HOMEWORK...

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NOTE:

Create a model means graph the solution region

b) $M = \frac{x}{3} + \frac{y}{2}$

Objective

$M = \frac{x}{3} + \frac{y}{4}$

6. Sung and Faith have weekend jobs at a marina, applying anti-fouling paint to the bottom of boats.
   - Sung can work no more than 14 h per weekend.
   - Faith is available no more than 18 h per weekend.
   - The marina will hire both of them for 24 h or less per weekend.
   - Sung paints one boat in 3 h, but Faith needs 4 h to paint one boat.

   The marina wants to maximize the number of boats that are painted each weekend.
   a) Create a model to represent this situation.
   b) Suppose that another employee, Frank, who can paint a boat in 2 h, replaced Faith for a weekend. How would your model change?

a) $x \rightarrow$ # of hours Sung works $\quad x \in W$

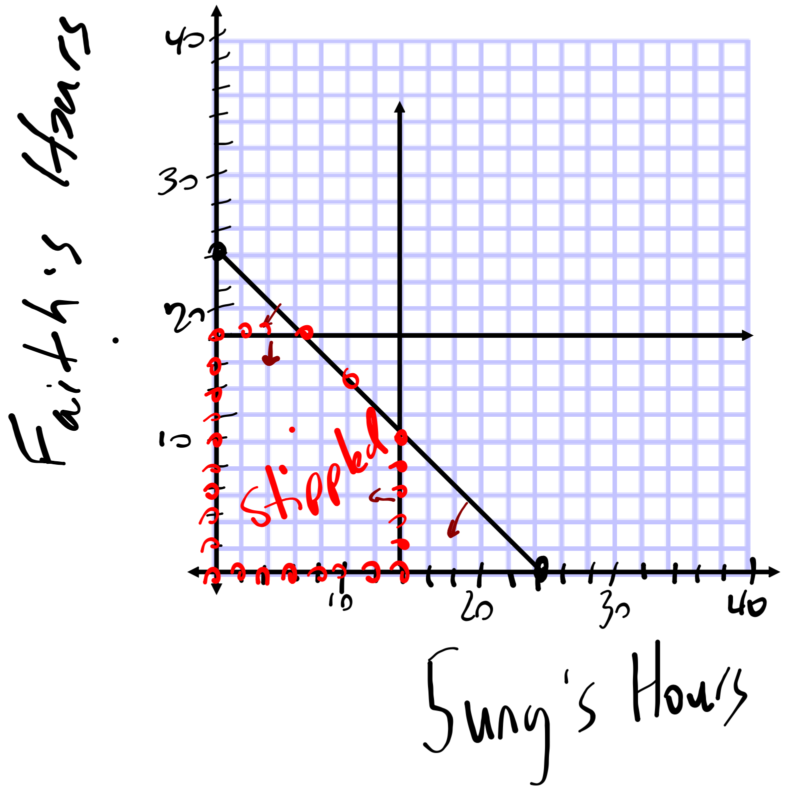
$y \rightarrow$ # of hours Faith works $\quad y \in W$

$x \leq 14 \qquad y \leq 18 \qquad x + y \leq 24$

$x + y = 24$

x int $(24, 0)$

y int $(0, 24)$



EXAMPLE of an OPTIMIZATION Problem...

Mick and Keith make MP3 covers to sell, using beads and stickers.

- At most, 45 covers with stickers and 55 bead covers can be made per day.
- Mick and Keith can make 45 or more covers, in total, each day.
- It costs \$0.75 to make a cover with stickers, \$1.00 to make one with beads.

Let x represent the number of covers with stickers and let y represent the number of bead covers.

Let C represent the cost of making the covers.

RESTRICTIONS: $x \in W$ $y \in W$

CONSTRAINTS: $x \le 45$ $y \le 55$ $x+y \ge 45$] Graph

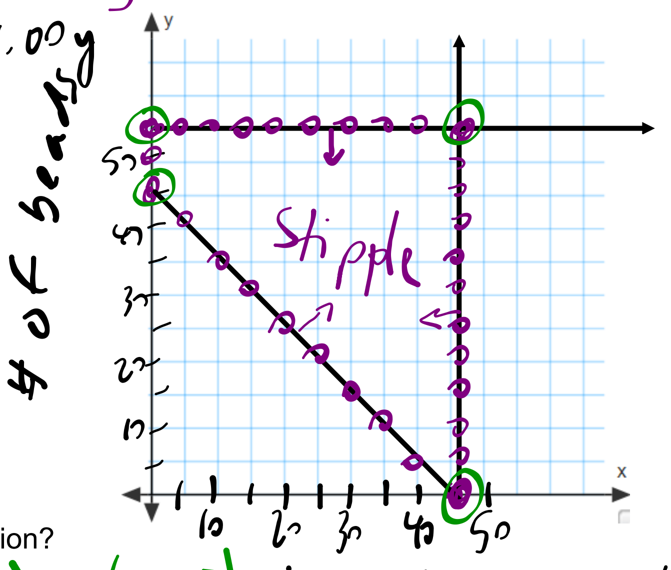
OBJECTIVE FUNCTION: $C = 0.75x + 1.00y$

a) Graph the solution set.

$x+y=45$

x int $(45,0)$

y int $(0,45)$



Intersections of overlap

b) What are the vertices of the feasible region?

$(45,0)$; $(45,55)$; $(0,55)$; $(0,45)$

c) Which point would result in the maximum value of the objective function?

d) Which point would result in the minimum value of the objective function?

