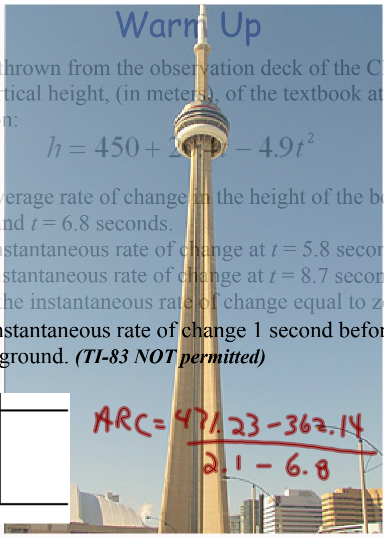
Warm Up



A math textbook is thrown from the observation deck of the CN Tower, 450 m above the ground. The vertical height, (in meters), of the textbook at time, t (in seconds), is given by the equation:

$$h = 450 + 20.4t - 4.9t^2$$

(a) Determine the average rate of change in the height of the book between $t = 2.1$ seconds and $t = 6.8$ seconds.
(b) Determine the instantaneous rate of change at $t = 5.8$ seconds. (***TI-83 NOT permitted***)
(c) Determine the instantaneous rate of change at $t = 8.7$ seconds. (***TI-83 permitted***)
(d) At what time is the instantaneous rate of change equal to zero? ***(TI-83 NOT permitted)***
(e) Determine the instantaneous rate of change 1 second before the book strikes the ground. ***(TI-83 NOT permitted)***

(a)

X (t)	Y_1 (h)
2.1	471.23
6.8	362.14

X=

471.23-362.14
109.09
Ans/(2.1-6.8)
-23.2106383

$$ARC = \frac{471.23 - 362.14 \text{ m}}{2.1 - 6.8 \text{ sec}} = -23.2 \text{ m/s}$$

(b) $t = 5.7$ & 5.9

X	Y_1
5.7	407.08
5.9	399.79

X=

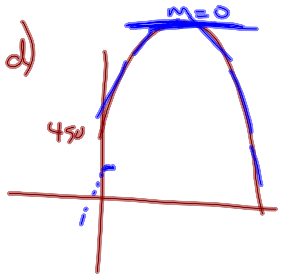
407.08-399.79
7.29
Ans/-.2
-36.45

$$IRC = \frac{407.08 - 399.79 \text{ m}}{5.7 - 5.9 \text{ sec}} = -36.45 \text{ m/s}$$

c)

$IRC = -64.86 \text{ m/s}$

d)



$h = -4.9t^2 + 20.4t + 450$

$h = -4.9(t^2 - 4.16t + 4.33) + 21.23 + 450$

$h = -4.9(t - 2.08)^2 + 471.23$

$V(2.08, 471.23)$

slope of a tangent to the curve equals "0" (vertex)

At 2.08 sec $IRC = 0$

e) $-4.9t^2 + 20.4t + 450 = 0$ ← ($h=0$ at ground)

$$t = \frac{-20.4 \pm \sqrt{(20.4)^2 - 4(-4.9)(450)}}{2(-4.9)}$$

$$t = \frac{-20.4 \pm 96.10}{-9.8}$$

$t = 11.89$ sec & $\cancel{-7.72}$

Want IRC @ $t = 10.89$ sec

X	Y_1
10.89	91.055
10.9	90.191

X=

$$IRC = \frac{91.055 - 90.191}{10.89 - 10.9}$$

91.055-90.191
.864
Ans/-.01
-86.4

Applications of Rate of Change

Functions must be developed...

Example:

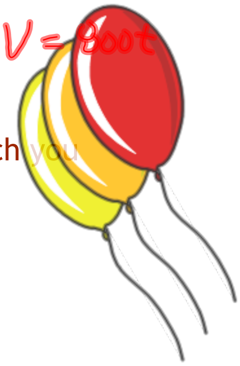
A spherical balloon is being inflated with helium at the rate of 800 cm^3/min. . ← change in Volume

(a) What is the instantaneous rate of change of the radius of this balloon 24 seconds after it has started to be inflated? 3.536 cm/min

(b) What is the instantaneous rate of change in the radius when the balloon has a surface area of 100π cm^2 ? 2.546 cm/min

Sphere: $V = \frac{4}{3}\pi r^3$ $\quad S.A. = 4\pi r^2$

$V = 800t$



- Rearrange formula to isolate the variable for which you are trying to determine the rate of change... in this case that would be "*r*".

