UNIT 1: ROOTS AND POWERS

SECTION 4.6: APPLYING THE EXPONENT LAWS

K. Sears
NUMBERS, RELATIONS AND FUNCTIONS 10

WHAT'S THE POINT OF TODAY'S LESSON?

We will continue working on the NRF 10 Specific Curriculum Outcome (SCO) "Algebra and Numbers 3" OR "AN3" which states:

"Demonstrate an understanding of powers with integral and rational exponents."

What does THAT mean???

SCO AN3 means that we will:

* **apply the 6 exponent laws you learned in grade 9:**
  $a^0 = 1$
  $(a^m)(a^n) = a^{m+n}$
  $a^m \div a^n = a^{m-n}$
  $(a^m)^n = a^{mn}$
  $(ab)^m = a^m b^m$
  $(a \div b)^n = a^n \div b^n$
* **use patterns to explain $a^{-n} = \frac{1}{a^n}$ and $a^{\frac{1}{n}} = \sqrt[n]{a}$**
* **apply all exponent laws to evaluate a variety of expressions**
* **express powers with rational exponents as radicals and vice versa**
* **identify and correct errors in work that involves powers**

Problems with the homework?

Page 227: **#3 to #16**
Page 228: **#17 to #21**

Page 233: **#3 TO #14**
Page 234: **#15 TO #17ab and #18 TO #20**

$(4)(21)$

Page 227 #4. a) $100^{0.5} = 100^{\frac{1}{2}} = 10$ b) $81^{0.25} = 81^{\frac{1}{4}} = 3$

c) $1024^{\frac{2}{10}} = 1024^{\frac{1}{5}} = 4$ d) $(-32)^{\frac{2}{10}} = -32^{\frac{1}{5}} = -2$

21. $T = 0.2R^{\frac{3}{2}}$
$T = 0.2(149)^{\frac{3}{2}}$ $T = 0.2(228)^{\frac{3}{2}}$
$= 364$ days $= 688$ days

Page 234

20. $\boxed{X}^{-n} = \frac{1}{X^n}$ $X > 0$

fraction is less than one

EXPONENT LAWS (separate sheet):

1. **Zero Exponent Law:** $a^0 = 1$
2. **Product of Powers:** $(a^m)(a^n) = a^{m+n}$
3. **Quotient of Powers:** $a^m \div a^n = a^{m-n}$
4. **Power of a Power:** $(a^m)^n = a^{mn}$
5. **Power of a Product:** $(ab)^m = a^m b^m$
6. **Power of a Quotient:** $(a \div b)^n = a^n \div b^n$

7. MULTIPLICATION PROPERTY OF RADICALS:

$\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$

EX.: $\sqrt{24}$ (Factors: 1, 2, 3, 4, 6, 8, 12, 24)

$= \sqrt{4 \cdot 6}$

$= \sqrt{4} \cdot \sqrt{6}$

$= 2 \cdot \sqrt{6}$

$= 2\sqrt{6}$ (MIXED RADICAL)

EX.: $\sqrt[3]{24}$ (ENTIRE RADICAL)

$= \sqrt[3]{8 \cdot 3}$

$= \sqrt[3]{8} \cdot \sqrt[3]{3}$

$= 2 \cdot \sqrt[3]{3}$

$= 2\sqrt[3]{3}$

8. POWERS WITH RATIONAL EXPONENTS WITH A NUMERATOR OF 1:

$$x^{\frac{1}{n}} = \sqrt[n]{x}$$

EX.:

$$8^{\frac{1}{3}}$$

$$= \sqrt[3]{8}$$

$$= 2$$

9. POWERS WITH RATIONAL EXPONENTS:

EXPONENT

$$x^{\frac{m}{n}} = \left(x^{\frac{1}{n}}\right)^m = \left(\sqrt[n]{x}\right)^m$$

INDEX

AND

EXPONENT

$$x^{\frac{m}{n}} = \left(x^m\right)^{\frac{1}{n}} = \sqrt[n]{x^m}$$

INDEX

EX.: Evaluate $16^{\frac{3}{2}}$.