Vertices	SUB OBJECTIVE $C = 0.75x + 1.00y$
$(0,45)$	45
Min $(45,0)$	33.75
$(0,55)$	55
Max $(45,55)$	88.75

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0.75*45+1.00*55
            88.75
0.75*0+1.00*55
               55
0.75*45
            33.75
█
```

45

GOAL

Solve optimization problems.

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EXPLORE...

• The following system of linear inequalities has been graphed below:

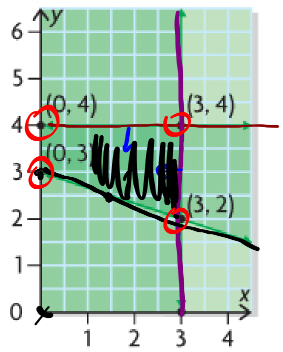
System of linear inequalities:
$y \geq 0$
$x \geq 0$
$y \leq 4$
$x \leq 3$
$3y \geq -x + 9$

Q1

Test (0, 0)
LS ≥ RS
0 | 9 No

$$\frac{3y}{3} = \frac{-x}{3} + \frac{9}{3}$$

$$y = -\frac{1}{3}x + 3$$



a) For each objective function, what points in the feasible region represent the minimum and maximum values?
i) $T = 5x + y$
ii) $T = x + 5y$

b) What do you notice about the optimal points for the two objective functions? Why do you think this happened?

SAMPLE ANSWER

a) i) For $T = 5x + y$,

If (x, y) is...	Then...	
(3, 2)	$T = 5(3) + 2$ $T = 17$	
(3, 4)	$T = 5(3) + 4$ $T = 19$	maximum
(0, 3)	$T = 5(0) + 3$ $T = 3$	minimum
(0, 4)	$T = 5(0) + 4$ $T = 4$	

ii) For $T = x + 5y$,

If (x, y) is...	Then...	
(3, 2)	$T = 3 + 5(2)$ $T = 13$	minimum
(3, 4)	$T = 3 + 5(4)$ $T = 23$	maximum
(0, 3)	$T = 0 + 5(3)$ $T = 15$	
(0, 4)	$T = (0) + 5(4)$ $T = 20$	

b) I noticed that the values of the coefficients of the variables and the values of the variables themselves all contribute to the value of the objective function. For $T = 5x + y$, the x-value is multiplied by 5 and the y-value is multiplied by 1. For $T = x + 5y$, the x-value is multiplied by 1 and the y-value is multiplied by 5. In each case, the greater the coordinate that is multiplied by 5, the greater the value of the objective function is. The converse is true for the least values.

EXAMPLE #1...

The vertices of the feasible region of a graph of a system of linear inequalities are (-4, -8); (5, 0) and (1, -6). Which point would result in the minimum value of the objective function C = 0.50x + 0.60y?

	$C = 0.50x + 0.60y$
min $(-4, -8)$	$0.50(-4) + 0.60(-8) = -6.8$
$(5, 0)$	$0.50(5) + 0.60(0) = 2.5$
$(1, -6)$	$0.50(1) + 0.60(-6) = -3.1$

* Sub your vertices into the objective to get max/min

EXAMPLE #2...

The following model represents an optimization problem. Determine the maximum solution.

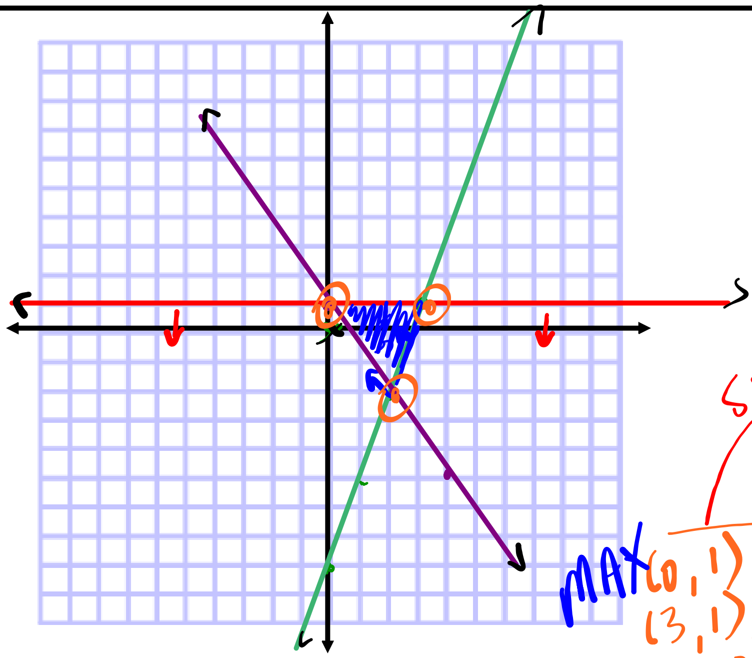
Restrictions: $x \varepsilon R \text{ and } y \varepsilon R$

Constraints: $y \le 1; 2y \ge -3x+2; y \ge 3x-8$

Objective Function: $D = -4x + 3y$

LS	$\ge$ RS
0	$3(0)-8$
	≥ -8 yes

LS	$\ge$ RS
$2(0)$	$-3(0)+2$
0	$\ge$ 2 No



$$\frac{2y}{2} = \frac{-3x}{2} + \frac{2}{2}$$

$$y = -\frac{3}{2}x + 1$$

SUB

	$D = -4x + 3y$
MAX (0, 1)	$-4(0) + 3(1) = 3$
(3, 1)	$-4(3) + 3(1) = -9$
(2, -2)	$-4(2) + 3(-2) = -14$

Max Solution is 3

HOMEWORK Questions...

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