(1) Midterm: <u>Dec. 5/2011</u>

Intervals of Increase and Decrease...

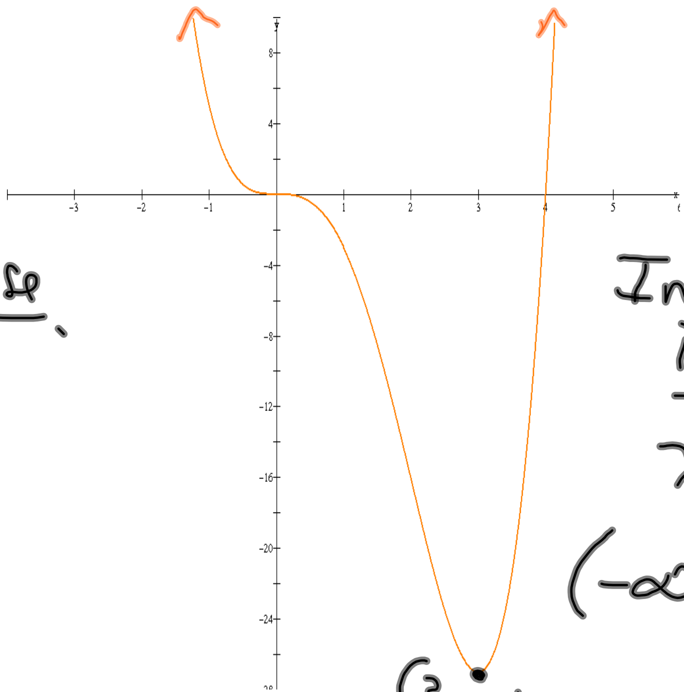
Given the function $f(x) = x^4 - 4x^3$, use the graph below to determine the intervals where $f(x)$ is increasing and where $f(x)$ is decreasing.

Graph...