$\frac{3}{2}$ (EXPONENT) (INDEX)

$$16^{\frac{3}{2}} = \left(\sqrt[2]{16}\right)^3 = 4^3 = 64$$

OR

$\frac{3}{2}$ (EXP.) (INDEX)

$$16^{\frac{3}{2}} = \sqrt[2]{16^3} = \sqrt{4096} = 64$$

10. POWERS WITH NEGATIVE EXPONENTS:

$$x^{-n} = \frac{1}{x^n} \quad AND \quad \frac{1}{x^{-n}} = x^n$$

EX.: $4^{-2} = \frac{1}{4^2} = \frac{1}{16}$

EX.: $\frac{1}{5^{-2}} = 5^2 = 25$

Basically, remember to take the reciprocal of the ENTIRE base and change the negative exponent to a positive exponent.

EX.: $\left(-\frac{3}{4}\right)^{-3}$ $\left(\frac{-4}{3}\right)^{3}$

$-\frac{64}{27}$

4.6 - APPLYING EXPONENT LAWS:

Let's build gradually on what we knew in grade 9...

For example:

$(2^2)(2^6)$
$= 2^8$
$= 256$

APPLYING EXPONENT LAWS:

Let's build gradually on what we knew in grade 9...

For example:

$$(2^2)(2^{-6})$$
$$= 2^{-4}$$
$$= \frac{1}{2^4}$$
$$= \frac{1}{16}$$

APPLYING EXPONENT LAWS:

Let's build gradually on what we knew in grade 9...

For example:

$\left[\left(2^2\right)\left(2^{-6}\right)\right]^{\frac{-3}{2}}$

$(2^{-4})^{-\frac{3}{2}} = 2^6 = 64$

$$= \left(2^{-4}\right)^{\frac{-3}{2}}$$
$$= \left(\frac{1}{2^4}\right)^{\frac{-3}{2}}$$
$$= \left(\frac{1}{16}\right)^{\frac{-3}{2}}$$
$$= 16^{\frac{3}{2}}$$
$$= \left(\sqrt{16}\right)^3$$
$$= 4^3$$
$$= 64$$

EXAMPLE:

a) $0.3^{-3} \cdot 0.3^{5} = 0.3^{2}$

b) $\left[\left(-\frac{3}{2}\right)^{-4}\right]^{2} \cdot \left[\left(-\frac{3}{2}\right)^{2}\right]^{3}$

c) $\frac{(1.4^{3})(1.4^{4})}{1.4^{-2}}$

$= 1.4^{3}\ 1.4^{4}\ 1.4^{2}$

$= 1.4^{9}$

d) $\left(\frac{7^{\frac{2}{3}}}{7^{\frac{1}{3}} \cdot 7^{\frac{5}{3}}}\right)^{6}$

$\left(\frac{7^{\frac{2}{3}}}{7^{2}}\right)^{6}$

$\frac{7^{4}}{7^{12}}$

$\frac{1}{7^{8}}$

$\left(-\frac{3}{2}\right)^{-8}\left(-\frac{3}{2}\right)^{6}$

$\left(-\frac{3}{2}\right)^{-2}$

$\left(-\frac{2}{3}\right)^{2}$

$\frac{4}{9}$

7^{-8}

SOLUTIONS:

a) $0.3^{-3} \cdot 0.3^{5} = 0.3^{(-3)+5}$

$= 0.3^{2}$

b) $\left[\left(-\frac{3}{2}\right)^{-4}\right]^{2} \cdot \left[\left(-\frac{3}{2}\right)^{2}\right]^{3} = \left(-\frac{3}{2}\right)^{-8} \cdot \left(-\frac{3}{2}\right)^{6}$

$= \left(-\frac{3}{2}\right)^{-2}$

$= \left(-\frac{2}{3}\right)^{2}$

c)
$$\frac{(1.4^3)(1.4^4)}{1.4^{-2}}$$
$$= \frac{1.4^{3+4}}{1.4^{-2}}$$
$$= \frac{1.4^7}{1.4^{-2}}$$
$$= 1.4^{7-(-2)}$$
$$= 1.4^9$$

d)
$$\left(\frac{7^{\frac{2}{3}}}{7^{\frac{1}{3}} \cdot 7^{\frac{5}{3}}}\right)^6$$
$$= \left(\frac{7^{\frac{2}{3}}}{7^{\frac{6}{3}}}\right)^6$$
$$= \left(7^{\frac{2}{3}-\frac{6}{3}}\right)^6$$
$$= \left(7^{-\frac{4}{3}}\right)^6$$
$$= 7^{-\frac{24}{3}}$$
$$= 7^{-8}$$
$$= \frac{1}{7^8}$$

