UNIT 1: ROOTS AND POWERS

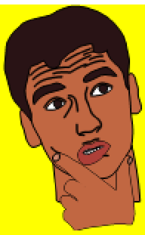
SECTION 4.4: FRACTIONAL EXPONENTS AND RADICALS

K. Sears
NUMBERS, RELATIONS AND FUNCTIONS 10

WHAT'S THE POINT OF TODAY'S LESSON?

We will continue working on the NRF 10 Specific Curriculum Outcome (SCO) "Algebra and Numbers 3" OR "AN3" which states:

"Demonstrate an understanding of powers with integral and rational exponents."



What does THAT mean???

SCO AN3 means that we will:

* apply the 6 exponent laws you learned in grade 9:
  $a^0 = 1$
  $(a^m)(a^n) = a^{m+n}$
  $a^m \div a^n = a^{m-n}$
  $(a^m)^n = a^{mn}$
  $(ab)^m = a^m b^m$
  $(a \div b)^n = a^n \div b^n$
* use patterns to explain $a^{-n} = \frac{1}{a^n}$ and $a^{\frac{1}{n}} = \sqrt[n]{a}$
* apply all exponent laws to evaluate a variety of expressions
* express powers with rational exponents as radicals and vice versa
* identify and correct errors in work that involves powers

ANY QUESTIONS BEFORE THE QUIZ??? (page 221, #1, #3, #4, #6a, #7b, #8, #9 & #11)

TIME FOR THE QUIZ! :)

(Use your list of the first 20 perfect squares, perfect cubes and perfect 4th powers if you have it. Otherwise, use your calculator.)

	Perfect Squares	Perfect Cubes	Perfect Fourth Powers
1	1	1	1
2	4	8	16
3	9	27	81
4	16	64	256
5	25	125	625
6	36	216	1296
7	49	343	2401
8	64	512	4096
9	81	729	6561
10	100	1 000	10 000
11	121	1 331	14 641
12	144	1 728	20 736
13	169	2 197	28 561
14	196	2 744	38 416
15	225	3 375	50 625
16	256	4 096	65 536
17	289	4 913	83 521
18	324	5 832	104 976
19	361	6 859	130 321
20	400	8 000	160 000

EXPONENT LAWS (separate sheet):

1. Zero Exponent Law: $a^0 = 1$
2. Product of Powers: $(a^m)(a^n) = a^{m+n}$
3. Quotient of Powers: $a^m \div a^n = a^{m-n}$
4. Power of a Power: $(a^m)^n = a^{mn}$
5. Power of a Product: $(ab)^m = a^m b^m$
6. Power of a Quotient: $(a \div b)^n = a^n \div b^n$

7. MULTIPLICATION PROPERTY OF RADICALS:

$\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$

EX.: $\sqrt{24}$ (Factors: 1, 2, 3, **4**, 6, 8, 12, 24)

$= \sqrt{4 \cdot 6}$
$= \sqrt{4} \cdot \sqrt{6}$
$= 2 \cdot \sqrt{6}$
$= 2\sqrt{6}$ (MIXED RADICAL)

EX.: $\sqrt[3]{\cancel{24}}$ (ENTIRE RADICAL)

$= \sqrt[3]{\cancel{8 \cdot 3}}$
$= \sqrt[3]{8} \cdot \sqrt[3]{3}$
$= 2 \sqrt[3]{3}$
$= 2\sqrt[3]{3}$

LET'S COPY AND COMPLETE THESE TABLES TOGETHER.

x	$x^{\frac{1}{2}}$ $(x^{0.5})$
1	$1^{\frac{1}{2}} =$ 1
4	$4^{\frac{1}{2}} =$ 2
9	$9^{\frac{1}{2}} = 3$
16	$16^{\frac{1}{2}} = 4$
25	$25^{\frac{1}{2}} = 5$

x	$x^{\frac{1}{3}}$ $[x^{(1\div 3)}]$
1	$1^{\frac{1}{3}} = 1$
8	$8^{\frac{1}{3}} = 2$
27	$27^{\frac{1}{3}} = 3$
64	$64^{\frac{1}{3}} = 4$
125	$125^{\frac{1}{3}} = 5$

WHAT DO YOU THINK THE EXPONENT $\frac{1}{2}$ MEANS?

WHAT DO YOU THINK THE EXPONENT $\frac{1}{3}$ MEANS?

