February 14, 2018

UNIT 1: ROOTS AND POWERS

SECTION 4.3: MIXED AND ENTIRE RADICALS

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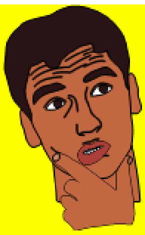
NUMBERS, RELATIONS AND FUNCTIONS 10

WHAT'S THE POINT OF TODAY'S LESSON?

We will begin working on the NRF 10 Specific Curriculum Outcome (SCO) "Algebra and Numbers 3" OR "AN3" which states:

"Demonstrate an understanding of powers with integral and rational exponents."



What does THAT mean???

SCO AN3 means that we will:

* apply the 6 exponent laws you learned in grade 9:
  $a^0 = 1$
  $(a^m)(a^n) = a^{m+n}$
  $a^m \div a^n = a^{m-n}$
  $(a^m)^n = a^{mn}$
  $(ab)^m = a^m b^m$
  $(a \div b)^n = a^n \div b^n$
* use patterns to explain $a^{-n} = \dfrac{1}{a^n}$ and $a^{\frac{1}{n}} = \sqrt[n]{a}$
* apply all exponent laws to evaluate a variety of expressions
* express powers with rational exponents as radicals and vice versa
* identify and correct errors in work that involves powers

HOMEWORK QUESTIONS???
(page 211, #3 TO #5, #7, #8, #10 and #12 TO #14)

RADICALS CAN BE WRITTEN AS PRODUCTS: ...

EX.: $\sqrt{16 \cdot 9}$ **AND** $\sqrt{16} \cdot \sqrt{9}$

$= \sqrt{144}$ $\qquad$ $= 4 \cdot 3$

$= 12$ $\qquad$ $= 12$

JUST AS WITH FRACTIONS, RADICALS CAN BE WRITTEN AS EQUIVALENT EXPRESSIONS:

EX.: $\sqrt[3]{8 \cdot 27}$ **AND** $\sqrt[3]{8} \cdot \sqrt[3]{27}$

$= \sqrt[3]{216}$ $\qquad$ $= 2 \cdot 3$

$= 6$ $\qquad$ $= 6$

EXPONENT LAWS (separate sheet):

1. **Zero Exponent Law:** $a^0 = 1$ $2^0 = 1$
2. **Product of Powers:** $(a^m)(a^n) = a^{m+n}$
   $3^2 \cdot 3^3 = 3^5$
3. **Quotient of Powers:** $a^m \div a^n = a^{m-n}$
4. **Power of a Power:** $(a^m)^n = a^{mn}$
5. **Power of a Product:** $(ab)^m = a^m b^m$ $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$
6. **Power of a Quotient:** $(a \div b)^n = a^n \div b^n$

$$m^{-2} = \frac{1}{m^2}$$

$$\frac{6^{-3}2^2}{4^{-2}} = \frac{2^2 \cdot 4^2}{6^3} = \frac{4 \times 16}{216} = \frac{64}{216} = \frac{8}{27}$$

7. MULTIPLICATION ~~PROPERTY~~ OF RADICALS:

$\boxed{\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}}$ $(ab)^{\frac{1}{n}} = a^{\frac{1}{n}} b^{\frac{1}{n}}$ (#5)

EX.: $\sqrt{24}$ (Factors: 1, 2, 3, 4, 6, 8, 12, 24)

$= \sqrt{4 \cdot 6}$

$= \sqrt{4} \cdot \sqrt{6}$

$= 2 \cdot \sqrt{6}$

$= 2\sqrt{6}$ (MIXED RADICAL) like mixed fraction $2\frac{1}{2}$

EX.: $\sqrt[3]{24}$ (ENTIRE RADICAL)

$= \sqrt[3]{8 \cdot 3}$

$= \sqrt[3]{8} \cdot \sqrt[3]{3}$

$= 2 \cdot \sqrt[3]{3}$

$= 2\sqrt[3]{3}$

secret '2'

$\sqrt{a} = a^{\frac{1}{2}}$

$\sqrt{9} = 9^{\frac{1}{2}}$ 9 y^x (1/2)

$\sqrt[3]{27} = 27^{\frac{1}{3}}$
$= 3$

$\sqrt[4]{2401} = 2401^{\frac{1}{4}}$
$= 7$

8. POWERS WITH RATIONAL EXPONENTS WITH A NUMERATOR OF 1:

$$x^{\frac{1}{n}} = \sqrt[n]{x}$$

EX.:

$$8^{\frac{1}{3}}$$
$$= \sqrt[3]{8}$$
$$= 2$$

9. POWERS WITH RATIONAL EXPONENTS:

EXPONENT

$$x^{\frac{m}{n}} = \left(x^{\frac{1}{n}}\right)^m$$
$$= \left(\sqrt[n]{x}\right)^m$$

INDEX

AND

EXPONENT

$$x^{\frac{m}{n}} = \left(x^m\right)^{\frac{1}{n}}$$
$$= \sqrt[n]{x^m}$$

INDEX

EX.: **Evaluate** $16^{\frac{3}{2}}$.

3 (EXPONENT)

2 (INDEX)

$$16^{\frac{3}{2}}$$
$$= \left(\sqrt[2]{16}\right)^3$$
$$= 4^3$$
$$= 64$$

OR

(EXP.)

