

Calculations...

$$P = \frac{I}{rt}$$

$$= \frac{56}{(0.05)(3)}$$

$$= \$373.33$$

$$P = \frac{I}{rt} \quad 120/.1/(5/12)$$

$$= \frac{120}{(0.10)\left(\frac{5}{12}\right)}$$

$$= \$2880.00$$

COMPOUND Interest...



Terminology Tango

Click on the picture to verify the match.



COMPOUND Interest

Interest is added to the principal periodically throughout the year. New interest may be paid on the principal plus the interest. The interest rate is stated per annum and is divided by the number of **compounding periods**.

$$A = P \left(1 + \frac{r}{n} \right)^{nt}$$

$$I = A - P$$

A = final value of the investment ...(principal + interest)

P = principal

r = annual interest rate

n = number of compounding periods in a year

t = term of the investment or loan in number of years

EXAMPLE #1: If \$1000 is invested at 8%/a compounded semi-annually for 2 years, how much will the investment be worth?

Using the simple interest formula...

$$I = 1000(0.08)(6/12) = \$40 \quad (\text{after 1st interest period})$$

$$\text{New principal} = 1000 + 40 = \$1040$$

$$I = 1040(0.08)(6/12) = \$41.60 \quad (\text{after 2nd interest period})$$

$$\text{New Principal} = 1040 + 41.60 = \$1081.60$$

$$I = 1081.60(0.08)(6/12) = \$43.26 \quad (\text{after 3rd interest period})$$

$$\text{New Principal} = 1081.60 + 43.26 = \$1124.86$$

$$I = 1124.86(0.08)(6/12) = \$44.99 \quad (\text{after 4th interest period})$$

$$\text{New Principal} = 1124.86 + 44.99 = \$1169.85$$

Using the formula...

$$A = P(1 + i)^{nt} = 1000(1 + 0.08/2)^{2 \times 2} = \$1169.86$$

EXAMPLE #2:

Calculate the final value of an initial investment of \$6000.00. Interest is paid at 4% per annum, compounded semi-annually, for three years.

- A = final value of the investment ...(principal + interest)
- P = principal
- r = annual interest rate
- n = number of compounding periods in a year
- t = term of the investment or loan in number of years

$$A = P \left(1 + \frac{r}{n}\right)^{nt}$$

$$A = 6000 \left(1 + \frac{0.04}{2}\right)^{(2)(3)}$$

$$A = 6000 \cdot (1 + 0.02)^6$$

$$A = 6000 \cdot (1.02)^6$$

$$A = 6756.98$$

EX #3: Maggie invests \$30 000 at 10% /a compounded quarterly for 20 years. Determine...

- How much will this investment be worth?
- How much interest did you earn?

$$\begin{aligned}
 A &= P\left(1 + \frac{r}{n}\right)^{nt} \\
 &= 30\,000\left(1 + \frac{0.10}{4}\right)^{4(20)} \\
 &= \$216\,287.03
 \end{aligned}$$

$$\begin{aligned}
 I &= A - P \\
 &= 216\,287.03 \\
 &\quad - 30\,000 \\
 &= \$186\,287.03
 \end{aligned}$$

EXAMPLE #4...

A keen MVHS student wants to save some money from their summer employment. They decide to take out a Canada Savings Bond which pays 2.5 % interest per year compounded monthly. If the student invests \$850 into the bond, how much interest will they earn if they don't touch the money for 3 years?

$$\begin{aligned}
 A &= P\left(1 + \frac{r}{n}\right)^{nt} \\
 &= 850\left(1 + \frac{0.025}{12}\right)^{12(3)} \\
 &= \$916.13
 \end{aligned}$$

$$\begin{aligned}
 I &= A - P \\
 &= \$916.13 - 850 \\
 &= \$66.13
 \end{aligned}$$

the RULE of 72

ACTIVITY 3.5
THE RULE OF 72

There is a quick way to estimate the time it takes for an investment compounded annually to double in value. This method is called the Rule of 72.

Rule of 72: a quick method of estimating the time it takes for an investment to double in value

To calculate the approximate length of time in years it takes for an investment to double, divide 72 by the annual interest rate expressed as a percentage. If you wanted to know approximately how long it would take an investment with an interest rate of 3.00% per annum to double in value, you would divide 72 by 3.

$$72 \div 3 = 24 \text{ years}$$

Using the Rule of 72, you can estimate that it would take about 24 years for the investment to double in value.

- Using the information above, write a formula that describes the Rule of 72. Use the formula to answer question 2.
- If you wanted to double your money in 10 years, at what rate of interest would you need to invest your money?

SOLUTIONS

- The Rule of 72 can be expressed with the following formula.