WHAT DO YOU THINK $a^{\frac{1}{4}}$ AND $a^{\frac{1}{5}}$ MEAN? (TEST YOUR PREDICTIONS WITH A CALCULATOR.)

WHAT DO YOU THINK $a^{\frac{1}{n}}$ MEANS?

In grade 9, you learned that for powers with integral bases and whole number exponents:
$a^m \cdot a^n = a^{m+n}$

WE CAN EXTEND THIS LAW TO POWERS WITH FRACTIONAL EXPONENTS WITH A NUMERATOR OF 1:

$$5^{\frac{1}{2}} \cdot 5^{\frac{1}{2}} \qquad \sqrt{5} \cdot \sqrt{5}$$

$5^{\frac{1}{2}}$ and $\sqrt{5}$ are equivalent expressions; that is, $5^{\frac{1}{2}} = \sqrt{5}$.

SIMILARLY...

$$5^{\frac{1}{3}} \cdot 5^{\frac{1}{3}} \cdot 5^{\frac{1}{3}} \qquad \sqrt[3]{5} \cdot \sqrt[3]{5} \cdot \sqrt[3]{5}$$

SO...

$$5^{\frac{1}{3}} = \sqrt[3]{5}$$

8. POWERS WITH RATIONAL EXPONENTS WITH A NUMERATOR OF 1:

$$x^{\frac{1}{n}} = \sqrt[n]{x}$$

EX.:

$$8^{\frac{1}{3}}$$
$$= \sqrt[3]{8}$$
$$= 2$$

EXAMPLE:

Evaluate each power without using a calculator.

a) $27^{\frac{1}{3}}$

$= 3$

b) $0.49^{\frac{1}{2}}$

$\left(\frac{49}{100}\right)^{\frac{1}{2}}$

$\frac{7}{10}$

0.7

c) $(-64)^{\frac{1}{3}}$

-4

d) $\left(\frac{4}{9}\right)^{\frac{1}{2}}$

$\frac{2}{3}$

YOU TRY!

Evaluate each power without using a calculator.

a) $1000^{\frac{1}{3}} = 10$ **b)** $0.25^{\frac{1}{2}}$ $\left(\frac{25}{100}\right)^{\frac{1}{2}} = \frac{5}{10}$

c) $(-8)^{\frac{1}{3}} = -2$ **d)** $\left(\frac{16}{81}\right)^{\frac{1}{4}} = \frac{2}{3}$

NOTE: Because a fraction can be written as a terminating or repeating decimal, we can interpret powers with decimal exponents.

EX.:

$$32^{0.2}$$
$$=32^{\frac{2}{10}}$$
$$=32^{\frac{1}{5}}$$
$$=\sqrt[5]{32}$$
$$= \mathbf{2}$$

YOU TRY!

Evaluate $100^{0.5}$.

$= 100^{\frac{5}{10}}$

$= 100^{\frac{1}{2}}$

$= 10$

To give meaning to a power such as $8^{\frac{2}{3}}$,

we use the exponent law $(a^m)^n = a^{mn}$.

EX.:

$$8^{\frac{2}{3}}$$
$$= 8^{\frac{1}{3} \cdot 2}$$
$$= \left(8^{\frac{1}{3}}\right)^2$$
$$= \left(\sqrt[3]{8}\right)^2$$
$$= 2^2$$
$$= 4$$

EX.:

$$8^{\frac{2}{3}}$$
$$= 8^{2 \cdot \frac{1}{3}}$$
$$= \left(8^2\right)^{\frac{1}{3}}$$
$$= \sqrt[3]{8^2}$$
$$= \sqrt[3]{64}$$
$$= 4$$

9. POWERS WITH RATIONAL EXPONENTS:

EXPONENT → m

$$x^{\frac{m}{n}} = \left(x^{\frac{1}{n}}\right)^m = \left(\sqrt[n]{x}\right)^m$$

INDEX → n

EXPONENT

[redacted]

EX.: **Evaluate** $16^{\frac{3}{2}}$.

$16^{\frac{3}{2}}$ 3 (EXPONENT), 2 (INDEX)

$= \left(\sqrt[2]{16}\right)^3$

$= 4^3$

$= 64$

OR

$16^{\frac{3}{2}}$ 3 (EXP.), 2 (INDEX)

$= \sqrt[2]{16^3}$

$= \sqrt{4096}$

$= 64$

EXAMPLE:

→ $\left(\sqrt[3]{40}\right)^2$ $\left(40^2\right)^{\frac{1}{3}}$

a) Write $40^{\frac{2}{3}}$ in radical form in 2 ways.

b) Write $\sqrt{3^5}$ and $\left(\sqrt[3]{25}\right)^2$ in exponent form.

