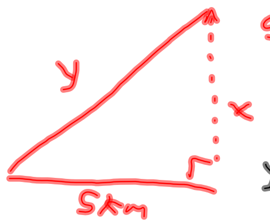
Warm Up

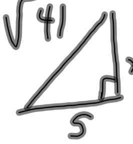
A rocket, rising vertically, is tracked by a radar station that is on the ground 5 km from the launch pad. How fast is the rocket rising when it is 4 km high, if its distance from the radar station is increasing at the rate of 2000 km/h?



$\frac{dy}{dt} = 2000\text{km/h}$

$x^2 + 25 = y^2$

$y = \sqrt{41}$



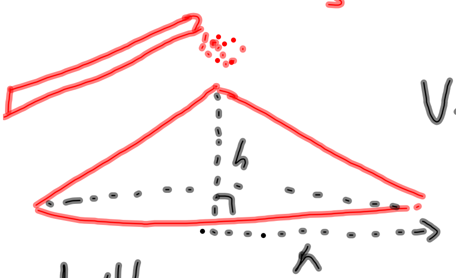
$2x\frac{dx}{dt} = 2y\frac{dy}{dt}$

$2(4)\frac{dx}{dt} = 2(\sqrt{41})(2000)$

$\frac{dx}{dt} = \frac{4000\sqrt{41}}{8} = 3201.6\text{km/h}$

Gravel is being dumped to the ground from a conveyor belt at the rate of 24 ft.^{3}/min. and its coarseness is such that it forms a pile in the shape of a cone whose base diameter is always four times the height of the pile. How fast is the area of the base of the pile increasing when the pile is 12 ft high?

$V = \frac{1}{3}\pi r^2 h$ $\rightarrow$ $A = \pi r^2$



$V = \frac{1}{3}\pi r^2\left(\frac{r}{2}\right)$

$V = \frac{1}{6}\pi r^3$

$\frac{1}{6}\pi\left(3r^2\frac{dr}{dt}\right)$

$d = 4h$

$2r = 4h$

$r = 2h$

$h = \frac{r}{2}$

$12 = \frac{r}{2}$

$24 = r$

$\frac{dV}{dt} = \frac{1}{2}\pi r^2\frac{dr}{dt}$

$24 = \frac{1}{2}\pi(24)^2\frac{dr}{dt}$

$\frac{dr}{dt} = \frac{\cancel{24}(2)}{(24)^{\cancel{2}}(\pi)}$

$\frac{dr}{dt} = \frac{1}{12\pi}$

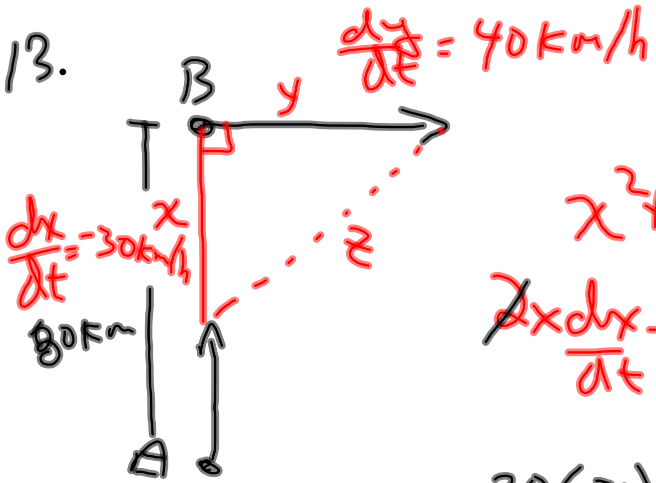
$A = \pi r^2$

$\frac{dA}{dt} = 2\pi r\frac{dr}{dt}$

$= 2\cancel{\pi}\left(\cancel{24}^{2}\right)\left(\frac{1}{\cancel{12\pi}}\right)$

$= 4\ \text{ft}^2/\text{min}$

13.

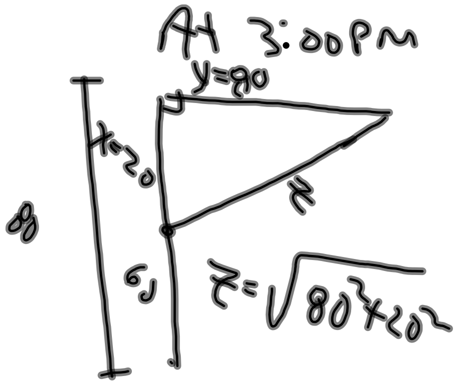


$$x^2 + y^2 = z^2$$

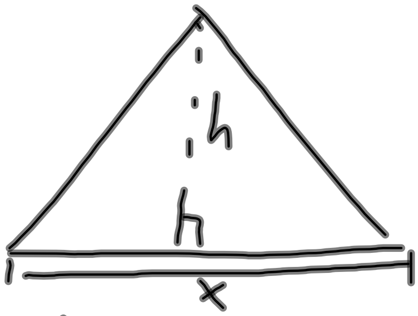
$$\cancel{2}x\frac{dx}{dt} + \cancel{2}y\frac{dy}{dt} = \cancel{2}z\frac{dz}{dt}$$

$$20(-30) + 80(40) = \left(\sqrt{80^2+20^2}\right)\frac{dz}{dt}$$

$$\frac{dz}{dt} = \frac{-600+3200}{\sqrt{80^2+20^2}} \text{ km/h}$$



9.



$$A = \frac{1}{2} x h$$

$$\frac{dA}{dt} = \frac{1}{2}\left(\frac{dx}{dt} h + x \frac{dh}{dt}\right)$$

$$\frac{dA}{dt} = 4 \text{ cm}^2/\text{min.}$$

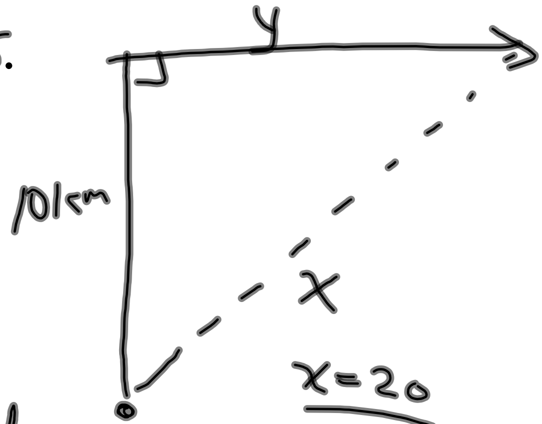
$$\frac{dx}{dt} = 1 \text{ cm}/\text{min.}$$

$$80 = \frac{1}{2} x h$$

$$80 = \frac{1}{2} x(20)$$

$$8 = x$$

15.



$\underline{x=20}$

$\frac{dy}{dt}=600km/h$

$$y^2+100=x^2$$

$$2y\frac{dy}{dt}=2x\frac{dx}{dt}$$

Bonus Problem

The minute hand on a watch is 8 mm long and the hour hand is 4 mm long. How fast is the distance between the tips of the hands changing at one o'clock?

(18.6mm/h)