Intervals of Increase

$x > 3$

$(3, \infty)$



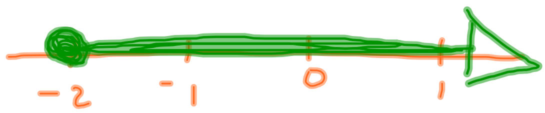
$(3, -27)$

Intervals of Decrease

$x < 3$

$(-\infty, 3)$

Bracket Notation



$x \geq -2$

$[-2, \infty)$

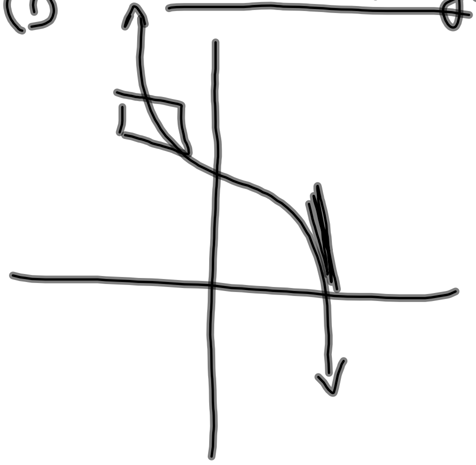
$(-1, 2]$

$(-\infty, 3)$

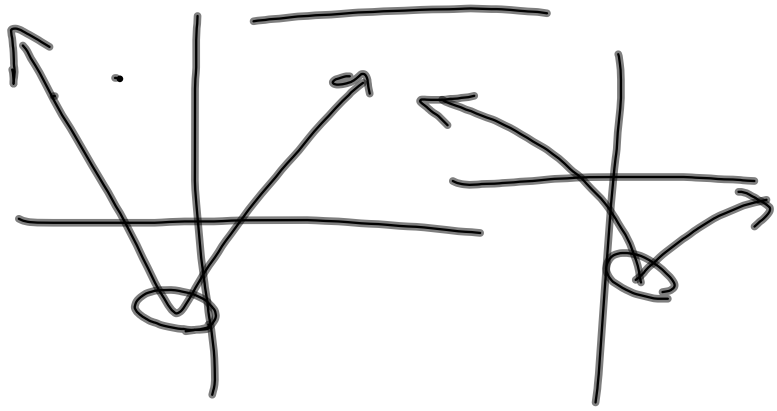
$(2, 3)$

$f'(x)$ undefined

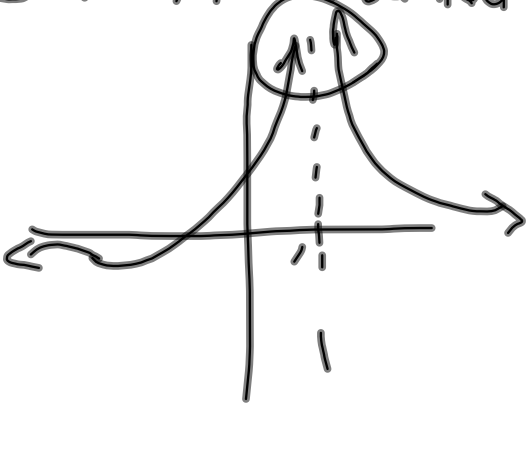
① Vertical tangent



② $f(x)$ comes to a corner

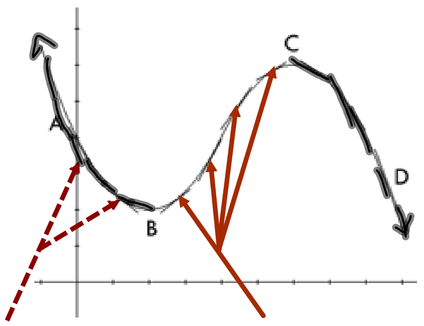


③ $f(x)$ is undefined



The Calculus of Intervals of Increase and Decrease

- Examine this graph for intervals of increase and decrease...



Intervals of increase?

(B, C)

Intervals of decrease?

$(-\infty, B)$ & (C, ∞)

What do you notice about the slopes of these tangents?

What do you notice about the slopes of these tangents?

*Critical Value(s):

Any value of x such that $f'(x) = 0$ or $f'(x)$ is undefined.

Where does $f(x)$ switch from increasing to decreasing? How would this tie in with Calculus?

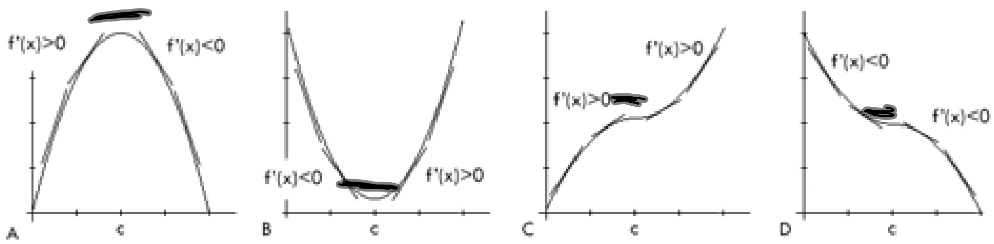
- *At the point where a function switches from increasing to decreasing, or decreasing to increasing, the derivative must be equal to 0 or undefined.*

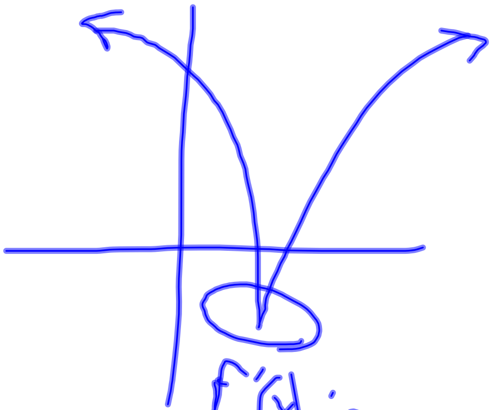
Let's summarize how Calculus could be used to identify regions of increase or decrease...

If $f'(x) > 0$ ------→ f(x) is increasing

If $f'(x) < 0$ ------→ f(x) is decreasing

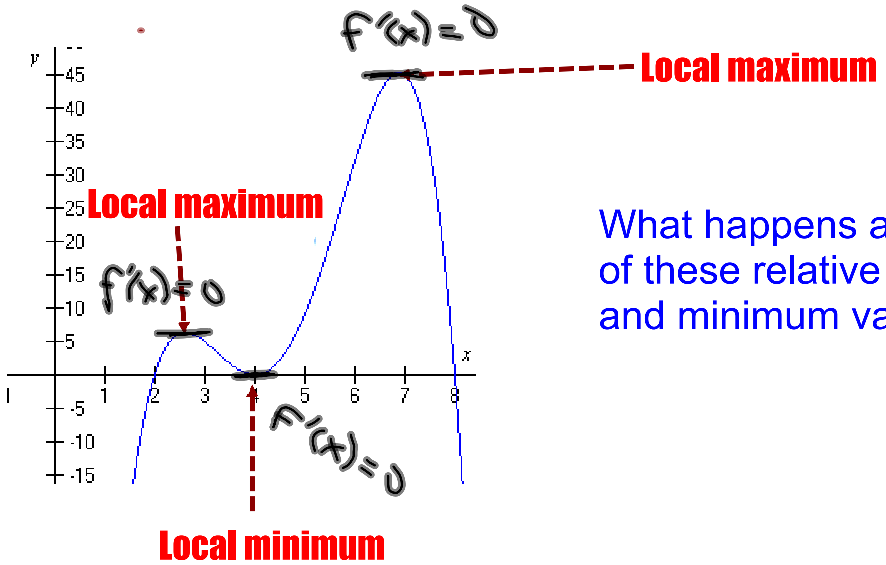
The graphs below illustrate the first derivative test.





