# Warm-Up

1. Prove that $\left(\frac{-1+i\sqrt{3}}{2}\right)^3 = 1$

2. Find all roots $z \in C$ of the equation $z^2 - 3iz + 10 = 0.$

3. Express the following in polar form:

(a) $(-1, -\sqrt{2})$ (b) $(0, -1)$

4. Convert the following to rectangular coordinates:

(a) $(3, 135°)$ (b) $(-8, -300°)$

1. Prove that $\left(\dfrac{-1+i\sqrt{3}}{2}\right)^3 = 1$

$(3+2)^2 \cancel{= 9+4}$

| LS | RS |
|---|---|
| $\dfrac{(-1+i\sqrt{3})^3}{2^3}$ | $1$ |
| $\dfrac{8}{8}$ | |
| $1$ | |

$(-1+i\sqrt{3})^2(-1+i\sqrt{3})^1$

$(1-2i\sqrt{3}+3i^2)(-1+i\sqrt{3})$

$(-2-2i\sqrt{3})(-1+i\sqrt{3})$

$= 2\cancel{-2i\sqrt{3}+2i\sqrt{3}}-2i^2(3)$

$= 8$

2. Find all roots $z \in C$ of the equation $z^2 - 3iz + 10 = 0$.

$$z = \frac{3i \pm \sqrt{9i^2 - 4(1)(10)}}{2(1)}$$

$$z = \frac{3i \pm \sqrt{-49}}{2}$$

$$z = \frac{3i \pm \sqrt{49i^2}}{2}$$

$$z = \frac{3i \pm 7i}{2}$$

$z = 5i$ OR $z = -2i$

$(z-5i)(z+2i) = 0$

$z = 5i$ OR $z = -2i$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$ax^2 + bx + c = 0$

3. Express the following in polar form:

(a) $(-1, -\sqrt{2})$ Q3

$r = \sqrt{1+2}$

$r = \sqrt{3}$

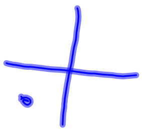
$\tan\theta = \sqrt{2}$

(Ref $\measuredangle$ 55°)

| $180-\theta$ | $\theta$ |
|---|---|
| $180+\theta$ | $360-\theta$ |

$\theta = 235°$

$(\sqrt{3}, 235°)$

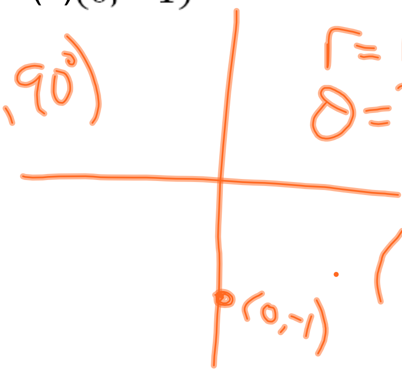


(b) $(0, -1)$

$(-1, 90°)$

$r = 1$

$\theta = 270°$




$(1, 270°)$

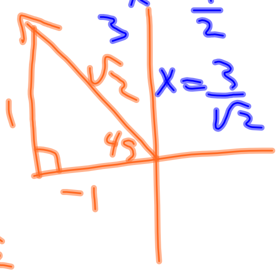
4. Convert the following to rectangular coordinates:

(a) $(3, 135°)$

$2x^2 = 9$

$x^2 = \frac{9}{2}$

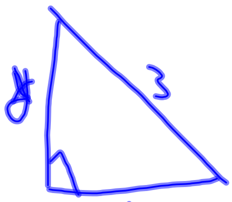
$x = \frac{3}{\sqrt{2}}$




$x = 3\cos 135°$

$x = 3\left(-\frac{1}{\sqrt{2}}\right)$

$x = -\frac{3}{\sqrt{2}} = \frac{-3\sqrt{2}}{2}$




$2x^2 = 9$

$y = 3\sin 135°$

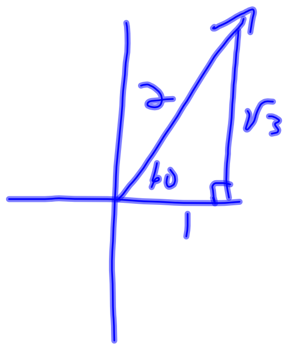
$y = 3\left(\frac{1}{\sqrt{2}}\right)$

$y = \frac{3}{\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right) = \frac{3\sqrt{2}}{2}$

