Gas is being pumped into a spherical balloon at the rate of 3 cm^3/s.

(a) How fast is the radius increasing when the radius is 15 cm?

(b) Without using the result from (a), find the rate at which the surface area of the balloon is increasing when the radius is 15 cm.

$V = \frac{4}{3}\pi r^3$ $\quad SA = 4\pi r^2$ $\quad V = 1125\pi$ $\quad V = \frac{4}{3}\pi r^3$

a) $\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$3 = 4\pi(15)^2 \frac{dr}{dt}$

$\frac{3}{900\pi} = \frac{dr}{dt}$

$\frac{dr}{dt} = 0.001 cm/s$

b) $SA = 4\pi\left(\frac{3V}{4\pi}\right)^{2/3}$ $\quad \frac{3V}{4\pi} = r^3$

$\left(\frac{3V}{4\pi}\right)^{1/3} = r$

$\frac{dSA}{dt} = \frac{8\pi}{3}\left(\frac{3V}{4\pi}\right)^{-1/3}\left(\frac{3}{4\pi}\frac{dV}{dt}\right)$

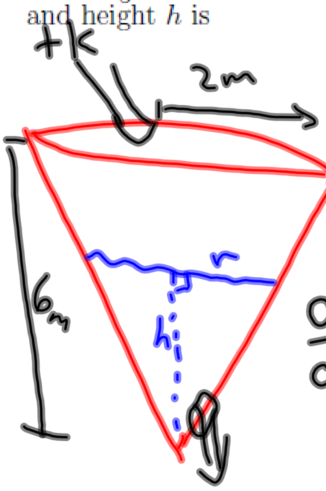
$= \frac{\cancel{8}^{2}\cancel{\pi}}{\cancel{3}}\left(\frac{3(1125\cancel{\pi})}{4\cancel{\pi}}\right)^{-1/3}\left(\frac{\cancel{3}}{\cancel{4}\cancel{\pi}}\right)(3)$

$= 6(3375)^{-1/3}$

$= 0.4\ cm^2/sec$

Water is leaking out of the bottom of an inverted conical tank at the rate of $\frac{1}{10}$ m^3/min, and at the same time is being pumped in the top at a constant rate of k m^3/min. The tank has height 6 m and the radius at the top is 2 m. Determine the constant k if the water level is rising at the rate of $\frac{1}{5}$ m/min when the height of the water is 2 m. Recall that the volume of a cone of radius r and height h is

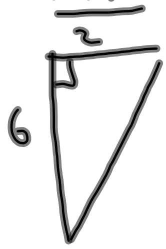
$$V = \frac{1}{3}\pi r^2 h.$$



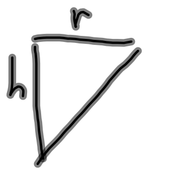
$$\frac{dh}{dt} = \frac{1}{5}\, m/min.$$

$$\frac{dV}{dt} = \left(k - \frac{1}{10}\right) m^3/min.$$

Tank



Water



$$\frac{2}{r} = \frac{6}{h}$$

$$\frac{2h}{6} = \frac{6r}{6}$$

$$r = \frac{1}{3}h$$

$$\frac{dr}{dt} = \frac{1}{3}\frac{dh}{dx}$$

$$V = \frac{1}{3}\pi\left(\frac{1}{3}h\right)^2 h$$

$$V = \frac{1}{27}\pi h^3$$

$$\frac{dV}{dt} = \frac{3}{27}\pi h^2 \frac{dh}{dt}$$

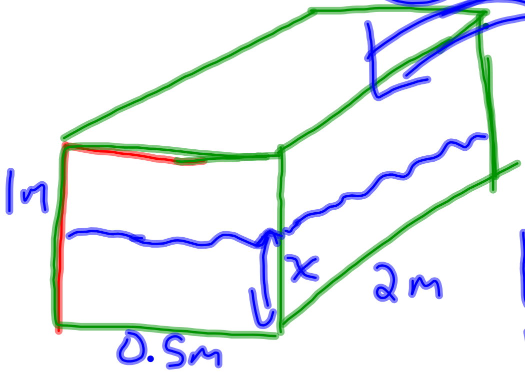
$$k - \frac{1}{10} = \frac{1}{9}\pi(2)^2\left(\frac{1}{5}\right)$$

$$k = \frac{4\pi}{45} + \frac{1}{10}$$

$$\underline{k = 0.379\, m^3/min}$$

A rectangular trough is 2 meter long, 0.5 meter across the top and 1 meter deep. At what rate must water be poured into the trough such that the depth of the water is increasing at 1 m/min when the depth of the water is 0.7 m?

$\frac{dV}{dt} = ?$



$V = \frac{1}{2}(2)x$

$V = x$

$\frac{dx}{dt} = 1 m/min.$

$\frac{dV}{dt} = \frac{dx}{dt}$

$\frac{dV}{dt} = 1 \, m^3/min.$