SEPTEMBER 13, 2016

UNIT 1: ROOTS AND POWERS

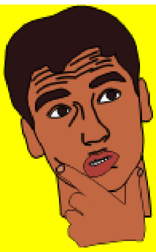
SECTION 3.2:
PERFECT SQUARES, PERFECT CUBES, AND THEIR ROOTS

K. SEARS
NUMBERS, RELATIONS AND FUNCTIONS 10

WHAT'S THE POINT OF TODAY'S LESSON?

We will continue working on the NRF 10 Specific Curriculum Outcome (SCO) "Algebra and Numbers 1" OR "AN1" which states:

"Demonstrate an understanding of factors of whole numbers by determining the prime factors, greatest common factor, least common multiple, square root and cube root."



What does THAT mean???

SCO AN1 means that we will:

* **find the prime factors of whole numbers like 8 (2^3 or $2 \cdot 2 \cdot 2$)**
* **determine the greatest common factor (GCF) of numbers like 28 and 49 (GCF = 7)**
* **determine the least common multiple (LCM) of numbers like 6 and 9 (LCM = 18)**
* **determine if a given whole number is a perfect square, like 25 ($5 \cdot 5$) or a perfect cube, like 8 ($2 \cdot 2 \cdot 2$)**
* **determine the square root of perfect squares, like 36 ($\sqrt{36} = 6$), and the cube root of perfect cubes, like 64 ($\sqrt[3]{64} = 4$)**

WARM-UP:

1. **Determine the GCF of 81 and 216.**

2. **Determine the LCM of 21 and 45.**

1. 81 → 9×9 → $3 \times 3 \times 3 \times 3$

216 → 2×108 → $2 \times 2 \times 54$ → $2 \times 2 \times 6 \times 9$ → $2 \times 2 \times 2 \times 3 \times 3 \times 3$

$GCF = 3 \times 3 \times 3$
$= 27$

2. 21 → 3×7

45 → 9×5 → $3 \times 3 \times 5$

$LCM = 3^2 \times 5 \times 7$
$= 315$

WARM-UP:

1. **Determine the GCF of 120, 960 and 1400. (40)**
2. **Determine the LCM of 15, 32 and 44. (5280)**

1.) 120 → 4 × 30 → 2 × 2 × 3 × 10 → 2 2 3 2×5 → $2^3 \times 3 \times 5$

960 → 10 × 96 → 2 × 5 × 4 × 24 → 2×5×2×2×4×6 → 2×5×2 2 2×2×2×3 → $2^6 \times 3 \times 5$

1400 → 14 × 100 → 2×7 × 10 × 10 → 2×7×2×5 2×5 → $2^3 \times 5^2 \times 7$

$GCF = 2^3 \times 5$
$= 8 \times 5$
$= 40$

2) 15 → 3 × 5

32 → 2 × 16 → 2 × 4 × 4 → 2 × 2×2×2×2 → 2^5

44 → 11 × 4 → 11 × 2 × 2 → $2^2 \times 11$

$LCM = 2^5 \times 3 \times 5 \times 11$
$= 5280$

WARM-UP:

Determine whether each number is a perfect square, a perfect cube or neither:

a) 3136 b) 4096 c) 5832 d) 2808

P.S. P.C. Both Neither

3136

4096

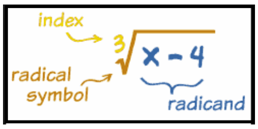
5832

2808

HOMEWORK QUESTIONS???
(pg 140, #6 TO #8, #15ad, #16af, #17 & #19)

VOCABULARY:

$\sqrt[a]{b}$



Cube root of "x-4"

1. Square Root: a number which, when multiplied by itself, results in a given number.
EX.: 5 is a square root of 25.

$\sqrt[3]{27}$

2. Cube Root: a number which, when raised to the exponent 3, results in a given number.
EX.: 5 ~~is~~ the cube root of 125.

3. Radical: An expression consisting of a radical sign, a radicand and an index~~.~~
EX.: $\sqrt[3]{64}$

4. Radicand: The number under a radical sign.
EX.: 64 is the radicand in $\sqrt[3]{64}$

5. Index: The number above the radical symbol that indicates which root is to be taken.
EX.: 3 is the index in $\sqrt[3]{64}$; if the index is not written, it is assumed to be 2.

GETTING STARTED:

Let's make a list on a separate sheet of loose-leaf of the first twenty perfect squares, perfect cubes and perfect fourth powers:

	Perfect Squares	Perfect Cubes	Perfect Fourth Powers
1	1	1	1
2	4	8	16
3	9	27	81
4	16	64	256
5	25	125	625
6	36	216	
7	49	343	
8	64	512	
9	81		
10	100		
11	121		
12	144		
13	169		
14	196		
15	225		
16	256		
17	289		
18	324		
19	351		
20	400		

GETTING STARTED:

Let's make a list on a separate sheet of loose-leaf of the first twenty perfect squares, perfect cubes and perfect fourth powers:

	Perfect Squares	Perfect Cubes	Perfect Fourth Powers
1	1	1	1
2	4	8	16
3	9	27	81
4	16	64	256
5	25	125	625
6	36	216	1296
7	49	343	2401
8	64	512	4096
9	81	729	6561
10	100	1 000	10 000
11	121	1 331	14 641
12	144	1 728	20 736
13	169	2 197	28 561
14	196	2 744	38 416
15	225	3 375	50 625
16	256	4 096	65 536
17	289	4 913	83 521
18	324	5 832	104 976
19	361	6 859	130 321
20	400	8 000	160 000

BUILDING ON WHAT WE KNOW:

We can use PRIME FACTORIZATION now to help us determine square roots and cube roots.

