

## Curriculum Outcomes:

(SS1) Solve problems and justify the solution strategy using circle properties, including: the perpendicular from the centre of a circle to a chord bisects the chord; the measure of the central angle is equal to twice the measure of the inscribed angle subtended by the same arc; the inscribed angles subtended by the same arc are congruent; a tangent to a circle is perpendicular to the radius at the point of tangency.

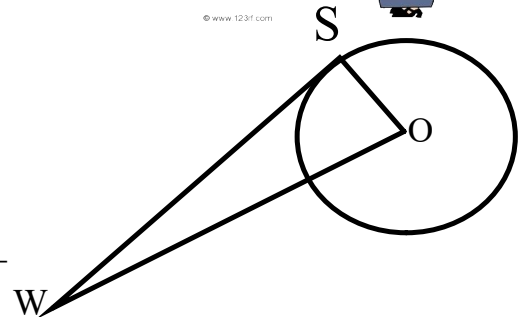
### **Student Friendly:**

How we can use the Chord properties to solve for unknown lengths. (Chord properties go hand and hand with Pythagorean theorem, angle sum of a triangle and isosceles triangles )

# Warm Up



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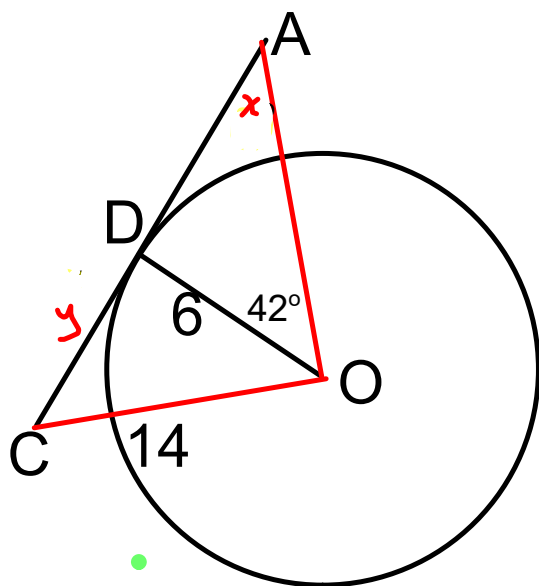


Fill in the blanks

- 1) The Tangent is \_\_\_\_\_
- 2) The center is labeled with the letter \_\_\_\_\_
- 3) The point of tangency is labeled with the letter \_\_\_\_\_
- 4) The radius is the line \_\_\_\_\_
- 5) Find the length of the radius if  $OW = 17$  and  $SW = 9$

# Warm Up

Determine the unknowns:



# Warm Up



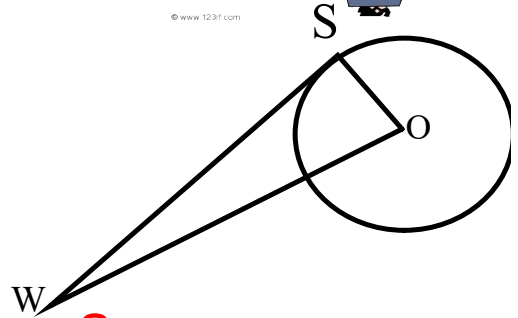
Fill in the blanks

1) The Tangent is SW

2) The center is labeled with the letter O

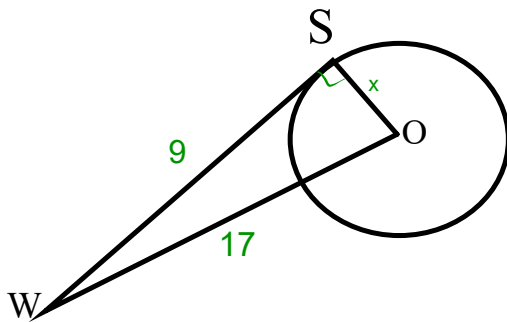
3) The point of tangency is labeled with the letter S

