Notes - Optimization Problems.pdf

5.4 Optimization Problems I: Creating the Model

inequality   *× equation*   *over lap*

optimization problem	constraint	objective function	feasible region
A problem where a quantity must be maximized or minimized following a set of guidelines or conditions.	A limiting condition of the optimization problem being modelled, represented by a linear inequality.	In an optimization problem, the equation that represents the relationship between the two variables in the system of linear inequalities and the quantity to be optimized.	The solution region for a system of linear inequalities that is modelling an optimization problem.

Need to Know — *Graph*

- You can create a model for an optimization problem by following these steps:

  Step 1. Identify the quantity that must be optimized. Look for key words, such as *maximize* or *minimize*, *largest* or *smallest*, and *greatest* or *least*.

  NEW

  Step 2. Define the variables that affect the quantity to be optimized. Identify any restrictions on these variables.

  ✓

  Step 3. Write a system of linear inequalities to describe all the constraints of the problem. Graph the system.

  ✓

  Step 4. Write an objective function to represent the relationship between the variables and the quantity to be optimized.

  NEW

APPLY the Math

EXAMPLE 1   Creating a model for an optimization problem with whole-number variables

p. 243

Three teams are travelling to a basketball tournament in cars and minivans.
- Each team has no more than 2 coaches and 14 athletes.
- Each car can take 4 team members, and each minivan can take 6 team members.
- No more than 4 minivans and 12 cars are available.

The school wants to know the combination of cars and minivans that will require the minimum and maximum number of vehicles. Create a model to represent this situation.

Juanita's Solution

Let m represent the number of minivans.
Let c represent the number of cars.

> The two variables in the problem are the number of cars and the number of minivans. The values of these variables are whole numbers.

$m \in W$ and $c \in W$

Constraints:
Number of cars available:
$c \leq 12$
Number of minivans available:
$m \leq 4$
Number of team members:
$4c + 6m \leq 48$

> I knew that this is an **optimization problem** because the number of vehicles has to be minimized and maximized.
>
> I wrote three linear inequalities to represent the three limiting conditions, or **constraints**.
>
> The maximum number of team members is the number of teams multiplied by the maximum number of coaches and athletes:
> $3(14) + 3(2) = 48$

optimization problem

A problem where a quantity must be maximized or minimized following a set of guidelines or conditions.

constraint

A limiting condition of the optimization problem being modelled, represented by a linear inequality.

Objective function:
Let V represent the total number of vehicles.
$V = c + m$

> I created an equation, called the **objective function**, to represent the relationship between the two variables (number of minivans and number of cars) and the quantity to be minimized and maximized (number of vehicles).

objective function

In an optimization problem, the equation that represents the relationship between the two variables in the system of linear inequalities and the quantity to be optimized.

Number of Cars and Minivans
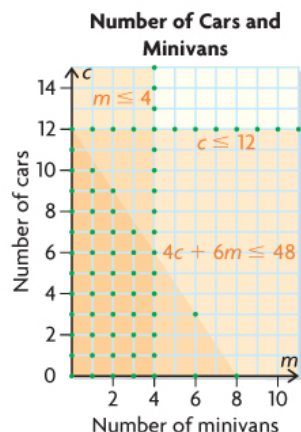
> I graphed the system of three inequalities.
>
> One of the solutions in the **feasible region** represents the combination of cars and minivans that results in the minimum total number of vehicles and another solution represents the maximum. I think I could use the objective function to determine each point, but I am not certain how yet.

feasible region

The solution region for a system of linear inequalities that is modelling an optimization problem.

APPLY the Math

EXAMPLE 1   Creating a model for an optimization problem with whole-number variables

p. 243

Three teams are travelling to a basketball tournament in cars and minivans.

TOTAL → $3 \times 16 = 48$

• Each team has no more than 2 coaches and 14 athletes.
• Each car can take 4 team members, and each minivan can take 6 team members.
• No more than 4 minivans and 12 cars are available.

The school wants to know the combination of cars and minivans that will require the minimum and maximum number of vehicles. Create a model to represent this situation.

Juanita's Solution

Variables   **Restrictions**

$x \to$ # of cars $x \in W$
$y \to$ # of minivans $y \in W$

Constraints...

