UNIT 1: ROOTS AND POWERS

SECTION 4.3: MIXED AND ENTIRE RADICALS

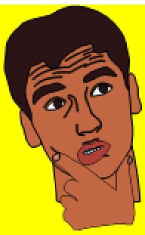
K. Sears

NUMBERS, RELATIONS AND FUNCTIONS 10

WHAT'S THE POINT OF TODAY'S LESSON?

We will begin working on the NRF 10 Specific Curriculum Outcome (SCO) "Algebra and Numbers 3" OR "AN3" which states:

"Demonstrate an understanding of powers with integral and rational exponents."



What does THAT mean???

SCO AN3 means that we will:

* apply the 6 exponent laws you learned in grade 9:
  $a^0 = 1$
  $(a^m)(a^n) = a^{m+n}$
  $a^m \div a^n = a^{m-n}$
  $(a^m)^n = a^{mn}$
  $(ab)^m = a^m b^m$
  $(a \div b)^n = a^n \div b^n$
* use patterns to explain $a^{-n} = \frac{1}{a^n}$ and $a^{\frac{1}{n}} = \sqrt[n]{a}$
* apply all exponent laws to evaluate a variety of expressions
* express powers with rational exponents as radicals and vice versa
* identify and correct errors in work that involves powers

HOMEWORK QUESTIONS??? (page 211, #3 TO #5, #7, #8, #10 and #12 TO #14)

RADICALS CAN BE WRITTEN AS PRODUCTS: ...

EX.: $\sqrt{16 \cdot 9}$ **AND** $\sqrt{16} \cdot \sqrt{9}$

$= \sqrt{144}$ $= 4 \cdot 3$

$= 12$ $= 12$

JUST AS WITH FRACTIONS, RADICALS CAN BE WRITTEN AS EQUIVALENT EXPRESSIONS:

EX.: $\sqrt[3]{8 \cdot 27}$ **AND** $\sqrt[3]{8} \cdot \sqrt[3]{27}$

$= \sqrt[3]{216}$ $= 2 \cdot 3$

$= 6$ $= 6$

$\sqrt{324} = \sqrt{36 \times 9}$

$= 6 \times 3$

$= 18$

EXPONENT LAWS (separate sheet):

1. **Zero Exponent Law:** $a^0 = 1$ $2^0 = 1$
2. **Product of Powers:** $(a^m)(a^n) = a^{m+n}$ $3^2 \cdot 3^3 = 3^5$
3. **Quotient of Powers:** $a^m \div a^n = a^{m-n}$
4. **Power of a Power:** $(a^m)^n = a^{mn}$
5. **Power of a Product:** $(ab)^m = a^m b^m$ $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$
6. **Power of a Quotient:** $(a \div b)^n = a^n \div b^n$

7. MULTIPLICATION ~~PROPERTY~~ OF RADICALS:

$\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$ $(ab)^{\frac{1}{n}} = a^{\frac{1}{n}} b^{\frac{1}{n}}$ (#5)

EX.: $\sqrt{24}$ **(Factors: 1, 2, 3, 4, 6, 8, 12, 24)**

$= \sqrt{4 \cdot 6}$

$= \sqrt{4} \cdot \sqrt{6}$

$= 2 \cdot \sqrt{6}$

$= 2\sqrt{6}$ **(MIXED RADICAL)** Like mixed fraction $2\frac{1}{2}$

EX.: $\sqrt[3]{24}$ **(ENTIRE RADICAL)**

$= \sqrt[3]{8 \cdot 3}$

$= \sqrt[3]{8} \cdot \sqrt[3]{3}$

$= 2 \cdot \sqrt[3]{3}$

$= 2\sqrt[3]{3}$

$\sqrt{24} = \sqrt{4 \times 6}$

$= 2\sqrt{6}$

1
4
9
16
25

$\sqrt{80} = \sqrt{4 \times 20}$

$= \sqrt{4 \times 4 \times 5}$

$= 2 \times 2\sqrt{5}$

$= 4\sqrt{5}$

c) $\sqrt{32} = \sqrt{16 \times 2}$

$= 4\sqrt{2}$

d) $\sqrt{600} = \sqrt{100 \times 6}$

$= 10\sqrt{6}$

8. POWERS WITH RATIONAL EXPONENTS WITH A NUMERATOR OF 1:

$$x^{\frac{1}{n}} = \sqrt[n]{x}$$

EX.:

$$8^{\frac{1}{3}}$$

$$= \sqrt[3]{8}$$

$$= 2$$

9. POWERS WITH RATIONAL EXPONENTS:

EXPONENT

$$x^{\frac{m}{n}} = \left(x^{\frac{1}{n}}\right)^m = \left(\sqrt[n]{x}\right)^m$$

INDEX

AND

EXPONENT

$$x^{\frac{m}{n}} = \left(x^m\right)^{\frac{1}{n}} = \sqrt[n]{x^m}$$

INDEX

EX.: Evaluate $16^{\frac{3}{2}}$.