YOU TRY!

a) $0.8^2 \cdot 0.8^{-7}$

b) $\left[\left(-\frac{4}{5}\right)^2\right]^{-3} \div \left[\left(-\frac{4}{5}\right)^4\right]^{-5}$

c) $\frac{(1.5^{-3})^{-5}}{1.5^5}$ $\quad \frac{(1.5)^{15}}{1.5^5} = (1.5)^{10}$

d) $\frac{9^{\frac{5}{4}} \cdot 9^{-\frac{1}{4}}}{9^{\frac{3}{4}}}$ $\quad \frac{9^{\frac{4}{4}}}{9^{\frac{3}{4}}} = 9^{\frac{1}{4}}$

a) 0.8^{-5}

$\frac{1}{0.8^5}$

b) $\left(-\frac{4}{5}\right)^{-6} \div \left(-\frac{4}{5}\right)^{-20}$

$\left(-\frac{4}{5}\right)^{-6-(-20)}$

$\left(-\frac{4}{5}\right)^{14}$

[Answers: a) $\frac{1}{0.8^5}$ b) $\left(-\frac{4}{5}\right)^{14}$

c) 1.5^{10} d) $9^{\frac{1}{4}}$]

EXAMPLE:

a) $(x^3y^2)(x^2y^{-4}) = x^5y^{-2}$

$= \frac{x^5}{y^2}$

b) $\frac{10a^5b^3}{2a^2b^{-2}}$

$5a^3b^5$

SOLUTIONS:

a) $(x^3y^2)(x^2y^{-4}) = x^3 \cdot y^2 \cdot x^2 \cdot y^{-4}$

$= x^3 \cdot x^2 \cdot y^2 \cdot y^{-4}$

$= x^{3+2} \cdot y^{2+(-4)}$

$= x^5 \cdot y^{-2}$

$= x^5 \cdot \frac{1}{y^2}$

$= \frac{x^5}{y^2}$

b) $\frac{10a^5b^3}{2a^2b^{-2}} = \frac{10}{2} \cdot \frac{a^5}{a^2} \cdot \frac{b^3}{b^{-2}}$

$= 5 \cdot a^{5-2} \cdot b^{3-(-2)}$

$= 5 \cdot a^3 \cdot b^5$

$= 5a^3b^5$

HOW I WOULD ACTUALLY SHOW MY WORK:

a) $(x^3y^2)(x^2y^{-4})$

b) $\dfrac{10a^5b^3}{2a^2b^{-2}}$

YOU TRY!

a) $m^4n^{-2} \cdot m^2n^3 = m^6n^1$

b) $\dfrac{6x^4y^{-3}}{14xy^2} = \dfrac{3x^3y^{-5}}{7} = \dfrac{3x^3}{7y^5}$

[Answers: a) m^6n b) $\frac{3x^3}{7y^5}$]

EXAMPLE:

a) $(8a^3b^6)^{\frac{1}{3}} = 8^{\frac{1}{3}}(a^3)^{\frac{1}{3}}(b^6)^{\frac{1}{3}}$

$= 2ab^2$

b) $(x^{\frac{3}{2}}y^2)(x^{\frac{1}{2}}y^{-1})$

c) $\frac{4a^{-2}b^{\frac{2}{3}}}{2a^2b^{\frac{1}{3}}}$

d) $\left(\frac{100a}{25a^5b^{-\frac{1}{2}}}\right)^{\frac{1}{2}}$

SOLUTIONS:

a) $(8a^3b^6)^{\frac{1}{3}} = 8^{\frac{1}{3}} \cdot a^{3\left(\frac{1}{3}\right)} \cdot b^{6\left(\frac{1}{3}\right)}$

