Slope as a Rate of Change

Rate of Change - a change in one quantity relative to the change in another quantity

$$\text{Rate of change} = \frac{\text{Vertical change}}{\text{Horizontal change}} = \frac{\text{Rise}}{\text{Run}} = \text{Slope}$$

Example
When you earn a wage, the money you receive is related to the number of hours you worked. How could you represent this rate of change?

$$\text{Rate of Change} = \frac{\text{money earned}}{\text{time worked}}$$

Oct 15-12:44 PM

Rate of Change from a Graph

Example

One year ago, Becky started working at a grocery store. She put $100 per month into her bank account. Becky already had $200 in the account. The table shows the growth in Becky's account

The horizontal values or 'the run' The vertical values or 'the rise'

Time (months)	Amount
0	200
1	300
2	400
3	500
4	600
5	700

a) Plot the data in the table on a graph

b) Determine the slope of the line that would connect the points

$$\frac{700-200}{5-0} = \frac{500}{5} = \frac{100}{1}$$

c) What is the rate of change in the account from month 1 to 3? 2 to 5?

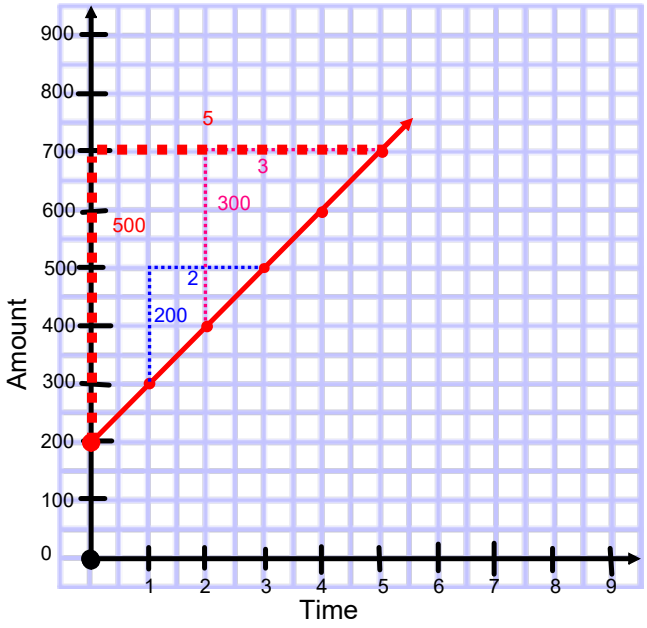
Months 1 to 3: $\frac{500-300}{3-1}$ Months 2-5: $\frac{700-400}{5-2}$

$\frac{200}{2}$ $\frac{300}{3}$

$= \frac{100}{1}$ $= \frac{100}{1}$

d) What do you notice about the rate of change for each period?

They are the same.



Rate of Change from a Table

Toothpicks are arranged to make the pattern shown

The table below represents the relationship between the number of squares and the number of toothpicks needed to make the squares. Once we recognize the rate of change, the table can be extended to represent additional squares not shown above.

(Run) Number of Squares	Total Number of Toothpicks (Rise)
1	4
2	7
3	10
4	13
5	16
6	19
7	22
8	25

Determine the rate of change between the number of squares

2-1=1

3-2=1

Determine the rate of change in the total number of toothpicks. What do you notice

7-4=3

10-7=3

What is the rate of change?

$\frac{3}{1}$ An increase of 3 toothpicks for every additional square.

Oct 15-12:46 PM

Things to consider before attempting the following questions:

Constant slope: the slope of a straight line is the same using any two points on that line, therefore in a table, the vertical values would have to have the same change between each row and the horizontal values would have to have the same change between each of its rows.

Interval: a piece of time that has a beginning time and an end time, often represented with a dash in between the start and finish (example: 12pm-1pm, 1pm-2pm etc.).

May 11-1:29 PM

Questions for practice

Pages 290 - 291 Questions 1-6

Pages 294-295 Questions 1-6

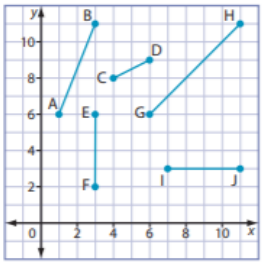
Oct 15-1:03 PM

Check Your Understanding

Try It

1. **a)** Determine the rise and run of each line segment.

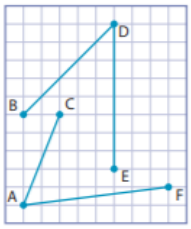
b) Determine the slope of each line segment.



2. Determine the slope of each line segment.

a) AC **b)** ED

c) BD **d)** AF



3. Examine the table of values.

x	y
0	4
1	6
2	8
3	10
4	12

a) What is the change in the x-values from one row to the next?

b) What is the change in the y-values?

c) What is the slope of the line that would connect these points on a graph?

Apply It

4. Josh drives non-stop from Deer Lake to Holyrood. His average speed is 90 km/h. The table shows the distance he drove during each hour of the trip.