Example: Use prime factorization to determine the square root of 1296.

$\sqrt{1296} = 1296^{\frac{1}{2}}$

y^x

$1296^{0.5}$

$= 36$

$$\begin{aligned}\sqrt{1296}: \quad & 1296 \\ &= 4 \cdot 324 \\ &= 2 \cdot 2 \cdot 4 \cdot 81 \\ &= 2 \cdot 2 \cdot 2 \cdot 2 \cdot 9 \cdot 9 \\ &= 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \\ &= (2 \cdot 2 \cdot 3 \cdot 3)(2 \cdot 2 \cdot 3 \cdot 3) \\ &= 36 \cdot 36\end{aligned}$$

$36^2 = 1296$, so $\sqrt{1296} = 36$.

BUILDING ON WHAT WE KNOW:

Example: YOU TRY! Use prime factorization to determine the square root of 1764.

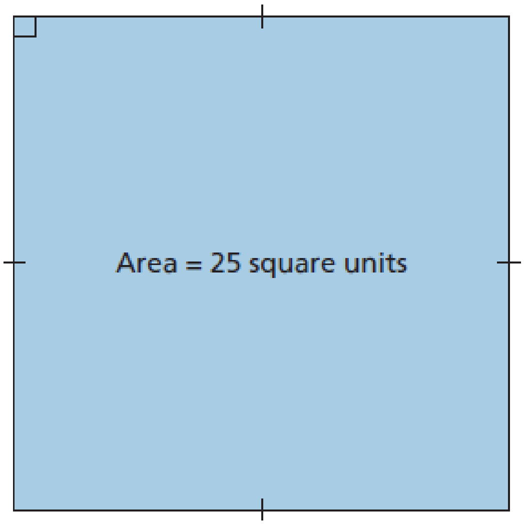
$\sqrt{1764}: \quad 1764^{0.5}$

$= 42$

CONCEPT REVIEW:

In grade 9, you explored the relationship between the area of a square and its side length:

$$\sqrt{25} = 25^{\frac{1}{2}}$$
$$= 25^{0.5}$$
$$= 5$$



BUILDING ON WHAT WE KNOW:

Example: Use prime factorization to determine the cube root of 1728.

$$\sqrt[3]{1728} = 1728^{\frac{1}{3}}$$
$$= 1728y^{x}$$
$$(1/3)$$

$\sqrt[3]{1728}$:

$$\begin{aligned} & 1728 \\ &= 4 \cdot 432 \\ &= 2 \cdot 2 \cdot 4 \cdot 108 \\ &= 2 \cdot 2 \cdot 2 \cdot 2 \cdot 4 \cdot 27 \\ &= 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 9 \\ &= 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 \\ &= (2 \cdot 2 \cdot 3)(2 \cdot 2 \cdot 3)(2 \cdot 2 \cdot 3) \\ &= 12 \cdot 12 \cdot 12 \end{aligned}$$

$12^3 = 1728$, so $\sqrt[3]{1728} = 12$.

BUILDING ON WHAT WE KNOW:

Example: YOU TRY! Use prime factorization to determine the cube root of 2744.

$\sqrt[3]{2744}$: 2744

2744 y^x (1/3) = 14

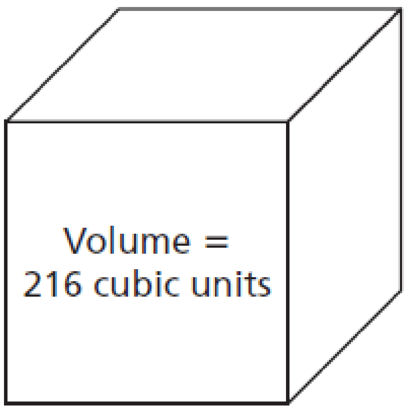
✓

NEW CONCEPT:

Based on what you know about squares, their area and their side length along with cube roots, can you now determine the edge length of a cube? What about its surface area?

$V = l \times l \times l$

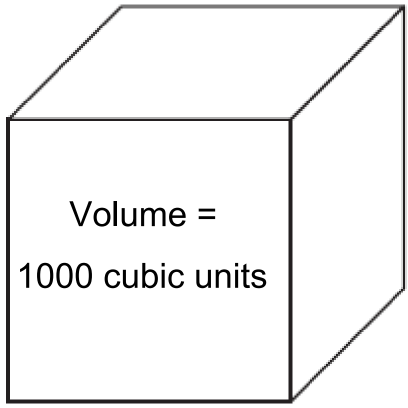
$\sqrt[3]{216} = 6$



YOU TRY!

Based on what you know about squares, their area and their side length along with cube roots, can you now determine the edge length of this cube? What about its surface area?

$$\sqrt[3]{1000} = 10$$



CONCEPT REINFORCEMENT:

FPCM 10

pages 140 / 141:   #3 TO #13, #15 TO #19a and #20

pages 146 / 147:   #4 TO #11, #13;
optional "challenge yourself questions", #16 TO #18;

QUIZ PREPARATION / PRACTICE:

FPCM 10:

page 149: **#1 to #8 and #10**
page 140: **Revisit #17 and #19a**
page 147: **Revisit #7 and #8**

QUIZ on sections 3.1 and 3.2