# Laws of Exponents

**Remember...** $b^x$ → **"b raised to the power of $x$" where, b – base**
**$x$ – exponent**

#1. PRODUCT - when multiplying...
"if the base is the same, then ADD the exponents."

$$b^m \times b^n = b^{m+n}$$

EXAMPLES:

$(x^7)(x^8) = x^{15}$

$(3)^7(3)^{10} = (3)^{17}$

$(3x-7)^{10}(3x-7)^4 = (3x-7)^{14}$

#2. QUOTIENT - when dividing...
"if the base is the same, then SUBTRACT the exponents."

$$\frac{b^m}{b^n} = b^{m-n}, \ b \neq 0$$

EXAMPLES:

$\frac{w^{12}}{w^4} = w^8$

$\frac{(12)^{10}}{(12)^1} = 12^9$

$\frac{(3a+4)^{17}}{(3a+4)^{10}} = (3a+4)^7$

#3. POWER - when raising a power to another power...
MULTIPLY the exponents."

$$(b^m)^n = b^{m \times n}$$

EXAMPLES:

$(x^7)^3 = x^{21}$

$(3^{-5})^3 = 3^{-15}$

$(w^2)^2 = w^4$

#4. POWER of a PRODUCT -

"when a product is raised to a power, each of the factors are raised to the power."

$$\boxed{(a \times b)^m = a^m \times b^m}$$

EXAMPLES:

$(x^3y^4)^6$

$= x^{18}y^{24}$

$(3x^6y^5)^2$

$= 9x^{12}y^{10}$

#5. POWER of a QUOTIENT -

"when a quotient is raised to a power, both the divisor and the dividend are raised to the power."

$$\boxed{\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}, \quad a \,\&\, b \neq 0}$$

EXAMPLES:

$\left(\frac{x^7}{y^{10}}\right)^3 = \frac{x^{21}}{y^{30}}$

$\left(\frac{s^3}{s^1}\right)^{10}$

$(s^2)^{10} = s^{20}$

$\frac{s^{30}}{s^{10}}$

$= s^{20}$

#6. ZERO -

"any base raised to the power of zero is <u>ALWAYS</u> equal to 1."

$$\boxed{b^0 = 1}$$

EXAMPLES:

$w^0 = 1$

$(99)^0 = 1$

$3w^0 = 3(1) = 3$

$(3w)^0 = 1$

$\frac{w^{10}}{w^{10}} = w^0 = 1$

#7. Negative -

"any base is raised to a negative exponent equals the reciprocal base (FLIP) raised to the positive exponent."

$$b^{-m} = \frac{1}{b^m} \quad \textbf{OR} \quad \left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^m = \frac{b^m}{a^m}$$

EXAMPLES:

$3^{-2}$

$= \frac{1}{9}$

$\left(\frac{3}{5}\right)^{-2} = \left(\frac{5}{3}\right)^2$

$= \frac{25}{9}$

$4 \cdot 3^{-2} = \frac{1}{4(3)^2}$

$= \frac{1}{36}$

NOTE:

$$(negative\ base)^{even\ exponent} \rightarrow postive\ answer$$

**AND**

$$(negative\ base)^{odd\ exponent} \rightarrow negative\ answer$$

$\frac{3}{4^{-1}} = 3(4)^1$

$= \frac{1}{12}$

$(-3)^{106} = 3^{106}$

$(-3)^{105} = -3^{105}$

$-3^2 = -9 \quad (-3)^2 = 9 \quad (-3^2) = -9$

## *Exponents and Radicals*

Earlier, you learned that powers with integral exponents have a special meaning. The exponent $\frac{1}{2}$ has a special meaning related to the principal square root of a number.

$\sqrt{2} = 2^{\frac{1}{2}}$ $\sqrt{3} = 3^{\frac{1}{2}}$ $\sqrt{5} = 5^{\frac{1}{2}}$ principal square roots

In order to learn mathematics, it is helpful to make comparisons.

- The cube of 2 is 8, since $2 \times 2 \times 2 = 8$.
- The cube of 2 is shown by the symbol $2^3 = 8$.
- The principal cube root of 8 is 2, since $2 \times 2 \times 2 = 8$.
- The principal cube root of 8 is shown by the radical symbol $\sqrt[3]{8} = 2$. means the principal cube root of 8.

Similarly, exponents that are rational have a special meaning.

*Using exponent laws*

$8^{\frac{1}{3}} \times 8^{\frac{1}{3}} \times 8^{\frac{1}{3}} = 8^{\frac{1}{3}+\frac{1}{3}+\frac{1}{3}}$
$= 8 \longleftarrow$ Compare. $\longrightarrow = 8$

*Using radicals*

$\sqrt[3]{8} \times \sqrt[3]{8} \times \sqrt[3]{8}$

Based on the above comparison, $\sqrt[3]{8}$ and $8^{\frac{1}{3}}$ behave in a similar way.

It seems reasonable to define $\sqrt[3]{8} = 8^{\frac{1}{3}}$

In general, the $n$th principal root of a number is shown by $\sqrt[n]{a} = a^{\frac{1}{n}}$

$\sqrt[13]{17} = 17^{\frac{1}{13}}$

$$\boxed{\sqrt[n]{a} = a^{\frac{1}{n}}}$$

**Fraction Exponents - To evaluate exponents that are fractions, the denominator of the fraction indicates which root to take and the numerator indicates which power the entire base is to be raised.**

$(a^{1/3})^2$

$a^{2/3}$

$$\boxed{a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m}$$

OR $\sqrt[n]{a^m}$

$(a^{1/n})^m$ $(a^m)^{1/n}$

Examples:

1. $8^{\frac{2}{3}} =$

$(\sqrt[3]{8})^2$

$= 4$

OR

$\sqrt[3]{8^2}$

2. $125^{-\frac{1}{3}} =$

$= \frac{1}{125^{1/3}}$

$= \frac{1}{\sqrt[3]{125}}$

$= \frac{1}{5}$

3. $32^{-\frac{7}{5}} =$

$= \frac{1}{32^{7/5}}$

$= \frac{1}{(\sqrt[5]{32})^7}$

$= \frac{1}{128}$

## More Examples...

$a)\ 125^{-\frac{2}{3}} = \dfrac{1}{\left(\sqrt[3]{125}\right)^2}$

$= \dfrac{1}{25}$

$b)\ 256^{0.375}$

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256^.375                8
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$= (256)^{3/8} = \left(\sqrt[8]{256}\right)^3$

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256^.375          8
256^(1/8)         2
8*√256            2
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$c)\ 8^{-\frac{1}{3}} = \dfrac{1}{\sqrt[3]{8}}$

$= \dfrac{1}{2}$

$d)\ (-8)^{\frac{1}{3}}$

$= \dfrac{1}{\sqrt[3]{-8}} = -\dfrac{1}{2}$

$e)\ 125^{2\frac{1}{3}}$

$= 125^{7/3}$

$= \left(\sqrt[3]{125}\right)^7$

$= 5^7$

$= 78125$

$f)\ \left(81^{-2}\right)^{-\frac{1}{4}}$

$81^{2/4}$

$81^{1/2}$

$= 9$