Years to double investment = 72 ÷ interest rate

$$y = 72 \div r$$

- $y = 72 \div r$

$$10 = 72 \div r$$

$$r = 72 \div 10$$

$$r = 7.2\%$$

You would need to invest your money at an interest rate of 7.2%.

$$D = \frac{72}{r}$$

$$10 = \frac{72}{r}$$

$$10r = 72$$

$$r = \frac{72}{10}$$

$$= 7.2\%$$

Time to double your money

$$D = \frac{72}{r}$$

→ as a percent

Interest rate = 10%

$$D = \frac{72}{10}$$

$$= 7.2$$

7.2 years to double

DISCUSS THE IDEAS

GUARANTEED INVESTMENT CERTIFICATES

Vyanjana has received a special gift of \$5000.00 from her grandparents, which she plans to invest for the future. She has researched investment options at her bank, and has decided to buy a Guaranteed Investment Certificate (GIC). GICs guarantee that the investor will receive his or her principal as well as a fixed amount of interest.

She has narrowed her choices down to three options:

Option 1: A GIC that offers 1.125% interest per annum, compounded monthly with a one-year term. This GIC cannot be redeemed before the end of the term so Vyanjana will not be able to access her money before the end of the one-year term.

Option 2: A GIC that offers 0.875% interest per annum, compounded monthly, with a one-year term. This GIC can be redeemed before the end of the term, but if Vyanjana wants to access her money before the end of the year, her investment will earn only 0.050% interest per annum.

Option 3: A GIC that offers 1.250% interest per annum, compounded annually, with a one-year term. The GIC cannot be redeemed before the end of the term.

Working in a small group, discuss Vyanjana's investment options.

1. Calculate how much interest Vyanjana would earn with each option. For option 2, calculate how much interest Vyanjana would earn after 6 months and after the full term of the investment.
2. Suggest reasons why Vyanjana might choose each of the three options.



SOLUTIONS

1. Calculate how much interest Vyanjana would earn with each option.

Option 1:

$$A = P \left(1 + \frac{r}{n} \right)^n$$

$$A = \$5000.00 \left(1 + \frac{0.01125}{12} \right)^{12}$$

$$A = \$5056.54$$

$$I = A - P$$

$$I = \$5056.54 - \$5000.00$$

$$I = \$56.54$$

Option 2a:

$$A = P \left(1 + \frac{r}{n} \right)^n$$

$$A = \$5000.00 \left(1 + \frac{0.00875}{12} \right)^{12}$$

$$A = \$5043.93$$

$$I = A - P$$

$$I = \$5043.93 - \$5000.00$$

$$I = \$43.93$$

Option 2b:

$$A = P \left(1 + \frac{r}{n} \right)^n$$

$$A = P \left(1 + \frac{0.0005}{12} \right)^6$$

$$A = \$5001.25$$

$$I = A - P$$

$$I = \$5001.25 - \$5000.00$$

$$I = \$1.25$$

Option 3:

$$A = P \left(1 + \frac{r}{n} \right)^n$$

$$A = \$5000.00 \left(1 + \frac{0.0125}{1} \right)^1$$

$$A = \$5000.00 (1.0125)^1$$

$$A = \$5062.50$$

$$I = A - P$$

$$I = \$5062.50 - \$5000.00$$

$$I = \$62.50$$

ACTIVITY 3.6
THE EFFECT OF DIFFERENT COMPOUNDING PERIODS

1. Calculate the interest and the final value for an investment of \$4000.00 at 3.00% per annum over 2 years for the following different compounding periods. Show your answers in a table like the one below. Use any method you wish to calculate your answers.
2. Which compounding period yields the greatest interest on the investment?

SOLUTIONS

1.

Interest period	Final value of investment (A)	Interest (I)
Annually	$\$4000.00 \left(1 + \frac{0.03}{1} \right)^{(1 \times 2)} \approx \4243.60	\$243.60
Semi-annually	$\$4000.00 \left(1 + \frac{0.03}{2} \right)^{(2 \times 2)} \approx \4245.45	\$245.45
Quarterly	$\$4000.00 \left(1 + \frac{0.03}{4} \right)^{(4 \times 2)} \approx \4246.40	\$246.40
Monthly	$\$4000.00 \left(1 + \frac{0.03}{12} \right)^{(12 \times 2)} \approx \4247.03	\$247.03
Daily	$\$4000.00 \left(1 + \frac{0.03}{365} \right)^{(365 \times 2)} \approx \4247.34	\$247.34

would knowing this affect your choice of

2. The daily compounding period yields the most interest. The annual compounding period yields the least interest. Knowing this, you would choose an investment which is compounded the most times per year to accumulate the most interest.

Hang on.... HOMEWORK!!!



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3.2 Build Your Skills Detailed Solutions.pdf



Compound Interest worksheet



Attachments

Assignment - Simple Interest.doc

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Compound Interest.pdf