frac{17}{19}$

c) What is the <u>LCM</u> of 340 and 380?

d) What is $\boxed{\frac{15}{340} + \frac{10}{380}}$?

a) <u>GCF</u>

340

$34 \cdot 10$

$\underline{\underline{2}} \cdot 17 \cdot \underline{\underline{2}} \cdot \underline{\underline{5}}$

380

$38 \cdot 10$

$\underline{\underline{2}} \cdot 19 \cdot \underline{\underline{2}} \cdot \underline{\underline{5}}$

$$\begin{aligned} GCF &= 2 \cdot 2 \cdot 5 \\ &= 4 \cdot 5 \\ &= 20 \end{aligned}$$

b) $\frac{340}{380} \begin{matrix} \div 20 \\ \div 20 \end{matrix}$

$= \frac{17}{19}$

c) LCM

$$340 = 34 \cdot 10 = 2 \cdot 17 \cdot 2 \cdot 5 \qquad 380 = 38 \cdot 10 = 2 \cdot 19 \cdot 2 \cdot 5$$

$$2^2 \cdot 5 \cdot 17 \qquad 2^2 \cdot 5 \cdot 19$$

$$\begin{aligned} LCM &= 2^2 \cdot 5 \cdot 17 \cdot 19 \\ &= 4 \cdot 5 \cdot 17 \cdot 19 \\ &= 20 \cdot 17 \cdot 19 \\ &= 340 \cdot 19 \\ &= 6460 \end{aligned}$$

d)

$$\frac{15 \times 19}{340 \times 19} + \frac{10 \times 17}{380 \times 17}$$

$$= \frac{285}{6460} + \frac{170}{6460}$$

$$= \frac{455 \div 5}{6460 \div 5}$$

$$= \frac{91}{1292}$$

VOCABULARY:

prime number: a whole number with exactly two factors, itself and 1.
EX.: 2, 3, 5, 7, 11, 13, 17, 19, 23, and 29 are the first 10 prime numbers.

composite number: a number with three or more factors.
EX.: 8 is a composite number; it has four factors (1, 2, 4 and 8).

prime factor: a prime number that is a factor of a number.
EX.: The prime factors of 30 are 2, 3, and 5.

prime factorization: writing a number as a product of its prime factors.
EX.: The prime factorization of 20 is $2 \cdot 2 \cdot 5$ or $2^2 \cdot 5$.

USING FACTOR TREES FOR PRIME FACTORIZATION:

3300

$33 \cdot 100$

$3 \cdot 11 \cdot 4 \cdot 25$

$3 \cdot 11 \cdot 2 \cdot 2 \cdot 5 \cdot 5$

$\text{P.F.} = 2 \cdot 2 \cdot 3 \cdot 5 \cdot 5 \cdot 11$

OR

$2^2 \cdot 3 \cdot 5^2 \cdot 11$

USING FACTOR TREES FOR PRIME FACTORIZATION:

YOU TRY! :)

2646

2×1323

$2 \times 3 \times 441$

$2 \times 3 \times 3 \times 147$

$2 \quad 3 \times 3 \times 3 \times 49$

$2 \quad 3 \times 3 \times 3 \times 7 \times 7$

$2 \times 3^3 \times 7^2$

DETERMINING THE GREATEST COMMON FACTOR:

EX.: **Determine the GCF of 12 and 15.**

12: **1, 2, 3, 4, 6, 12**

15: **1, 3, 5, 15**

GCF = 3

DETERMINING THE GREATEST COMMON FACTOR:

EX.: **You try! Determine the GCF of 18 and 24.**

18: **1, 2, 3, 6, 9, 18**

24: **1, 2, 3, 4, 6, 8, 12, 24**

GCF = 6

DETERMINING THE GREATEST COMMON FACTOR FOR LARGER NUMBERS: PRIME FACTORIZATION

EX.: **Determine the GCF of 138 and 198.**

138

2×69

$2 \times 3 \times 23$

198

2×99

$2 \times 9 \times 11$

$2 \times 3 \times 3 \times 11$

$2 \times 3^2 \times 11$

$GCF = 2 \times 3$
$= 6$

$LCM = 2 \times 3^2 \times 11 \times 23$
$= 4554$

Once you have the prime factorization of the numbers, multiply all the prime factors they have in common to determine their GCF.