4) The radius is the line OS



SHOW YOUR WORK

5) Find the length of the radius if  $OW = 17$  and  $SW = 9$



$\angle OSW = 90^\circ$  (TangP)

os  $\Rightarrow$  radius  $\Rightarrow$  leg

$$a^2 = c^2 - b^2$$

$$a^2 = 17^2 - 9^2$$

$$a^2 = 289 - 81$$

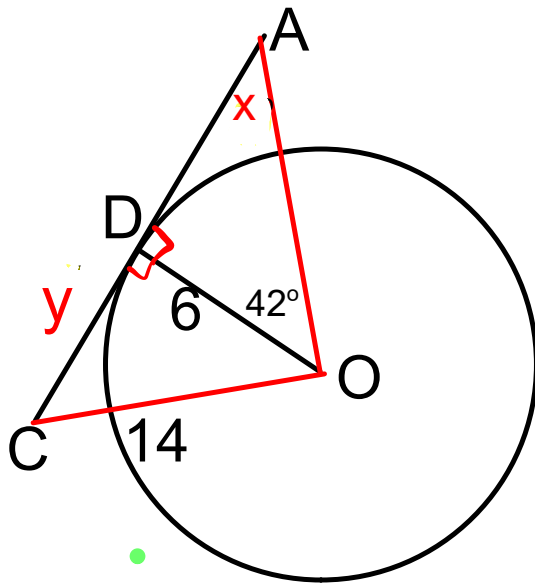
$$a^2 = 208$$

$$a = \sqrt{208}$$

$$a = 14.4$$

# Warm Up

Determine the unknowns:



$$\angle ADO = 90^\circ \text{ (TangP)}$$

$$\angle CDO = 90^\circ \text{ (TangP)}$$

$$X = 180 - 90 - 42$$

$$X = 48^\circ \text{ (SATT)}$$

$$y \Rightarrow \text{leg}$$

$$a^2 = c^2 - b^2$$

$$a^2 = 14^2 - 6^2$$

$$a^2 = 196 - 36$$

$$a^2 = 160$$

$$a = \sqrt{160}$$

$$a = 12.6 \text{ cm}$$

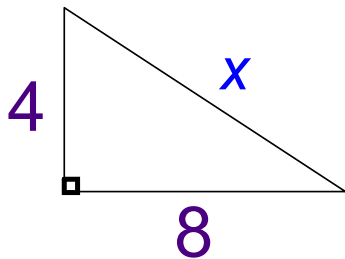
## Calculating with Tangents We First

Identify the  $90^\circ < \_ \_ \_ = 90^\circ$  (Tang P)

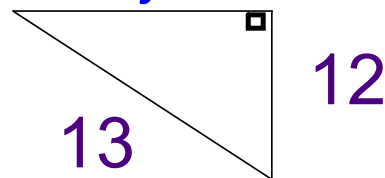
Then we use ...

### 1) Pythagorean Theorem

finding the hypotenuse  $\rightarrow c^2 = a^2 + b^2$



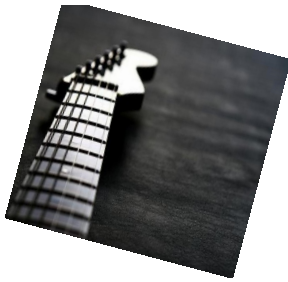
finding a side  $\rightarrow a^2 = c^2 - b^2$



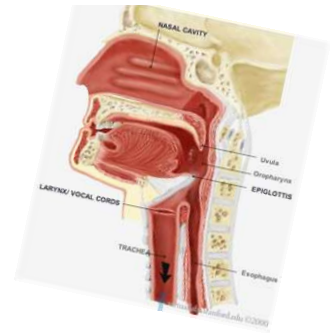
or

### 2) Angle Sum of Triangle

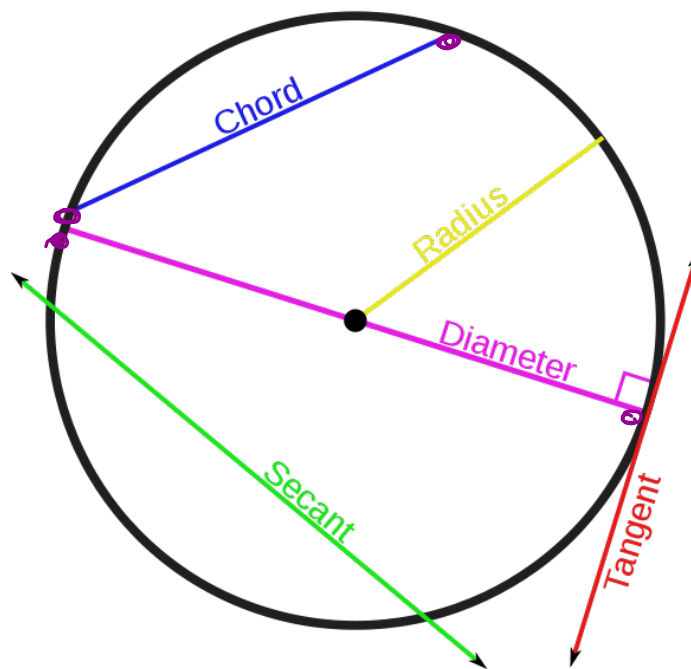
Unknown Angle =  $180^\circ - 90^\circ - \text{known angle}$



