Unit 2

Section 1 - Uniform Circular Motion

Uniform Circular Motion

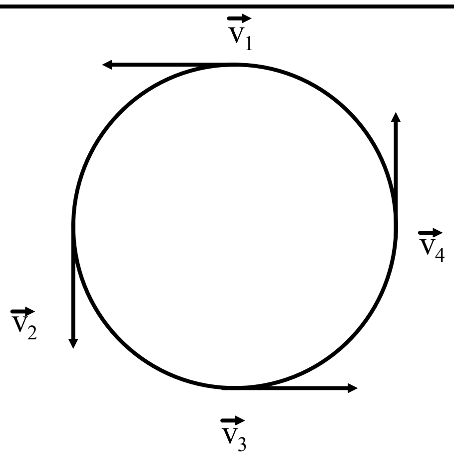
An object with uniform circular motion is an object that travels at constant speed in a circular path.

Horizontal Circular Motion

Imagine you are looking down on a circular track with an object travelling counterclockwise.

CCW

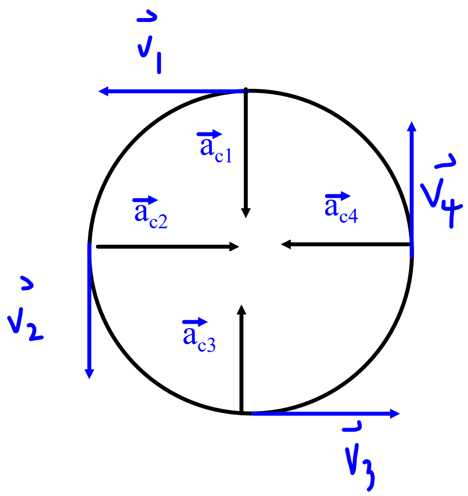
The object's velocity at any point is always drawn tangent to the circular path. The object's speed is constant.



<u>Centripetal acceleration</u> is always directed toward the center of the circular path.

centripetal -> center-seeking

CCW

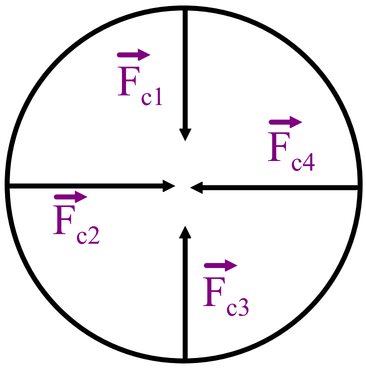


Centripetal Force

Centripetal force is a net force that causes centripetal acceleration. $\mathbf{F_c}$ may be a tension, force of friction, force of gravity or a combination of force components that point along the radial direction.

$\underbrace{\vec{F}_{net} = m\vec{a}}$ (2nd Law)

$\vec{F}_c = m\vec{a}_c$



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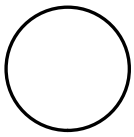
Formulas - Horizontal Circular Motion

Constant Speed: $v = \frac{d}{t}$



$d = C = 2\pi r$

$t = T$ period

(time for 1 revolution)

$$\boxed{v = \frac{2\pi r}{T}}$$

v -> speed (m/s)
r -> radius (m)
T -> period (s)

$\boxed{T = \frac{1}{f}}$ where f -> frequency (Hz) and $1\text{Hz} = 1\text{s}^{-1}$ $\boxed{f = \frac{\#}{t}}$

$\frac{\#\ \text{bwls}}{10s}$

$\frac{\#}{s} = \# s^{-1}$

$V = \frac{2\pi r}{T} = \frac{2\pi r}{\frac{1}{f}} = 2\pi r f$

→ $$\boxed{v = 2\pi r f}$$

v -> speed (m/s)
r -> radius (m)
f -> frequency (Hz)

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a_c -> magnitude of centripetal acceleration (m/s^2)
v -> speed (m/s)
r -> radius (m)
T -> period (s)
f -> frequency (Hz)

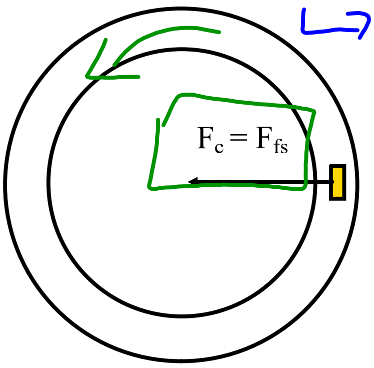
$$\boxed{\boxed{a_c = \frac{v^2}{r}} = \frac{\left(\frac{2\pi r}{T}\right)^2}{r} = \boxed{\frac{4\pi^2 r}{T^2}} = \boxed{4\pi^2 r f^2}}$$