(b) $(-8, -300°)$

$x = -8\left(\frac{1}{2}\right)$

$x = -4$

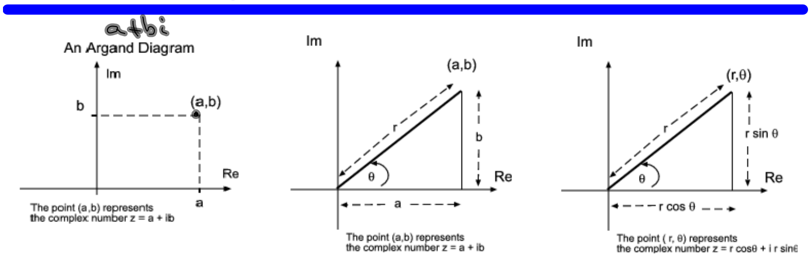



$y = -8\left(\frac{\sqrt{3}}{2}\right)$

$y = -4\sqrt{3}$

$(-4, -4\sqrt{3})$

# Complex Numbers in Polar Form




- to put into polar form, we know that.. a+bi

$\cos\theta = \frac{a}{r}$ **AND** $\sin\theta = \frac{b}{r}$ **WITH** $r = \sqrt{a^2+b^2}$

so, $a = r\cos\theta$ so, $b = r\sin\theta$

$$a + bi \Rightarrow r\cos\theta + (r\sin\theta)i = r$$

$$\therefore \quad polar\ form\ is\ldots \quad r\,cis\,\theta = r(\cos\theta + i\sin\theta)$$

Shorthand

$In\ general,$

$$z = r[\cos(\theta + 2\pi k) + i\sin(\theta + 2\pi k)],\ k \in I$$

$a+bi = r(\cos\theta + i\sin\theta)$ $\quad r\,cis\,\theta = r(\cos\theta + i\sin\theta)$

## EXAMPLES...

#1. Express each of the following into polar form...

$a)\ \sqrt{3} - i$

$(\sqrt{3}, -1)$ Q4

$r = \sqrt{3+1}$

$r = 2$

$\tan\theta = \frac{1}{\sqrt{3}}$

(Ref∢ = 30°, Q4)

$\theta = 330°$

$= 2(\cos(330°) + i\sin(330°))$

$= 2\,cis\,330°$

$b)\ -1 + i$ $(-1, 1)$ Q2

$r = \sqrt{1+1}$

$r = \sqrt{2}$

$\tan\theta = 1$

(Ref∢ = 45°, Q2)

$\theta = 135°$

$= \sqrt{2}(\cos 135° + i\sin 135°)$

$= \sqrt{2}\,cis\,135°$

$c)\ -4i$ $(0, -4)$

$r = \sqrt{0+16}$

$r = 4$

$\theta = 270°$

$= 4(\cos 270° + i\sin 270°)$

$= 4\,cis\,270°$

#2. Express each of the following into the form "a + b*i*"...

$a)\ \sqrt{3}\,cis(-60°)$

$= \sqrt{3}(\cos(-60) + i\sin(-60))$

$= \sqrt{3}\left(\frac{1}{2} + \left(-\frac{\sqrt{3}}{2}\right)i\right)$

$= \frac{\sqrt{3}}{2} - \frac{3}{2}i$

$b)\ \sqrt{2}\,cis\left(\frac{7\pi}{4}\right)$

$= \sqrt{2}\left(\frac{1}{\sqrt{2}} + \left(-\frac{1}{\sqrt{2}}\right)i\right)$

$= 1 - i$

# Homework...

Worksheet - Converting Polar_Rectangular Coordinates.doc

Should be able to complete ALL questions on this sheet

Attachments

Worksheet - Converting Polar_Rectangular Coordinates.doc