## Section 8.2

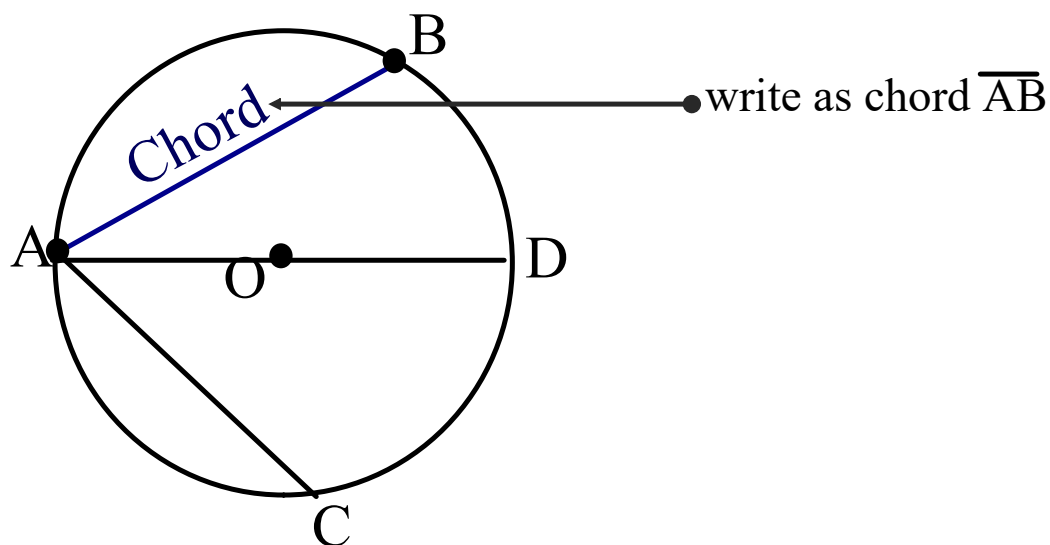
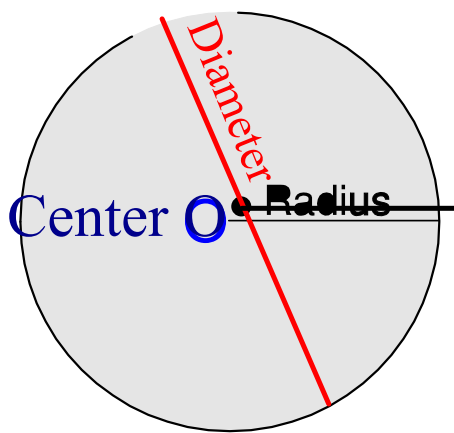


# Properties of Chords in Circles



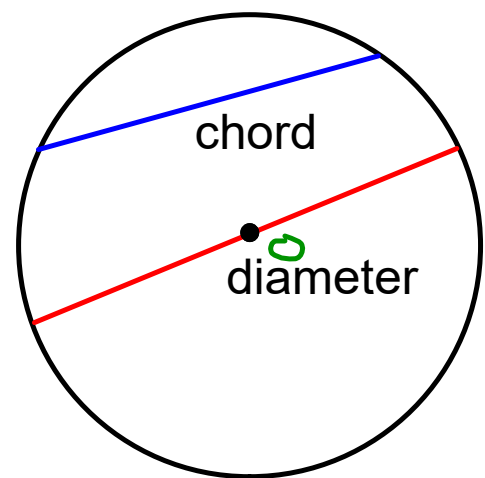
## Properties of Circles & Terminology:

**Circle** - the set of all points that are equidistant from a fixed point.





- A line segment that joins two points on a circle is a chord.
- A diameter of a circle is a chord through the centre of the circle. It's the longest chord.