$r = \sqrt[3]{\frac{3V}{4\pi}}$

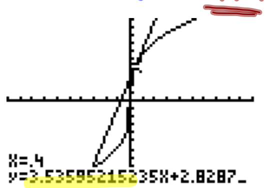
- Declare a variable for time and express the changing quantity as a varying expression... Let "t" Rep. time in minutes
  ie. The volume of air in this balloon will equal 800t cm^3

Volume $= 800t$ cm^3

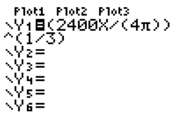
$r = \sqrt[3]{\frac{3(800t)}{4\pi}} = \sqrt[3]{\frac{600}{\pi}t}$

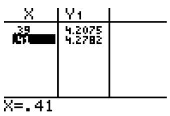
$24\text{ sec} \times \frac{1\text{ min}}{60\text{ s}} = \frac{2}{5} = 0.4\text{ min}$

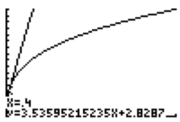
IRC @ $\underline{0.4\text{min}}$.



TI-83 solution...







IRC = 3.54 cm/min

- Must determine the time that the instantaneous rate of change is being calculated in part (b)

$\frac{100\pi}{4\pi} = \frac{4\pi r^2}{4\pi}$

$25 = r^2$

$r = 5\text{ cm}$

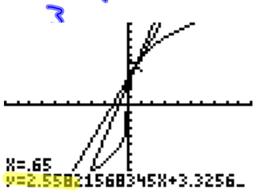
$V = \frac{4}{3}\pi(5)^3$

$V = \frac{500\pi}{3}\text{ cm}^3$

$V = 800t$

$\frac{500\pi}{3} = 800t$

$t =$ 500π/3 523.5987756 Ans/800 .6544984695



$t = 0.65$ min.

$= 2.56$ cm/min

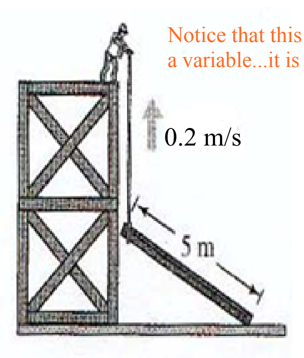
Example 2:

A construction worker pulls a 5 m plank up the side of a building under construction by means of a rope tied to one end of the plank (see diagram). Assume the opposite end of the plank follows a path perpendicular to the wall of the building and that the worker is able to pull the rope at a rate of 0.2 m/s. What is the rate of change of the angle between the plank and the building when it is 3 m from the base of the building ?

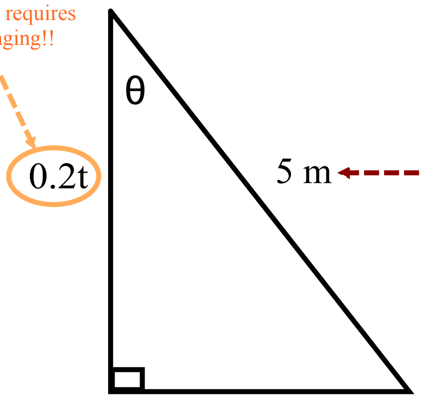
decreasing
at 3.82°/sec

Let t represent the time elapsed after plank is started to be pulled up the side of the building

Bonus



Notice that this side requires a variable...it is changing!!



Length of this ladder will never change...measurements that do not change will be placed on the diagram

What rate of change are we given?

How does this relate to this diagram?

What rate of change are we trying to determine?

Determine a function that will relate the changing quantities.

Determine the exact "TIME" when we would like this instantaneous rate of change