Check Up Time...

1. Solve each of the following:

$(a)\ 3x^2+8=10x$ $\qquad(b)\ x^2-5x>14$ $\qquad(c)\ x^3+3x^2-24x=26$

2. Simplify each of the following:

$(a)\ \left(2-3\sqrt{8}\right)^2$ $\qquad(b)\ \dfrac{2\sqrt{3}}{3-\sqrt{8}}$

3. Evaluate the following:

$$27^{-\frac{2}{3}}+3w^0-\frac{2}{3^{-2}}-3^{-2}+\frac{2}{5^{-2}}$$

$= \cancel{\frac{1}{9}} + 3 - 2(9) - \cancel{\frac{1}{9}} + 2(25)$

$= 3 - 18 + 50$

$= 35$

Complex Numbers

Look at the following equation...

$x + 1 = 0$, $x \in W$ ⟵ No Solution over the whole numbers

If we extend to the integers or real number systems then there will be a solution.

What about the equation $x2 + 1 = 0$, $x \in R$?

$$x^2 = -1$$

$$x = \sqrt{-1} \;\; ???$$

There is no solution over the real number system, therefore we extend into a new number system...the Complex Numbers.

$a + bi$, $a, b \in R$

Real Part   Imaginary Part

So what about this "*i*" that appears?

Most Important prinicple in complex number system

$$i^2 = -1$$

$$i = \sqrt{-1}$$

What is $\sqrt{-36}$?

Basic Operations involving Complex Numbers

I. Addition and Subtraction

$(a+bi)+(c+di)=(a+c)+(b+d)i$

Collect "real" terms

Collect "imaginary" terms

$=3-7i$

Example:

Express the following complex expression in standard form:

$$2(3-5i)-(7-5i)+2(-1+i)$$

$= 6-10i-7+5i-2+2i$

$\boxed{=-3-3i}$

II. Multiplication and Powers

$i^2=-1$

Examples:

a) $(2-i)(-1+3i)$ $= a+bi$

$=-2+6i+i-3i^2$

$=-2+7i-3(-1)$

$\boxed{=1+7i}$

b) $3-2(5-2i)^2$

$=3-2(25-20i+4i^2)$

$=3-50+40i-8i^2$

$\boxed{=-39+40i}$

c) $2i^5-i^8+(2i^3)^5$

$=2i^5-i^8+32i^{15}$

$=2(i^2)^2 i-(i^2)^4+32(i^2)^7 i$

$=2i-1-32i$

$\boxed{=-1-30i}$

i^{20}

$=(i^2)^{10}$

$=1$

$i^n=(i^2)^{8}i^1$

$=(1)i$

$=i$

III. Division

Before we can divide we must first review the concept of conjugates...

Conjugates

$$a+bi \Leftrightarrow a-bi$$

Means to take conjugate

Examine what happens when you multiply complex conjugates...

$$(2-5i)(2+5i)$$

$=4-25i^2=29$

Now we are ready to try division...

Multiply the numerator and denominator by the conjugate of the denominator

Example:

a) $\dfrac{2+4i}{1-i}\left(\dfrac{1+i}{1+i}\right)$ conjugate

$=\dfrac{2+2i+4i+4i^2}{1-i^2}$

$=\dfrac{-2+6i}{2}$

$\boxed{=-1+3i}$

b) $\dfrac{(2-i)(-1+3i)}{(-3+2i)^2}$ $\dfrac{-79}{169}+\dfrac{47}{169}i$

$=\dfrac{-2+6i+i-3i^2}{9-12i+4i^2}$

$=\dfrac{1+7i}{5-12i}\left(\dfrac{5+12i}{5+12i}\right)$

$=\dfrac{5+12i+35i+84i^2}{25-144i^2}$

$=\dfrac{-79+47i}{169}$

$=\dfrac{-79}{169}+\dfrac{47}{169}i$

$\cancel{3-\sqrt{3}\,i}$

$3-i\sqrt{3}$

Principle of Equality - "Comparison"

- comparison of left side versus right side.
- real parts must equal eachother and the imaginary parts must be equal.

EXAMPLE #1: $3 + i + 2i = 6i - (2x + yi)$

$a + bi = c + di$

$a = c$ & $b = d$

Real = Real   Im = Im

$3 + i = 6i - 2x - yi$

Re = Re

$3 = -2x$

$x = \frac{3}{-2}$

Im = Im

$1 = 6 - y$

$y = 5$

EXAMPLE #2: $4i(3x - y) = 3 - (3 - yi)i$

$12xi - 4yi = 3 - 3i + yi^2$ $i^2 = -1$

$12xi - 4yi = 3 - 3i - y$

Re = Re

$0 = 3 - y$

$y = 3$

Im = Im

$12x - 4y = -3$

$12x - 4(3) = -3$

$12x = 9$

$x = \frac{9}{12}$

$x = \frac{3}{4}$