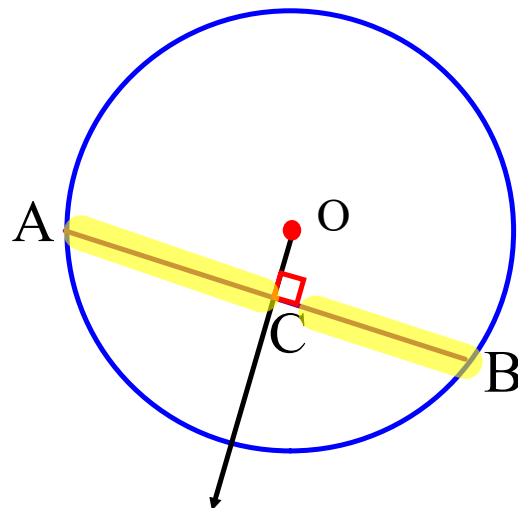
Perpendicular bisector:

→ line that cuts a chord into two equal pieces at  $90^\circ$  angle

### Perpendicular to a Chord Property 1

- A line drawn from the centre of a circle that is perpendicular to a chord bisects the chord. (It cuts the chord into two equal parts.)

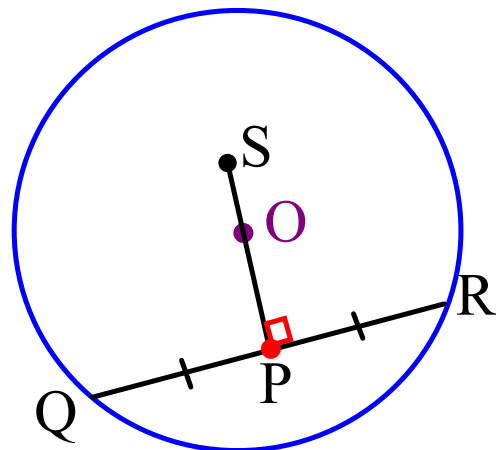
If  $OC$  is perpendicular to  $AB$   
Then  $AC = CB$  (Chord P1)



## Perpendicular to a Chord Property 2

- The perpendicular bisector of a chord in a circle passes through the centre of the circle.

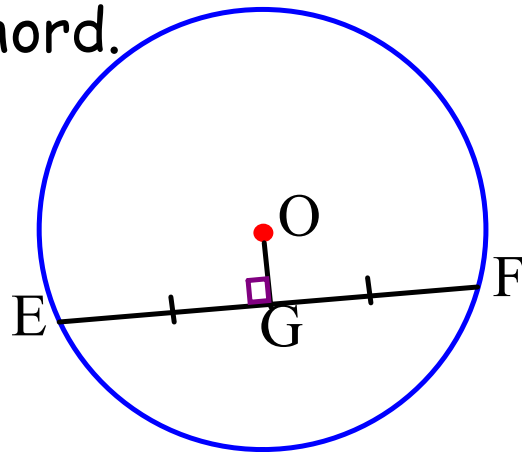
A perpendicular bisector of a chord must go through the centre.



### Perpendicular to a Chord Property 3

- A line that joins the centre of a circle and the midpoint of a chord is perpendicular to the chord.

If  $O$  is the centre and  
 $EG = GF$ , then  
 $\angle OGE = \angle OGF = 90^\circ$ .  
(Chord P3)



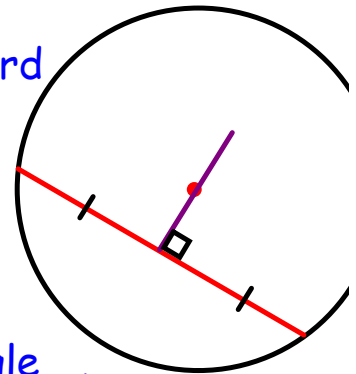
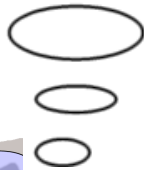
# STOP!



Yes!  
We know  
that a

Aren't they  
all saying the  
same thing?

