HOMEWORK Questions???

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NOTE:

Create a model means graph the solution region

5. A football stadium has 50 000 seats.
   - Two-fifths of the seats are in the lower deck.
   - Three-fifths of the seats are in the upper deck.
   - At least 30 000 tickets are sold per game.
   - A lower deck ticket costs \$120, and an upper deck ticket costs \$80.

   Create a model that could be used to determine a combination of tickets for lower-deck and upper-deck seats that should be sold to maximize revenue.

$\frac{2}{5}(50000) = 20000$

30 000

objective

$R = 120x + 80y$

x → # of lower deck seats. $x \in W$

y → # of upper deck seats. $y \in W$

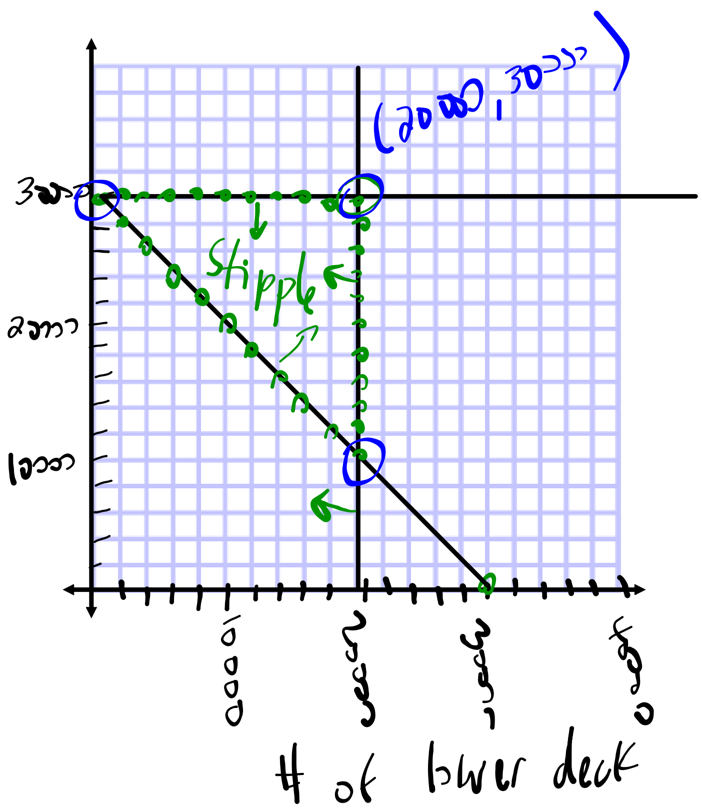
Graph

$x \leq 20000$ $y \leq 30000$ $x + y \geq 30000$

$x + y = 30000$

x int $(30000, 0)$

y int $(0, 30000)$



EXAMPLE of an OPTIMIZATION Problem...

Mick and Keith make MP3 covers to sell, using beads and stickers.

- At most, 45 covers with stickers and 55 bead covers can be made per day.
- Mick and Keith can make 45 or more covers, in total, each day. Objective
- It costs \$0.75 to make a cover with stickers, \$1.00 to make one with beads.

Let x represent the number of covers with stickers and let y represent the number of bead covers.

Let C represent the cost of making the covers.

RESTRICTIONS: $x \in W$ $y \in W$

CONSTRAINTS: $x \le 45$ $y \le 55$ $x + y \ge 45$ ← Graph

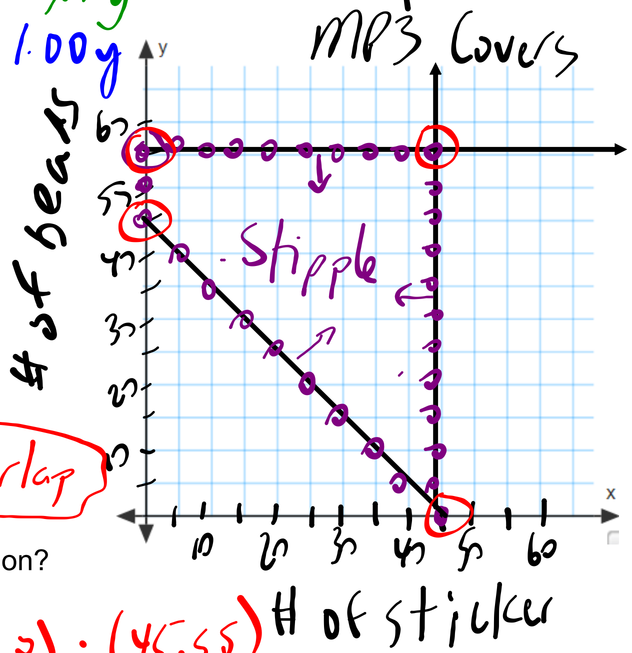
OBJECTIVE FUNCTION: $C = 0.75x + 1.00y$

a) Graph the solution set.

$x + y = 45$

x int $(45, 0)$

y int $(0, 45)$



Intersections of overlap

b) What are the vertices of the feasible region?

$(0, 45)$; $(0, 55)$; $(45, 0)$; $(45, 55)$

c) Which point would result in the maximum value of the objective function?

$(45, 55)$

d) Which point would result in the minimum value of the objective function?

$(45, 0)$

SUB

Vertex	$C = 0.75x + 1.00y$
$(0, 45)$	$0.75(0) + 1.00(45) = \$45$
$(0, 55)$	$0.75(0) + 1.00(55) = \$55$
Min $(45, 0)$	$0.75(45) + 1.00(0) = \$33.75$
Max $(45, 55)$	$0.75(45) + 1.00(55) = \$88.75$

GOAL

Solve optimization problems.