F_c -> magnitude of centripetal force (N)
m -> mass (kg)
a_c -> magnitude of centripetal accleration (m/s ^{2})
v -> speed (m/s)
r -> radius (m)
T -> period (s)
f -> frequency (Hz)

$$\boxed{\boxed{F_c = ma_c} = \frac{mv^2}{r} = \frac{4m\pi^2 r}{T^2} = 4m\pi^2 r f^2}$$

Unbanked Curves and Centripetal Force

↳ Flat



When a car travels around a flat curve (unbanked curve), the centripetal force keeping the car on the curve comes from the static friction between the road and the tires. (It is static and not kinetic friction because the tires are not slipping with respect to the radial direction.) If static friction is insufficient given the speed and radius of the turn, the car will skid off the road.

$$\boxed{F_c = F_{fs}}$$

$$\frac{mv^2}{r} = \mu_s N$$

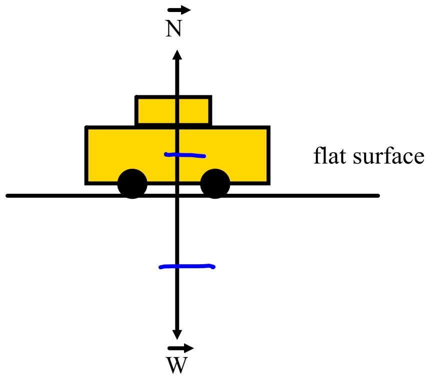
$$\frac{mv^2}{r} = \mu_s W$$

$$\frac{\cancel{m}v^2}{r} = \mu_s \cancel{m}g$$

$$v^2 = \mu_s gr$$

$$\boxed{v = \sqrt{rg\mu_s}}$$

$V = \sqrt{r\mu_s g}$



v -> speed (m/s) μ_s -> coefficient of static friction r -> radius of curve (m) $g = 9.80\ m/s^2$

Example:

If the maximum speed at which a car can safely navigate an unbanked turn of radius 50.0 m is 21.0 m/s, what is the coefficient of static friction? ($\mu_s = 0.900$)

$V = 21.0\ m/s$
$r = 50.0\ m$
$\mu_s = ?$

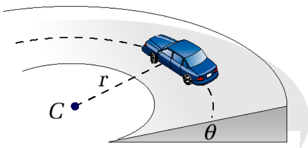
$V = \sqrt{rg\mu_s}$
$V^2 = rg\mu_s$
$\mu_s = \frac{v^2}{rg}$
$\mu_s = 0.900$

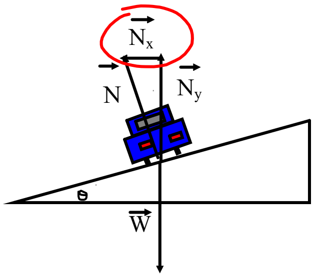
ws

Banked Curves and Centripetal Force

The reliance on friction can be eliminated completely for a given speed if a curve is banked at an angle relative to the horizontal.

* Assume a friction-free banked curve.





We'll need two equations to derive a formula for this type of problem.

$$\boxed{F_c = N_x} \qquad W = N_y$$

$$\frac{mv^2}{r} = N\sin\theta \qquad mg = N\cos\theta$$

$$m = \frac{rN\sin\theta}{v^2} \qquad m = \frac{N\cos\theta}{g}$$

$$\frac{rN\sin\theta}{v^2} = \frac{N\cos\theta}{g}$$

$$\frac{v^2}{gr} = \frac{\sin\theta}{\cos\theta}$$

$$\frac{v^2}{gr} = \tan\theta$$

$$v^2 = gr\tan\theta$$

$$\boxed{v = \sqrt{rg\tan\theta}}$$

Example:

The turns in a track have a maximum radius at the top of 316 m and are banked steeply. If a car travels at a speed of 43 m/s, what is the angle of the curve with respect to the horizontal? Assume the turn is frictionless. (31°)

$r = 316\,m$

$v = 43\,m/s$

θ -- ?

$v = \sqrt{rg\tan\theta}$

$v^2 = rg\tan\theta$

$\tan\theta = \frac{v^2}{rg}$

$\theta = 31°$

WS

Attachments

Lab - Path Projectile Results.jpg

Physics 122 - Circular Motion - Curves.doc

Mass on spring demo.notebook

p122- h proj lab.jpg

p122- h proj lab 2.jpg

p122- h proj lab 3.jpg

Physics 122 - Hdout Projectile Problems.doc

Song - Harry.jpg

Phys - Circular Motion Basic.jpg

Physics 122 - C11 - Circular Motion Problems.doc

Phys - Kepler Graph.jpg

Physics 122 - Worksheet - Unbanked and Banked Curve Problems.docx