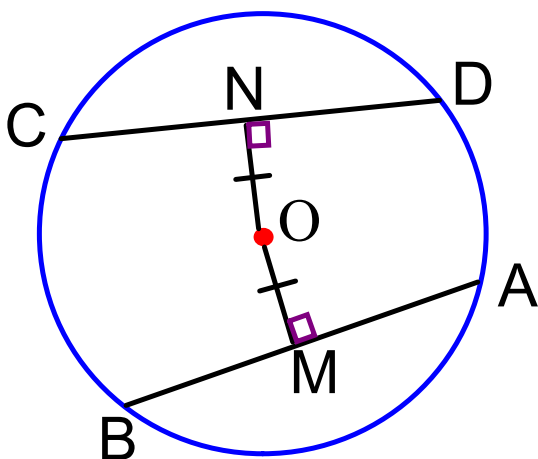
perpendicular bisector of a cord



hits the cord at a 90 degree angle ,  
the chord is cut in two equal pieces,  
and passes through the centre.

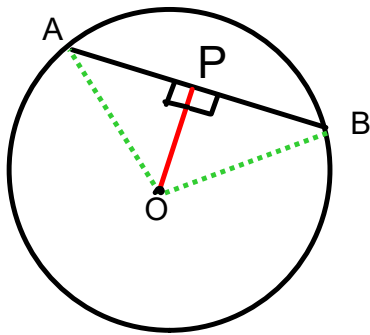
### Perpendicular to a Chord Property 4

- Two chord that are equal distance from the center must be the same



If  $OM = ON$ ,  
then  $AB = CD$   
OR  
If  $AB = CD$ ,  
then  $OM = ON$

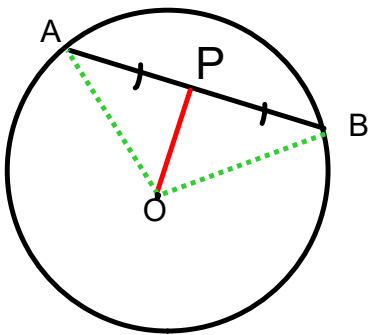
## Chord Properties:



If given the  $90^\circ$   
 $\angle OPB = \angle OPA = 90^\circ$  (Given)

then

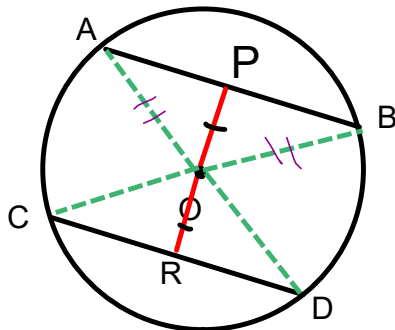
$$AP = PB \text{ (Chord P 1)}$$



If the chord cut in two equal  
 pieces  $AP = PB$  (Given)

then

$$\angle APO = \angle BPO = 90^\circ \text{ (Chord P 3)}$$



If given the perpendicular  
 bisectors

$$OP = OR \text{ (Given)}$$

then

$$AB = CD \text{ (Chord P 4)}$$

If given two equal chords

$$AB = CD \text{ (Given)}$$

then

$$OP = OR \text{ (Chord P 4)}$$

To Solve use:

$$\text{Angle} = \text{___}^\circ \text{ (SATT) or (ITT)}$$

$$\text{Side} = \text{___} \text{ cm (Pythagorean theorem)}$$

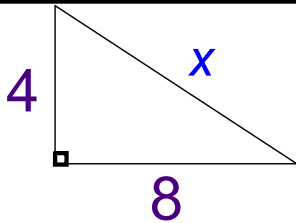


## Working With Chords Lengths We Only Use ...

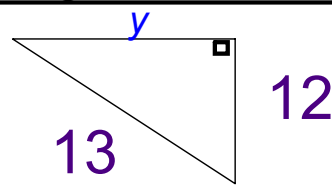
\* **Note: the only reason they give you diameter is so you can use the radius** \*

### 1) Pythagorean Theorem

hypotenuse  $c^2 = a^2 + b^2$



leg  $a^2 = c^2 - b^2$



or

### 2) Angle Sum of Triangle (SATT)

Unknown Angle =  $180^\circ - 90^\circ - \text{known angle}$

or

### 3) Isosceles Triangle (ITT)

$OT = OP \Rightarrow \text{radii}$

$\angle OTP = \angle OPT$  (ITT)

Isosceles Triangle Theorem (ITT)