$= (2^3)^{\frac{1}{3}} \cdot a^1 \cdot b^2$

$= 2ab^2$

b) $(x^{\frac{3}{2}}y^2)(x^{\frac{1}{2}}y^{-1}) = x^{\frac{3}{2}} \cdot x^{\frac{1}{2}} \cdot y^2 \cdot y^{-1}$

$= x^{\frac{3}{2}+\frac{1}{2}} \cdot y^{2+(-1)}$

$= x^2y$

$$\text{c)}\ \frac{4a^{-2}b^{\frac{2}{3}}}{2a^{2}b^{\frac{1}{3}}} = \frac{4}{2}\cdot\frac{a^{-2}}{a^{2}}\cdot\frac{b^{\frac{2}{3}}}{b^{\frac{1}{3}}}$$

$$= 2\cdot a^{(-2)-2}\cdot b^{\frac{2}{3}-\frac{1}{3}}$$

$$= 2\cdot a^{-4}\cdot b^{\frac{1}{3}}$$

$$= \frac{2b^{\frac{1}{3}}}{a^{4}}$$

$$\text{d)}\ \left(\frac{100a}{25a^{5}b^{-\frac{1}{2}}}\right)^{\frac{1}{2}} = \left(\frac{100}{25}\cdot\frac{a^{1}}{a^{5}}\cdot\frac{1}{b^{-\frac{1}{2}}}\right)^{\frac{1}{2}}$$

$(a^{\frac{4}{1}})^{\frac{1}{2}}$

$(\frac{1}{2})(\frac{1}{2})$

$$= \left(4\cdot a^{1-5}\cdot b^{\frac{1}{2}}\right)^{\frac{1}{2}}$$

$$= \left(4\cdot a^{-4}\cdot b^{\frac{1}{2}}\right)^{\frac{1}{2}}$$

$$= 4^{\frac{1}{2}}\cdot a^{(-4)\left(\frac{1}{2}\right)}\cdot b^{\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)}$$

$$= 2\cdot a^{-2}\cdot b^{\frac{1}{4}}$$

$$= \frac{2b^{\frac{1}{4}}}{a^{2}}$$

HOW I WOULD ACTUALLY SHOW MY WORK:

a) $(8a^3b^6)^{\frac{1}{3}}$

b) $(x^{\frac{3}{2}}y^2)(x^{\frac{1}{2}}y^{-1})$

HOW I WOULD ACTUALLY SHOW MY WORK:

c) $\dfrac{4a^{-2}b^{\frac{2}{3}}}{2a^2b^{\frac{1}{3}}}$

d) $\left(\dfrac{100a}{25a^5b^{-\frac{1}{2}}}\right)^{\frac{1}{2}}$

YOU TRY!

a) $(25a^4b^2)^{\frac{3}{2}}$ $125a^6b^3$

b) $(x^3y^{-\frac{3}{2}})(x^{-1}y^{\frac{1}{2}}) = x^2y^{-1} = \frac{x^2}{y}$

c) $\frac{12x^{-5}y^{\frac{5}{2}}}{3x^{\frac{1}{2}}y^{-\frac{1}{2}}}$ $4x^{-\frac{11}{2}}y^3 = \frac{4y^3}{x^{\frac{11}{2}}}$

d) $\left(\frac{50x^2y^4}{2x^4y^7}\right)^{\frac{1}{2}}$ $\frac{5}{xy^{\frac{3}{2}}}$

[Answers: a) $125a^6b^3$ b) $\frac{x^2}{y}$

c) $\frac{4y^3}{x^{\frac{11}{2}}}$ d) $\frac{5}{xy^{\frac{3}{2}}}$]

CONCEPT REINFORCEMENT:

FPCM 10:
Page 241: #3 to #6
Page 242: #7 to #11, #14 to #17 & #19
Page 243: #21 & #22

Page 236: #1 to #8 (ALL!)

UNIT 1 TEST PREPARATION

FPCM 10:
Page 197: Skills Summary (3.1 / 3.2)
Page 198: Review Questions (3.1 / 3.2)
Page 201: Practice Test (#1 & #3)

Page 244: Study Guide
Page 245: Skills Summary
Pages 246 to 248: Review Questions
Page 249: Practice Test