(INDEX)

$$16^{\frac{3}{2}}$$
$$= \sqrt[2]{16^3}$$
$$= \sqrt{4096}$$
$$= 64$$

NOTE: THERE ARE SOME RADICALS THAT CANNOT BE SIMPLIFIED.

EX.: $\sqrt[4]{24}$ **(Factors: 1, 2, 3, 4, 6, 8, 12, 24)**

24 HAS NO FACTORS OTHER THAN 1 THAT CAN BE WRITTEN AS A FOURTH POWER; THEREFORE, IT CANNOT BE SIMPLIFIED (WRITTEN AS A MIXED RADICAL).

$\sqrt[4]{24} = \sqrt[4]{2 \times 12}$

$2 \quad 2 \times 6$

$2 \times 2 \times 2 \times 3$

$= \sqrt[4]{24}$

$\sqrt{24} = 2 \times 12$

$2 \quad 2 \times 6$

$2 \times 2 \times 2 \times 3$

$2\sqrt{6}$

WE CAN ALSO USED PRIME FACTORIZATION TO SIMPLIFY A RADICAL.

EX.: Simplify each radical.

a) $\sqrt{80}$

8×10

$2 \times 4 \times 2 \times 5$

$2 \cdot 2 \; 2 \times 2 \times 5$

$2 \times 2 \sqrt{5}$

$4\sqrt{5}$

b) $\sqrt[3]{144}$

12×12

$3 \times 4 \times 3 \times 4$

$3 \times 2 \; 2 \times 3 \times 2 \times 2$

$2\sqrt[3]{18}$

c) $\sqrt[4]{162}$

$\sqrt[4]{2} \times \sqrt[4]{81}$

$3\sqrt[4]{2}$

YOU TRY!

EX.: Simplify each radical.

a) $\sqrt{63}$

9×7

$3\sqrt{7}$

b) $\sqrt[3]{108}$

2×54

$2 \times 2 \times 27$

$2 \quad 2 \quad 9 \times 3$

$2 \times 2 \times 3 \times 3 \times 3$

$3\sqrt[3]{4}$

c) $\sqrt[4]{128}$

4×32

$2 \times 2 \times 4 \times 8$

$2 \; 2 \; 2 \times 2 \times 2 \times 2 \times 2$

$2\sqrt[4]{8}$

LET'S TRY TO SIMPLIFY RADICALS WITHOUT USING PRIME FACTORIZATION, IF POSSIBLE.

EX.: Write each radical in simplest form, if possible.

a) $\sqrt[3]{40}$ b) $\sqrt{26}$ c) $\sqrt[4]{32}$

a) $\sqrt[3]{8 \times 5}$

$= 2\sqrt[3]{5}$

b) $= \sqrt{26}$

c) $= \sqrt[4]{16 \times 2}$

$= 2\sqrt[4]{2}$

YOU TRY TO SIMPLIFY RADICALS WITHOUT USING PRIME FACTORIZATION, IF POSSIBLE.

EX.: Write each radical in simplest form, if possible.

a) $\sqrt{30}$ b) $\sqrt[3]{32}$ c) $\sqrt[4]{48}$

a) $\sqrt{30}$

b) $\sqrt[3]{8 \times 4}$

$2\sqrt[3]{4}$

c) $\sqrt[4]{16 \times 3}$

$2\sqrt[4]{3}$

YOU TRY **Write as a simplified radical.**

1. $\sqrt[3]{256}$

2×128

$2 \times 2 \times 64$

$2 \quad 2 \times 8 \times 8$

$2 \quad 2 \quad 2 \times 4 \quad 2 \times 4$

$2 \quad 2 \quad 2 \quad 2 \times 2 \quad 2 \quad 2 \times 2$

$4\sqrt[3]{4}$

2. $\sqrt[4]{48}$

6×8

$2 \times 3 \quad 2 \times 4$

$2 \quad 3 \quad 2 \times 2 \times 2$

$2\sqrt[4]{3}$

3. $\sqrt{108}$

$\sqrt{3} \times \sqrt{36}$

$6\sqrt{3}$

WRITING MIXED RADICALS AS ENTIRE RADICALS:

EX.: Write each mixed radical as an entire radical.

a) $4\sqrt{3}$

$= \sqrt{4 \times 4 \times 3}$

$= \sqrt{48}$

b) $3\sqrt[3]{2}$

$\sqrt[3]{3 \times 3 \times 3 \times 2}$

$\sqrt[3]{54}$

c) $2\sqrt[5]{2}$

$\sqrt[5]{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}$

$\sqrt[5]{64}$

YOU TRY WRITING MIXED RADICALS AS ENTIRE RADICALS:

EX.: Write each mixed radical as an entire radical.

a) $7\sqrt{3}$

$=\sqrt{7\times7\times3}$

$=\sqrt{147}$

b) $2\sqrt[3]{4}$

$\sqrt[3]{2\times2\times2\times4}$

$\sqrt[3]{32}$

c) $2\sqrt[5]{3}$

$=\sqrt[5]{2\times2\times2\times2\times2\times3}$

$=\sqrt[5]{96}$

CONCEPT REINFORCEMENT:

***FPCM 10*:**

Page 211: 3, 4, 11
Page 218: 4, 5, 10 - 12, 14, 15, 16, 18
Page 219: 21, 22b, 24

QUIZ PREPARATION:

***FPCM 10*:**

Page 221: #1, #3, #4, #6a, #7b, #8, #9 & #11

https://mathmalfunctions.wordpress.com/exponential-functions/exponent-laws/