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EXPLORE...

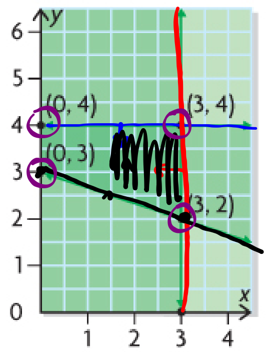
- The following system of linear inequalities has been graphed below:

System of linear inequalities:

$y \geq 0$
$x \geq 0$ Q1
$y \leq 4$
$x \leq 3$
$3y \geq -x + 9$

LS ≥ RS

0	9



$\frac{3y}{3} = \frac{-x}{3} + \frac{9}{3}$

$y = -\frac{1}{3}x + 3$

* Sub vertex into the objective to find min/max

a) For each objective function, what points in the feasible region represent the minimum and maximum values?
i) $T = 5x + y$
ii) $T = x + 5y$

b) What do you notice about the optimal points for the two objective functions? Why do you think this happened?

SAMPLE ANSWER

a) i) For $T = 5x + y$,

If (x, y) is...	Then...	
(3, 2)	$T = 5(3) + 2$ $T = 17$	
(3, 4)	$T = 5(3) + 4$ $T = 19$	maximum
(0, 3)	$T = 5(0) + 3$ $T = 3$	minimum
(0, 4)	$T = 5(0) + 4$ $T = 4$	

ii) For $T = x + 5y$,

If (x, y) is...	Then...	
(3, 2)	$T = 3 + 5(2)$ $T = 13$	minimum
(3, 4)	$T = 3 + 5(4)$ $T = 23$	maximum
(0, 3)	$T = 0 + 5(3)$ $T = 15$	
(0, 4)	$T = (0) + 5(4)$ $T = 20$	

b) I noticed that the values of the coefficients of the variables and the values of the variables themselves all contribute to the value of the objective function. For $T = 5x + y$, the x-value is multiplied by 5 and the y-value is multiplied by 1. For $T = x + 5y$, the x-value is multiplied by 1 and the y-value is multiplied by 5. In each case, the greater the coordinate that is multiplied by 5, the greater the value of the objective function is. The converse is true for the least values.

EXAMPLE #1...

The vertices of the feasible region of a graph of a system of linear inequalities are (-4, -8); (5, 0) and (1, -6). Which point would result in the minimum value of the objective function C = 0.50x + 0.60y?

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	vertex	$C = 0.50x + 0.60y$
Min	$\boxed{(-4,-8)}$	$0.5(-4) + 0.6(-8) = -6.8$
	$(5,0)$	$0.5(5) + 0.6(0) = 2.5$
	$(1,-6)$	$0.5(1) + 0.6(-6) = -3.1$

EXAMPLE #2...

The following model represents an optimization problem. Determine the maximum solution.

Restrictions: $x \varepsilon R$ and $y \varepsilon R$

Constraints: $y \le 1$; $2y \ge -3x + 2$; $y \ge 3x - 8$

Objective Function: $D = -4x + 3y$

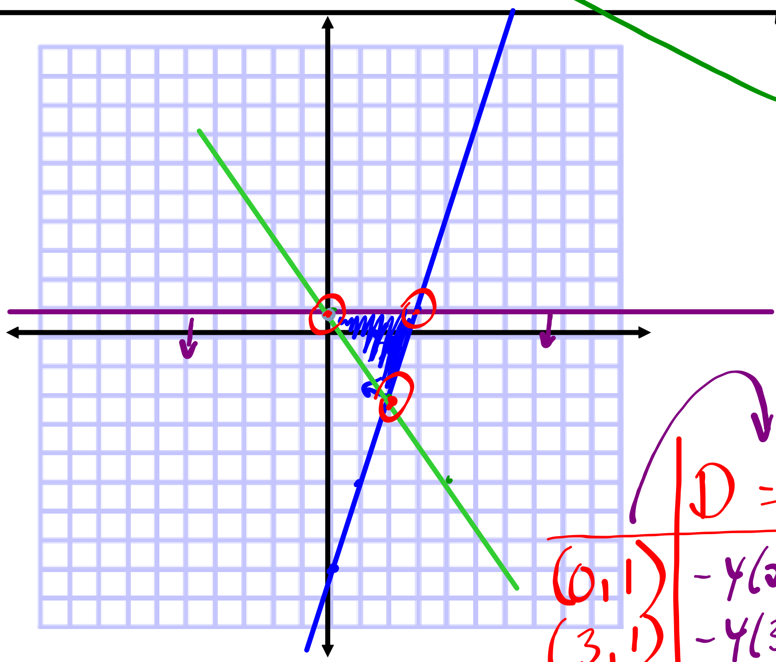
LS	$\ge$ RS
0	$3(0) - 8$
$>$	-8

$\frac{2y}{2} = \frac{-3x}{2} + \frac{2}{2}$

$y = -\frac{3}{2}x + 1$

LS	$\ge$ RS
$2(0)$	$-3(0) + 2$
$0 \ge$	2

No



sub

	$D = -4x + 3y$
$(0,1)$	$-4(0) + 3(1) = 3$ Max Solution
$(3,1)$	$-4(3) + 3(1) = -9$
$(2,-2)$	$-4(2) + 3(-2) = -14$

HOMEWORK Questions...

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