SOLUTION:

a) $40^{\frac{2}{3}} = \left(\sqrt[3]{40}\right)^2$ or $\sqrt[3]{40^2}$

b) $\sqrt{3^5} = 3^{\frac{5}{2}}$ AND $\left(\sqrt[3]{25}\right)^2 = 25^{\frac{2}{3}}$

YOU TRY!

$\rightarrow \left(\sqrt[5]{26}\right)^2$ or $\left(26^2\right)^{\frac{1}{5}}$

a) Write $26^{\frac{2}{5}}$ in radical form in 2 ways.

$\rightarrow 6^{\frac{5}{2}}$ $\quad 19^{\frac{3}{4}}$

b) Write $\sqrt{6^5}$ and $\left(\sqrt[4]{19}\right)^3$ in exponent form.

SOLUTION:

a) $\left(\sqrt[5]{26}\right)^2$ or $\sqrt[5]{26^2}$ $\quad \left(26^2\right)^{\frac{1}{5}}$ $\quad \sqrt[5]{26^2}$

b) $6^{\frac{5}{2}}, 19^{\frac{3}{4}}$

EXAMPLE:

Evaluate.

a) $0.04^{\frac{3}{2}}$

$\left(\frac{4}{100}\right)^{\frac{3}{2}} = \left(\frac{2}{10}\right)^3 = \frac{8}{1000} \rightarrow \frac{1}{125}$

b) $27^{\frac{4}{3}}$ $= (3)^4$ $= 81$

c) $(-32)^{0.4}$

$= -32^{\frac{4}{10}}$

$= -32^{\frac{2}{5}}$

$= (-2)^2$

$= 4$

d) $1.8^{1.4}$

$= 2.3$

SOLUTION:

a) $0.04^{\frac{3}{2}} = \left(0.04^{\frac{1}{2}}\right)^3$

$= \left(\sqrt{0.04}\right)^3$

$= 0.2^3$

$= 0.008$

c) The exponent $0.4 = \frac{4}{10}$ or $\frac{2}{5}$

So, $(-32)^{0.4} = (-32)^{\frac{2}{5}}$

$= \left[(-32)^{\frac{1}{5}}\right]^2$

$= \left(\sqrt[5]{-32}\right)^2$

$= (-2)^2$

$= 4$

d) $1.8^{1.4}$

Use a calculator.

$1.8^{1.4} = 2.2770\ldots$

YOU TRY!

Evaluate.

a) $0.01^{\frac{3}{2}}$ $\left(\frac{1}{100}\right)^{\frac{3}{2}}$ $\left(\frac{1}{10}\right)^{3}$ $\frac{1}{1000}$

b) $(-27)^{\frac{4}{3}}$ $(-3)^4$ $= 81$

c) $81^{\frac{3}{4}} = 3^3 = 27$

d) $0.75^{1.2} = 0.70\ldots$

SOLUTION:

a) 0.001 b) 81 c) 27 d) 0.7080…

EXAMPLE:

Biologists use the formula $b = 0.01m^{\frac{2}{3}}$ to estimate the brain mass, b kilograms, of a mammal with body mass m kilograms. Estimate the brain mass of each animal.

a) a husky with a body mass of 27 kg

b) a polar bear with a body mass of 200 kg

SOLUTION:

Use the formula $b = 0.01m^{\frac{2}{3}}$.

a) Substitute: $m = 27$

$b = 0.01(27)^{\frac{2}{3}}$

$b = 0.01(\sqrt[3]{27})^2$

$b = 0.01(3)^2$

$b = 0.01(9)$

$b = 0.09$

Use the order of operations. Evaluate the power first.

The brain mass of the husky is approximately 0.09 kg.

b) Substitute: $m = 200$

$b = 0.01(200)^{\frac{2}{3}}$

Use a calculator.

200 y^x (2/3) * 0.01

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0.01(200)^(2/3)
       0.341995189
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The brain mass of the polar bear is approximately 0.34 kg.

YOU TRY!

Use the formula $b = 0.01m^{\frac{2}{3}}$ to estimate the brain mass of each animal.

a) a moose with a body mass of 512 kg

b) a cat with a body mass of 5 kg

SOLUTION:

a) approximately 0.64 kg

b) approximately 0.03 kg

CONCEPT REINFORCEMENT:

***FPCM 10*:**

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