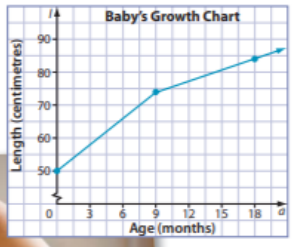
Time (h)	Distance (km)
1	90
2	180
3	270
4	360
5	450
6	540

a) Graph the data. Connect the points with a straight line.

b) What is the slope of the line?

c) How is the slope related to the rate of change in distance?

5. A baby's growth is plotted for the first 18 months of her life. Determine the rate of change in growth from age 9 months to age 18 months.



6. Julie earns $20 for babysitting for 2 h.

a) Copy and complete the table.

Time Worked (h)	Amount Earned ($)	Rate of Change
0	0	
1	10	10 − 0 = 10
2	20	
3		
4		
5		

b) What is the rate of change in Julie's earnings? Explain what this means.

c) Suppose Julie gets an increase of $5 for 2 h. What is her new rate of change in earnings?

d) Graph both of Julie's earnings on one grid. What is similar about the graphs? Explain.

e) Describe any differences between the two graphs.

Check Your Understanding

Try It

1. Determine the slope of the line in a graph of the data.

x	y
0	1
2	4
4	7
6	10
8	13

2. Does a graph of the data in each table show a constant slope? Explain.

Table A

x	y
0	4
1	6
2	8
3	10
4	12

Table B

x	y
0	3
1	4
2	6
3	9
4	13

Apply It

3. Suppose Toshi drives at a constant rate for 4 h. After 1 h, he has travelled 100 km. After 3 h, he has travelled 300 km. Determine the rate of change in distance, and explain what it means.

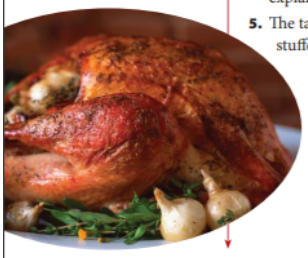
4. Olivia babysits for the Youngs and always receives the same rate of pay. She babysat for 2 hours and received $16. She was paid $40 for 5 hours. Determine the rate of change in pay, and explain what it means.

5. The table shows the time to cook stuffed turkeys of varying sizes.

Mass (kg)	Time (h)
3	3.5
5	4
7	4.5
9	5

a) Graph the data.

b) Determine the slope of the line. Explain what it means.



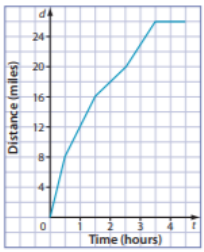
294 MHR • Chapter 6

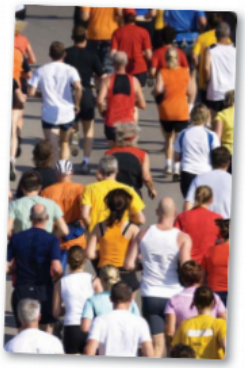
May 11-1:47 PM

F.Y.I.

There have been over 60 runnings of the Huffin' Puffin Marathon, which used to be called the Newfoundland Provincial Marathon. It is the oldest marathon in Canada and the third oldest in North America.

6. Chelsea plans to run in the Huffin' Puffin Marathon. The results of her last marathon are shown in the graph.





a) Identify the intervals on the graph that have constant slope.

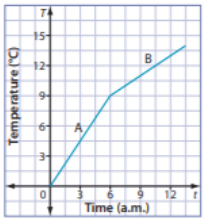
b) Determine the slope of each of these intervals.

c) Explain the slope as a rate of change.

d) During which interval did the distance Chelsea ran not change? Explain what this means.

e) What is the rate of change for this interval?

7. The graph shows the hourly temperature for the morning.



a) Determine the slopes of intervals A and B.

b) How are the slopes the same? How are they different?

c) Describe the slope of interval A as a rate of change.

d) Describe the slope of interval B as a rate of change.

6.3 Slope as Rate of Change • **MHR 295**

May 11-1:48 PM

11. a) Example: When Andrea sees the slope is 2, she will likely realize that the slope is too steep for the line on the graph.

b) $\text{slope} = \frac{\text{rise}}{\text{run}}$

$= \frac{(5-4)}{(2-0)}$

$= \frac{1}{2}$

tan (angle of elevation) = slope

tan (angle of elevation) $= \frac{1}{2}$

$\tan^{-1}\left(\frac{1}{2}\right)$ = angle of elevation

26.565... = angle of elevation

The angle of elevation is about 27°.

6.3 Slope as Rate of Change, pages 286 to 297

On the Job 1 Check Your Understanding, pages 290 to 291

1. a) AB: rise 5, run 2; CD: rise 1, run 2; EF: rise −4, run 0; GH: rise 5, run 5; IJ: rise 0, run 4

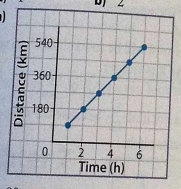
b) AB: 2.5; CD: 0.5; EF: undefined; GH: 1; IJ: 0

2. a) 2.5 b) undefined

c) 1 d) 0.125

3. a) 1 b) 2 c) 2

4. a)



b) 90

c) The slope is equal to the rate of change in distance.