DETERMINING THE GREATEST COMMON FACTOR FOR LARGER NUMBERS: PRIME FACTORIZATION

EX.: **You try! Determine the GCF of 126 and 144.**

126

6×21

$2 \times 3 \times 3 \times 7$

$2 \times 3^2 \times 7$

144

12×12

$3 \times 4 \times 3 \times 4$

$3 \times 2 \times 2 \times 3 \times 2 \times 2$

$2^4 \times 3^2$

$GCF = 2 \times 3 \times 3$
$= 18$

$LCM = 2^4 \times 3^2 \times 7$
$= 1008$

DETERMINING THE LOWEST COMMON MULTIPLE:

EX.: **Determine the LCM of 8 and 12.**

8: **8, 16, 24, 32, 40**

12: **12, 24, 36, 48**

LCM = 24

DETERMINING THE LOWEST COMMON MULTIPLE:

EX.: **You try! Determine the LCM of 6 and 10.**

6: **6, 12, 18, 24, 30, 36**

10: **10, 20, 30, 40, 50**

LCM = 30

DETERMINING THE LOWEST COMMON MULTIPLE FOR LARGER NUMBERS OR GROUPS OF NUMBERS: PRIME FACTORIZATION

EX.: Determine the LCM of 18, 20 and 30.

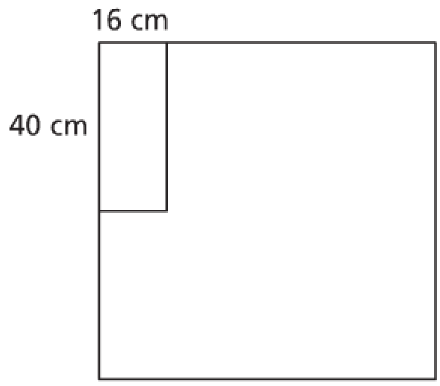
Once you have the prime factorization of the numbers, multiply the greatest power of each of the prime factors to determine the LCM.

DETERMINING THE LOWEST COMMON MULTIPLE FOR LARGER NUMBERS OR GROUPS OF NUMBERS: PRIME FACTORIZATION

EX.: You try! Determine the LCM of 28, 42 and 63.

SOLVING PROBLEMS THAT INVOLVE GCF AND LCM:

EX.: **What is the side length of the SMALLEST square that could be tiled with rectangles that measure 16 cm by 40 cm? (Assume the tiles cannot be cut.) Sketch the square and the rectangular tiles.**



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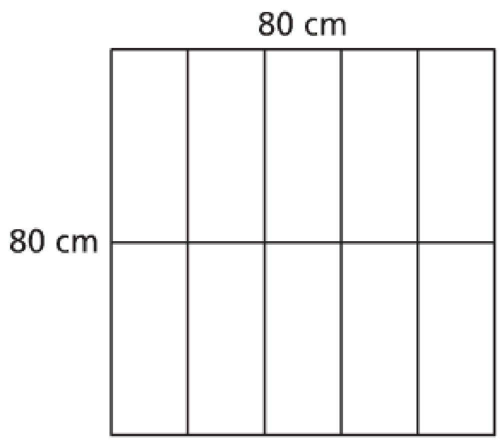
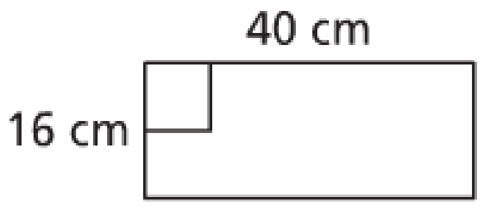


SOLVING PROBLEMS THAT INVOLVE GCF AND LCM:

EX.: **What is the side length of the LARGEST square that could be used to tile a rectangle that measures 16 cm by 40 cm? (Assume the tiles cannot be cut.) Sketch the rectangle and the square tiles.**



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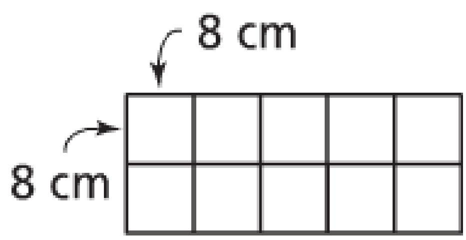

CONCEPT REINFORCMENT:

"FOUNDATIONS AND PRE-CALCULUS - MATHEMATICS 10" (FPCM 10)

pages 140 / 141: #3 TO #13, #15 TO #19a and #20