① $4x + 6y \leq 48$

② $x \leq 12$ ③ $y \leq 4$

$x = 12$ $y = 4$
vertical   horizontal

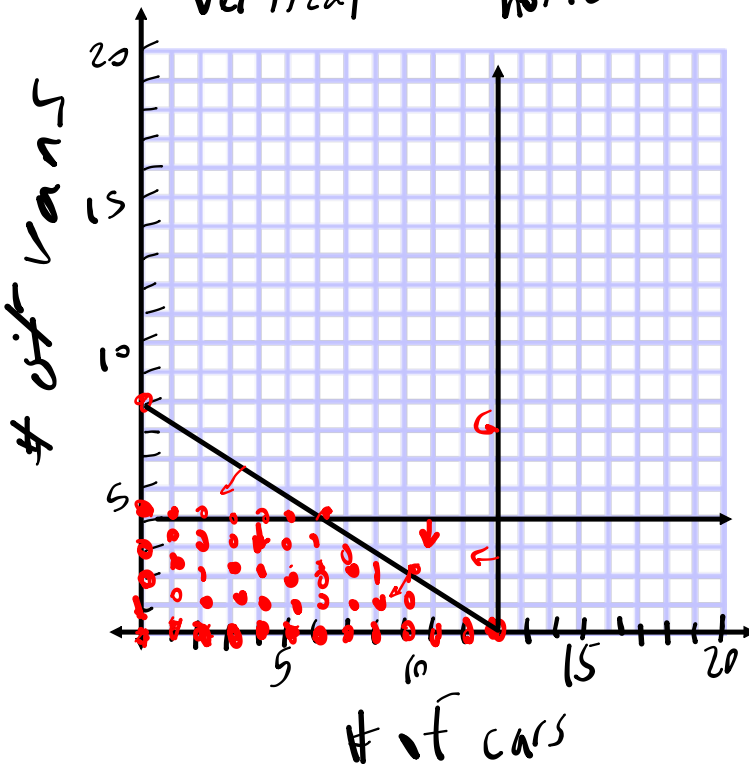**Graph Boundary**

$4x + 6y = 48$

x int (let $y = 0$)

$\frac{4x + 6(0)}{4} = \frac{48}{4}$

$x = 12$
$(12, 0)$

y int (let $x = 0$)

$\frac{4(0) + 6y}{6} = \frac{48}{6}$

$y = 8$
$(0, 8)$

# of vans (y-axis, values 5, 10, 15, 20)

# of cars (x-axis, values 5, 10, 15, 20)

EXAMPLE 2   Creating a model for a maximization problem with positive
real-number variables

p. 245

A refinery produces oil and gas.
- At least 2 L of gasoline is produced for each litre of heating oil.
- The refinery can produce up to 9 million litres of heating oil and 6 million litres of gasoline each day.
- Gasoline is projected to sell for $1.10 per litre. Heating oil is projected to sell for $1.75 per litre.

The company needs to determine the daily combination of gas and heating oil that must be produced to maximize revenue. Create a model to represent this situation.

Umberto's Solution

Let h represent the number of litres of heating oil.
Let g represent the number of litres of gasoline.

Restrictions:
$h \geq 0$ and $g \geq 0$, where $h \in R$ and $g \in R$

> I knew that this is an optimization problem because the total revenue has to be maximized.
>
> The two variables in the problem are the volume of heating oil and the volume of gasoline, both in litres. Litres are measured using positive real numbers.

Constraints:
Ratio of gasoline produced to oil produced:
$g \geq 2h$
Amount of gasoline that can be produced:
$g \leq 6\ 000\ 000$
Amount of oil that can be produced:
$h \leq 9\ 000\ 000$

> I created inequalities to represent the five constraints of the problem.
>
> I treated the restrictions on each variable as a constraint.

Let R represent total revenue from sales of gasoline and heating oil.
Objective function to maximize:
$R = 1.10g + 1.75h$

> I wrote an objective function to represent the relationship between the two variables (volume of heating oil and volume of gasoline) and the quantity to be maximized (total revenue).

Production of Gasoline and Heating Oil

(3, 6)
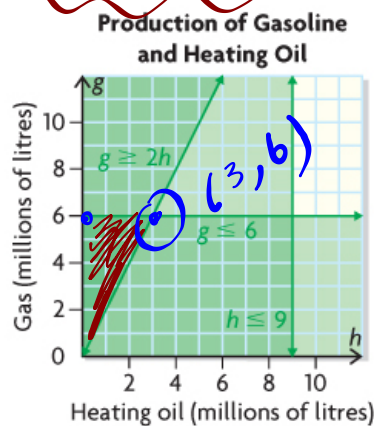
> I graphed the system of inequalities in the 1st quadrant because of the restrictions on the variables. The feasible region is a right triangle and includes all points on its boundaries.
>
> I think I can use the objective function to determine which point in the feasible region represents the combination of oil and gas that will result in the maximum revenue, but I am not sure how yet.