$16^{\frac{3}{2}}$ 3 (EXPONENT) 2 (INDEX)

$= \left(\sqrt[2]{16}\right)^3$

$= 4^3$

$= 64$

OR

$16^{\frac{3}{2}}$ 3 (EXP.) 2 (INDEX)

$= \sqrt[2]{16^3}$

$= \sqrt{4096}$

$= 64$

NOTE: THERE ARE SOME RADICALS THAT CANNOT BE SIMPLIFIED.

EX.: $\sqrt[4]{24}$ **(Factors: 1, 2, 3, 4, 6, 8, 12, 24)**

24 HAS NO FACTORS OTHER THAN 1 THAT CAN BE WRITTEN AS A FOURTH POWER; THEREFORE, IT CANNOT BE SIMPLIFIED (WRITTEN AS A MIXED RADICAL).

WE CAN ALSO USED PRIME FACTORIZATION TO SIMPLIFY A RADICAL.

EX.: Simplify each radical.

$2\times2\times2\times2$

a) $\sqrt{80}$

$\sqrt{4\times20}$

$\sqrt{4\times4\times5}$

$= 2\times2\sqrt{5}$

$= 4\sqrt{5}$

b) $\sqrt[3]{144}$

$\sqrt[3]{8\times18}$

$2\sqrt{18}$

c) $\sqrt[4]{162}$

$\sqrt[4]{2\times81}$

$3\sqrt[4]{2}$

YOU TRY!

EX.: Simplify each radical.

a) $\sqrt{63}$ b) $\sqrt[3]{108}$ c) $\sqrt[4]{128}$

LET'S TRY TO SIMPLIFY RADICALS WITHOUT USING PRIME FACTORIZATION, IF POSSIBLE.

EX.: Write each radical in simplest form, if possible.

a) $\sqrt[3]{40}$ **b)** $\sqrt{26}$ **c)** $\sqrt[4]{32}$

YOU TRY TO SIMPLIFY RADICALS WITHOUT USING PRIME FACTORIZATION, IF POSSIBLE.

EX.: Write each radical in simplest form, if possible.

a) $\sqrt{30}$ **b)** $\sqrt[3]{32}$ **c)** $\sqrt[4]{48}$

WRITING MIXED RADICALS AS ENTIRE RADICALS:

EX.: Write each mixed radical as an entire radical.

a) $4\sqrt{3}$

$= \sqrt{4 \times 4 \times 3}$

$= \sqrt{48}$

b) $3\sqrt[3]{2}$

$= \sqrt[3]{3 \times 3 \times 3 \times 2}$

$= \sqrt[3]{54}$

c) $2\sqrt[5]{2}$ $\sqrt[5]{2 \times 2 \times 2 \times 2 \times 2 \times 2}$

$\sqrt[5]{64}$

YOU TRY WRITING MIXED RADICALS AS ENTIRE RADICALS:

EX.: Write each mixed radical as an entire radical.

a) $7\sqrt{3}$

$\sqrt{7 \times 7 \times 3}$

$= \sqrt{147}$

b) $2\sqrt[3]{4}$

$\sqrt[3]{2 \times 2 \times 2 \times 4}$

$= \sqrt[3]{32}$

c) $2\sqrt[5]{3}$

$\sqrt[5]{2 \times 2 \times 2 \times 2 \times 2 \times 3}$

$\sqrt[5]{96}$

CONCEPT REINFORCEMENT:

FPCM 10:

Page 211: 3, 4, 11
Page 218: 4, 5, 10 - 12, 14, 15, 16, 18
Page 219: 21, 22b, 24

pages 146 / 147: #4 TO #11, #13;
optional "challenge yourself questions", #16 TO #18;

page 211: #3 TO #5, #7, #8, #10, #12 and #13 TO #15
page 212: #16a

QUIZ PREPARATION:

FPCM 10:

Page 221: #1, #3, #4, #6a, #7b, #8, #9 & #11

https://mathmalfunctions.wordpress.com/exponential-functions/exponent-laws/