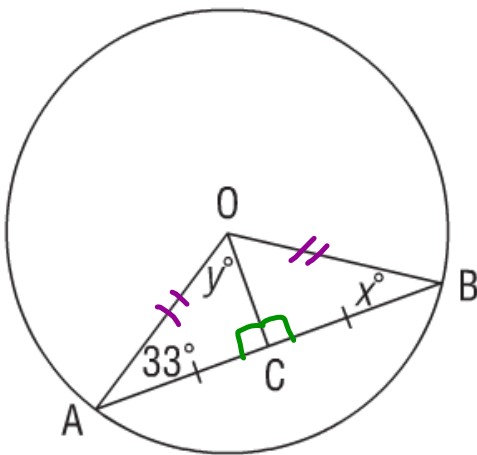
-Base angles in an isosceles triangles are equal

$b = 180^\circ - a^\circ - a^\circ$

$a^\circ = \frac{180^\circ - b^\circ}{2}$

## Determining the Measure of Angles in a Triangle

Example #1. Determine the values of  $x^\circ$  and  $y^\circ$ .



**$AC = BC$  (Given)**

**$\angle OCA = \angle OCB = 90^\circ$  (Chord P)**

**$OB = OA$  (Radii)**

**$y^\circ = \angle AOC = 57^\circ$  (SATT)**

**$x^\circ = \angle OBC = 33^\circ$  (ITT)**

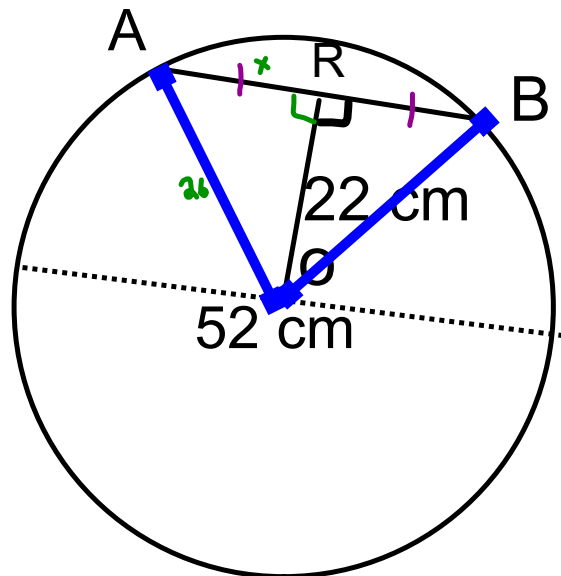
$y^\circ = \angle AOC$

What is the length of the cord AB?

$\angle ORB = \angle ORA = 90^\circ$  (GIVEN)

$AR = BR$  (Chord P)

$OB = OA$  (Radii) = 26 cm



$AR \Rightarrow \text{leg}$

$$a^2 = c^2 - b^2$$

$$a^2 = 26^2 - 22^2$$

$$a^2 = 676 - 484$$

$$a^2 = 192$$

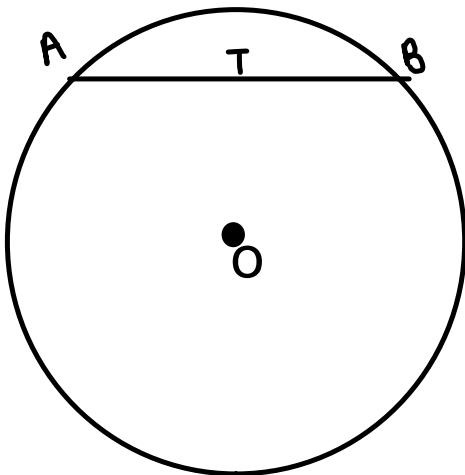
$$a = \sqrt{192}$$

$$a = 13.9$$

$$\begin{aligned} AB &= 2(13.9) \\ &= 27.8 \text{ cm} \end{aligned}$$

## EXAMPLE...

A chord  $AB$  that is 16.8 cm in length, is drawn in a circle that has a diameter of 21 cm. How far is the chord from the center of the circle? ↪



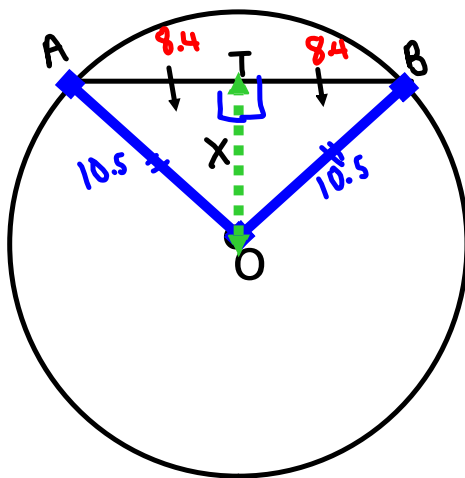
$$\angle \text{---} = \angle \text{---} = 90^\circ$$