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EXAMPLE 2   Creating a model for a maximization problem with positive real-number variables

p. 245

A refinery produces oil and gas.
- At least 2 L of gasoline is produced for each litre of heating oil.
- The refinery can produce up to 9 million litres of heating oil and 6 million litres of gasoline each day.
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Dependency

$I \rightarrow x$ (oil)
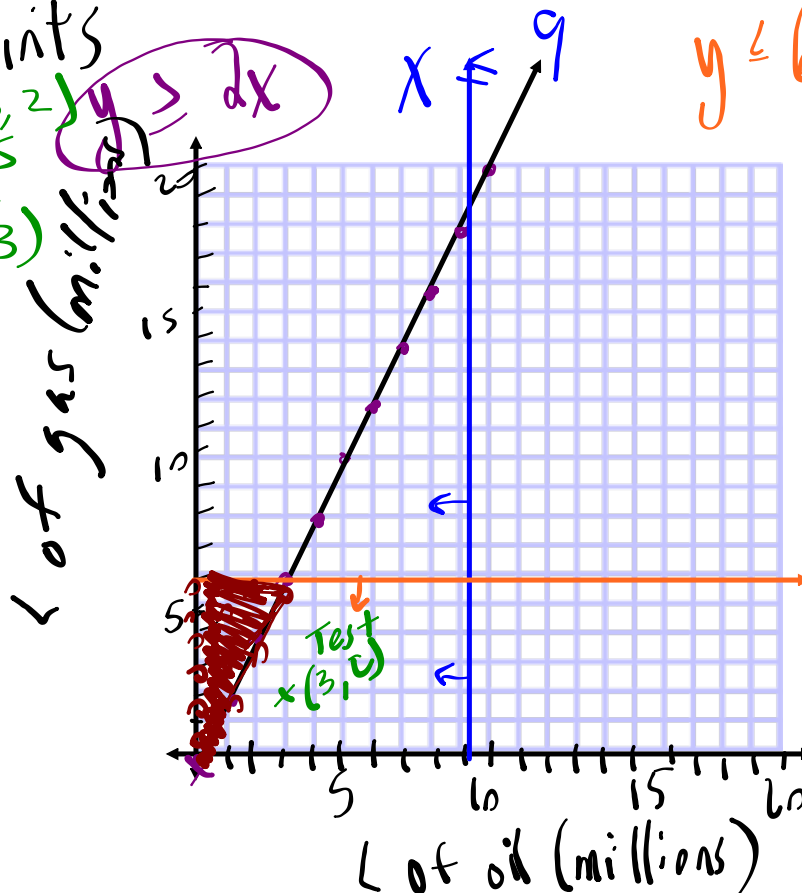$D \rightarrow y$ (gas)
Gas depends on oil

Variables

$x \rightarrow$ millions of liters of oil
$y \rightarrow$ millions of liters of gas

Restrictions
$x \in R$ } Quad 1
$y \in R$

Constraints

Test (3,2) $y \geq 2x$
LS $\geq$ RS

2	2(3)
	≥ 6

NO

$x \leq 9$ $y \leq 6$

$y = 2x$

x	y
0	0
10	20
5	10

L of gas (millions)

Test
x (3,0)

L of oil (millions)

p.247

In Summary

Key Ideas

- To solve an optimization problem, you need to determine which combination of values of two variables results in a maximum or minimum value of a related quantity.
- When creating a model, the first step is to represent the situation algebraically. An algebraic model includes these parts:
  - a defining statement of the variables used in your model
  - a statement describing the restrictions on the variables
  - a system of linear inequalities that describes the constraints
  - an objective function that shows how the variables are related to the quantity to be optimized
- The second step is to represent the system of linear inequalities graphically.
- In optimization problems, any restrictions on the variables are considered constraints. For example, if you are working with positive real numbers, $x \geq 0$ and $y \geq 0$ are constraints and should be included in the system of linear inequalities.

Need to Know

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  Step 2. Define the variables that affect the quantity to be optimized. Identify any restrictions on these variables.

  Step 3. Write a system of linear inequalities to describe all the constraints of the problem. Graph the system.

  Step 4. Write an objective function to represent the relationship between the variables and the quantity to be optimized.

HOMEWORK...

omit 1d ←

Page 248: #1, #2, #3, #5, #6

> **NOTE:**
>
> Create a model means graph the solution region

Attachments

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