5. $\frac{10}{9}$

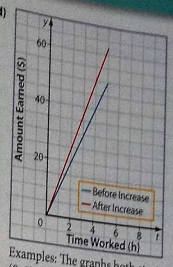
6. a)

Time Worked (h)	Amount Earned ($)	Rate of Change
0	0	
1	10	10 − 0 = 10
2	20	20 − 10 = 10
3	30	30 − 20 = 10
4	40	40 − 30 = 10
5	50	50 − 40 = 10

b) The rate of change is 10. This means that for each additional hour that Julie works, she is paid an additional $10.

c) 12.5

d)



Examples: The graphs both start at the point (0, 0), they both have a positive slope, and they are both only valid for positive numbers.

e) Example: The graphs have different slopes.

On the Job 2 Check Your Understanding, pages 294 to 295

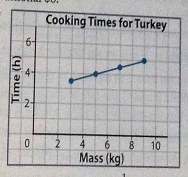
1. $\frac{3}{2}$

2. Table A shows a constant slope of 2. Table B does not show a constant slope; the y-values change by different amounts.

3. The rate of change in distance is 200. This means that for each hour that Toshi drove, he travelled an additional 100 km.

4. The rate of change in pay is 8. This means that for each hour that Olivia babysits, she earns an additional $8.

5. a)



b) The slope of the line is $\frac{1}{4}$. This means that for each additional kilogram in mass of a turkey, an additional $\frac{1}{4}$ of an hour is required to cook it.

6. a) There are five intervals that have constant slope. Label them as A: 0–0.5 h, B: 0.5–1.5 h, C: 1.5–2.5 h, D: 2.5–3.5 h, and E: 3.5–4.5 h in order from the origin.

b) A: 16; B: 8; C: 4; D: 6; E: 0

c) Each slope represents the miles travelled per hour during that interval, or speed.

d) In interval E, Chelsea's distance did not change, so she had stopped running.

e) 0

7. a) A: $\frac{3}{2}$; B: $\frac{2}{3}$

b) Example: The slopes are both positive. The slope of interval A has a rise of 3 and a run of 2, while the slope of interval B has a rise of 2 and a run of 3.

c) The temperature increases 3 degrees for each 2-hour interval that passes.

d) The temperature increases 2 degrees for each 3-hour interval that passes.

Work With It, pages 296 to 297

1. a) Jim is running faster during some intervals than during others.

b) Label intervals from A to E, starting at the origin. A: 80; B: 200; C: 66$\frac{2}{3}$; D: 0; E: −400

c) Jim has stopped running.

2. a)

Number of Tables	Number of People
1	4
2	6
3	8
4	10
5	12

b) Two additional people are seated for each additional table.

c) 2

d) For each additional table, two more people are seated.

3. a) Between points A and B, the rate of change in height is $\frac{3}{2}$; this means that the height increases 3 m for each 2 m travelled in distance. Between points B and C, the rate of change in height is 0; this means that the roller coaster is travelling along a flat section. Between points C and D, the rate of change in height is $-\frac{3}{4}$; this means that the roller coaster is descending 3 m vertically for every 4 m of distance travelled. Between points D and E, the rate of change in height is 0; this means that the roller coaster is travelling along a flat section.

b) The steepest part is the ascent between points A and B.

4. Example: The rate of change is the change in the y-values divided by the change in the x-values. If Joe is paid $15 per hour to cut grass, the rate of change is 15.

5. a) For each 12 horizontal units, the height of the roof increases by 5 vertical units.

b) A roof with pitch $\frac{12}{5}$ rises by 12 vertical units for each 5 horizontal units; a roof with pitch $\frac{12}{5}$ is much steeper than a roof with pitch $\frac{5}{12}$.

Chapter 6 Skill Check, pages 298 to 299

1. AB: $\frac{1}{4}$, CD: −1, EF: 3

2.

	Rise	Run	Slope
a)	0	6 m	0
b)	2 in.	3 in.	$\frac{2}{3}$
c)	2 m	200 cm	1
d)	4 m	0	undefined
e)	60 cm	3 m	$\frac{1}{5}$

3. a) 34° b) 79° c) 14°

4. 27°

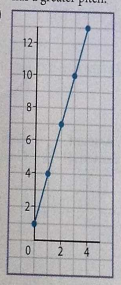
5. a) 0° b) 3° c) 10°

6. a) Example: The upper roof is steeper than the lower roof.

b) The upper roof will be more difficult to walk on because it is steeper.

c) The upper roof has a pitch of $\frac{12}{6}$ and the lower roof has a pitch of $\frac{6}{12}$, because the steeper roof has a greater pitch.

7. a)



b) The slope of the line is 3. This slope represents the change in the y-value for each incremental change in the x-value.