$$\text{---} = \text{---} = \text{--- cm (Radii)}$$

$$\text{---} = \text{---} = \text{--- cm (Chord P)}$$

## EXAMPLE...

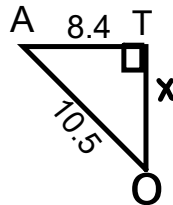
A chord  $ATB$  that is 16.8 cm in length, is drawn in a circle that has a diameter of 21 cm. How far is the chord from the center of the circle?  $\hookrightarrow r = 10.5$



$$\angle OTA = \angle OTB = 90^\circ \text{ (Chord P)}$$

$$OB = OA = 10.5 \text{ cm (Radii)}$$

$$AT = BT = 8.4 \text{ cm (Chord P)}$$



$$OT \Rightarrow \text{leg}$$

$$a^2 = c^2 - b^2$$

$$a^2 = 10.5^2 - 8.4^2$$

$$a^2 = 110.25 - 70.56$$

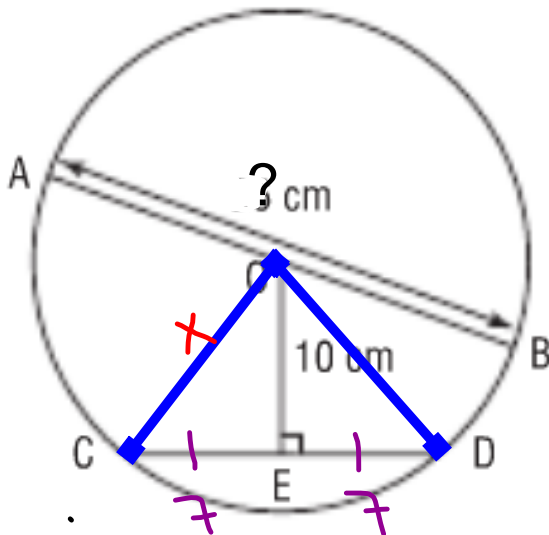
$$a^2 = 39.69$$

$$a = \sqrt{39.69}$$

$$a = 6.3 \text{ cm}$$

## Using the Pythagorean Theorem in a Circle

Example #2. What is the length of chord AB, to the nearest tenth, if CD is 14 cm?

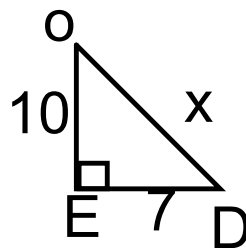


$\angle OED = 90^\circ$  (GIVEN)

$CE = ED$  (Chord P)

$OC = OD$  (Radii)

$OD \Rightarrow \text{Hyp}$



$$c^2 = a^2 + b^2$$

$$c^2 = 7^2 + 10^2$$

$$c^2 = 49 + 100$$

$$c^2 = 149$$

$$c = \sqrt{149}$$

$$c = 12.2 \text{ cm}$$

$AB \Rightarrow \text{Diameter}$

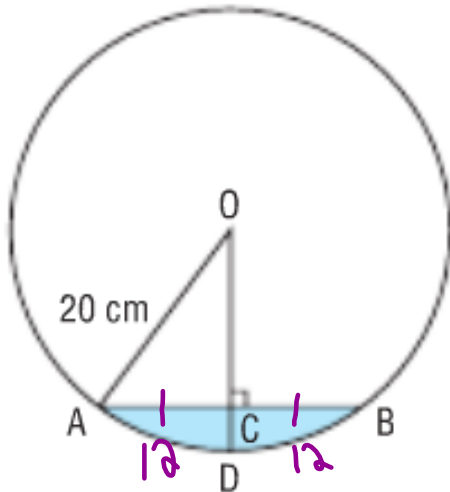
$$= 2(\text{radius})$$

$$= 2(12.2)$$

$$= 24.4 \text{ cm}$$

## Solving Problems Using the Property of a Chord and its Perpendicular

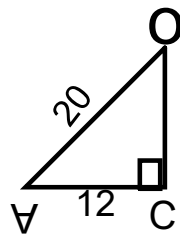
Example #3. Determine the length of CD, if the chord AB = 24cm