$f'(x)$ is undefined

Relative Extrema: (local maximum or local minimum)



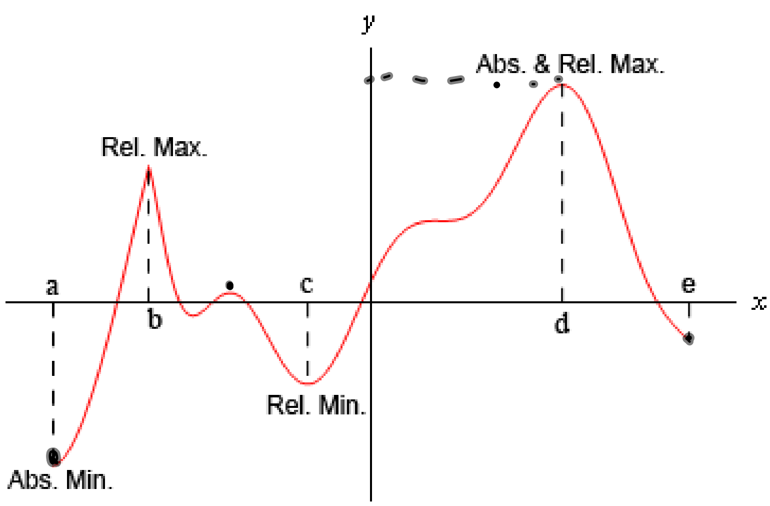
What happens at each of these relative maximum and minimum values?

We say that *f(x)* has a **relative (or local) maximum** at $x = c$ if $f(x) \le f(c)$ for every x in some open interval around $x = c$.

- function switches from increasing to decreasing.

We say that *f(x)* has a **relative (or local) minimum** at $x = c$ if $f(x) \ge f(c)$ for every x in some open interval around $x = c$.

- function switches from decreasing to increasing.



First Derivative Sign Table:

Intervals of increase and decrease can be organized using a first derivative sign table...

Example:

Determine the intervals of increase and decrease for the function ...

$$f(x) = x^4 - 4x^3 + 2$$

① Critical Values:

$f'(x) = 4x^3 - 12x^2$

$0 = 4x^2(x-3) = f'(x)$

Critical Values: $x = 0$ or $x = 3$

Sign Table

Interval	factors $4x^2$	factors $x-3$	$f'(x)$	$f(x)$
$(-\infty, 0)$	+	−	−	Dec
$(0, 3)$	+	−	−	Dec
$(3, \infty)$	+	+	+	Inc

LOCAL MAX.

None

LOCAL MIN.

$(3, -25)$

Warm-Up

Complete the following question using a TI-83 graphing calculator.

8. Consider the function $f(x) = x^4 - 4x^3$. *(UNB Final Exam: 2005)*

   (a) Determine the intervals where $f(x)$ is increasing, where $f(x)$ is decreasing, and all local maxima and minima of $f(x)$, if any.

Warm Up

University of British Columbia: 2004

For the function

$$f(x) = \frac{x^2}{x^2 - 1}$$

determine the critical points, local maxima and minima, and intervals where f(x) is increasing or decreasing.

$$f'(x) = \frac{2x(x^2-1) - x^2(2x)}{(x^2-1)^2}$$

$$f'(x) = \frac{2x^3 - 2x - 2x^3}{(x^2-1)^2}$$

$$f'(x) = \frac{-2x}{(x^2-1)^2}$$

Critical Values:

$f'(x) = 0$

$-2x = 0$

$x = 0$

f'(x) undefined (Set Den. = 0)

$(x^2-1)^2 = 0$

$[(x-1)(x+1)]^2 = 0$

$x = 1, -1$

	$-2x$	$(x^2-1)^2$	f'	f
$(-\infty, -1)$	+	+	+	Inc
$(-1, 0)$	+	+	+	Inc
$(0, 1)$	−	+	−	Dec
$(1, \infty)$	−	+	−	Dec

Local Max.

(0, 0)

Local Min.

None