$$\angle OCA = \angle OCB = 90^\circ \text{ (GIVEN)}$$

$$OD = OB = OA = 20 \text{ cm (Radii)}$$

$$AB = BC = 12 \text{ (Chord P)}$$



OT  $\Rightarrow$  leg

$$a^2 = c^2 - b^2$$

$$a^2 = 20^2 - 12^2$$

$$a^2 = 400 - 144$$

$$a^2 = 256$$

$$a = \sqrt{256}$$

$$a = 16 \text{ cm}$$

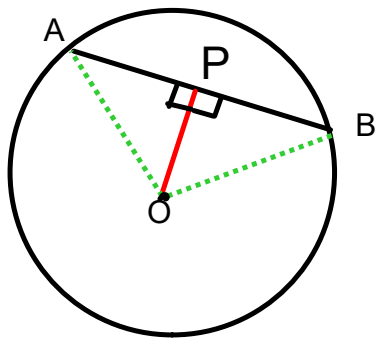
$$x \Rightarrow CD$$

$$= OD - OC$$

$$= 20 - 16$$

$$= 4 \text{ cm}$$

## Chord Properties:

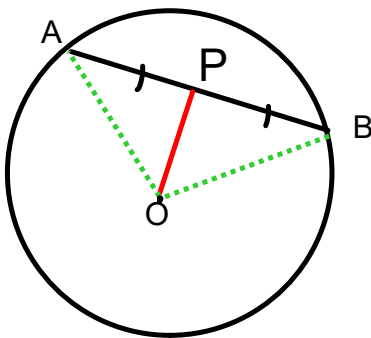


If given the  $90^\circ$

$$\angle OPB = \angle OPA = 90^\circ \text{ (Given)}$$

then

$$AP = PB \text{ (Chord P 1)}$$

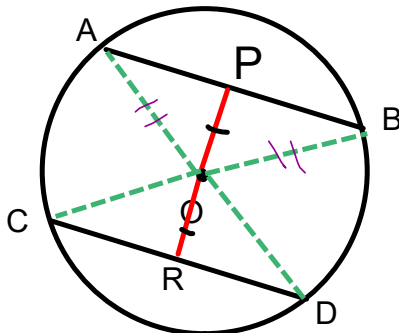


If the chord cut in two equal pieces

$$AP = PB \text{ (Given)}$$

then

$$\angle APO = \angle BPO = 90^\circ \text{ (Chord P 3)}$$



If given the perpendicular bisectors

$$OP = OR \text{ (Given)}$$

then

$$AB = CD \text{ (Chord P 4)}$$

If given two equal chords

$$AB = CD \text{ (Given)}$$

then

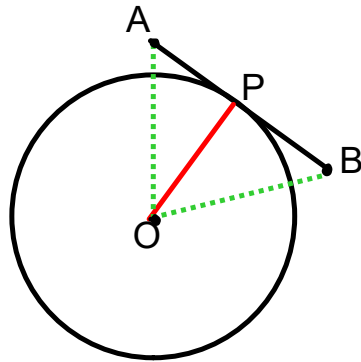
$$OP = OR \text{ (Chord P 4)}$$

To Solve use:

$$\text{Angle} = \underline{\quad}^\circ \text{ (SATT) or (ITT)}$$

$$\text{Side} = \underline{\quad} \text{ cm (Pythagorean theorem)}$$





Tangent Properties:

$$\angle APO = 90^\circ \text{ (Tang P)}$$

$$\angle BPO = 90^\circ \text{ (Tang P)}$$

To Solve use:

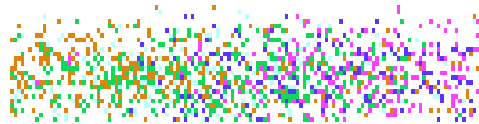
$$\text{Angle} = \underline{\hspace{2cm}}^\circ \text{ (SATT) } 180^\circ - 90^\circ - \text{given angle}$$

$$\text{Side} = \underline{\hspace{2cm}} \text{ cm (Pythagorean theorem)}$$

$$c = \sqrt{a^2 + b^2} \text{ Hypotenuse}$$

$$a = \sqrt{c^2 - b^2} \text{ Leg}$$

# Class/Homework



-click on the "Homework" link on my teachers page for optional review questions

- If you have any questions you can contact me on the

Remind app

or

through email:

[melanie.burns@nbed.nb.ca](mailto:melanie.burns@nbed.nb.ca)

