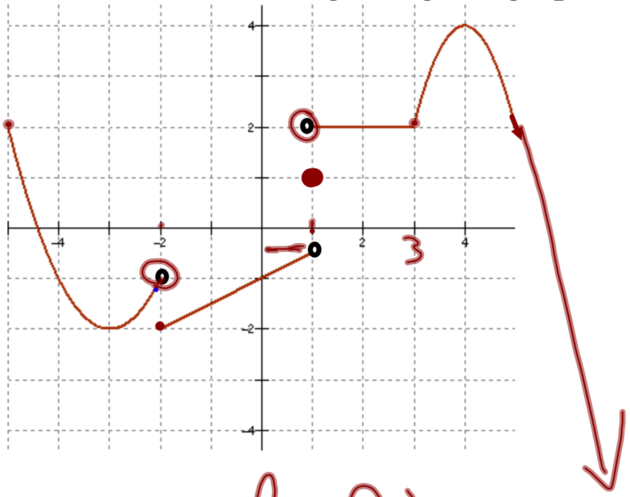
EXAMPLE...

Evaluate the following using the graph of $f(x)$ shown below...



$\lim_{x\to 3} f(x) = 2$

1. $\lim_{x\to -2^-} f(x) = -1$
2. $\lim_{x\to -2^+} f(x) = -2$
3. $\lim_{x\to -2} f(x) =$ DNE

Diff

4. $f(-2) = -2$
5. $f(1) = 1$
6. $\lim_{x\to 1^-} f(x) = -\frac{1}{2}$
7. $\lim_{x\to 1^+} f(x) = 2$

Domain: $\{x \mid x \geq -5, x \in R\}$ $[-5, \infty)$

Range: $\{y \mid y \leq 4, y \in R\} \rightarrow (-\infty, 4]$

BRACKET NOTATION

$[-2, 3)$ ← up to... but not including

Includes

$-7 < x < 1$

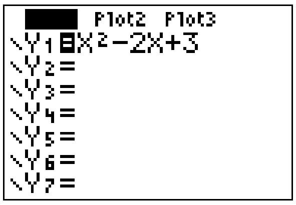
$(-7, 1)$

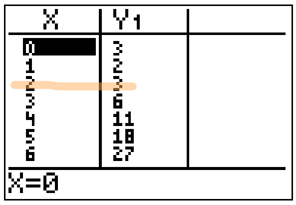
-2   3

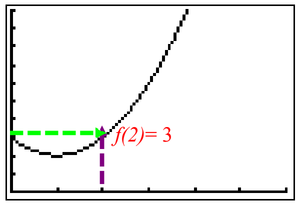
$-2 \leq x < 3$

Limit of a Function

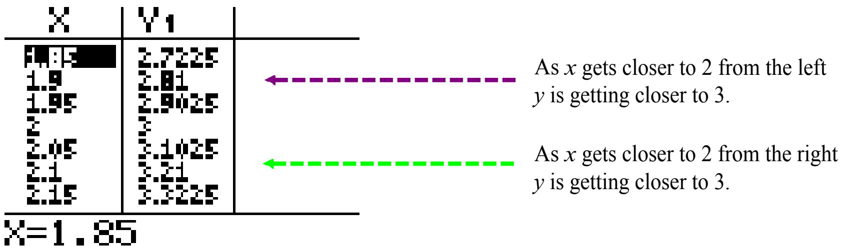
Let's examine the function $f(x) = x^2 - 2x + 3$



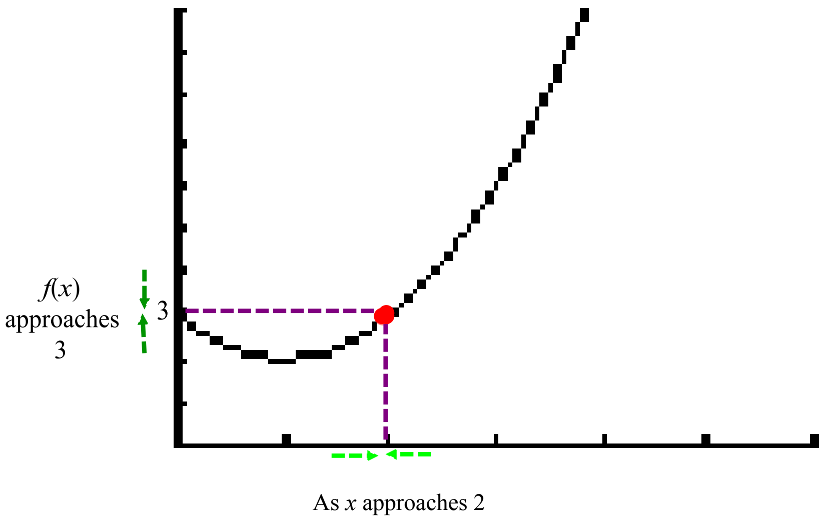




We can see that $f(2) = 3$...let's check the behaviour of f as we get closer and closer to $x = 2$.



From the above, the notion of the limit of a function arises...



Notation: $\lim_{x \to 2} f(x) = 3$

"The limit of the function $f(x)$ as x approaches 2 is equal to 3."

Evaluating Limits

I. Using a Graph:

- We looked at this in the previous two examples

II. Algebraically:

- Direct Substitution...

Examples:

$$\lim_{x\to -2} \frac{x^2-2x+1}{x+3}$$

$$= \frac{(-2)^2-2(-2)+1}{-2+3}$$

$$= \frac{9}{1}$$

$$= 9$$

$$\lim_{x\to 3}\left(16-x^2\right)$$

$$= 16-(3)^2$$

$$= 7$$

- Indeterminate limits... $\Rightarrow$ Direct substitution leads to $\frac{0}{0}$

⇒ Factor
⇒ Rationalize
⇒ Expand
⇒ Find Common Denominators

Examples:

$$\lim_{x\to 3} \frac{x^2-6x+9}{9-x^2} = \frac{9-18+9}{9-9} = \frac{0}{0}$$

$$\lim_{x\to 3} \frac{(x-3)^2}{(3-x)(3+x)} \Rightarrow \frac{\overset{-1}{\cancel{(x-3)}}(x-3)}{\cancel{(3-x)}(3+x)}$$

$$\lim_{x\to 3} \frac{-(x-3)}{3+x}$$

$$= \frac{-(0)}{6}$$

$$= 0$$

$$\lim_{h\to 0} \frac{\sqrt{4+h}-2}{h} = \frac{0}{0}$$

conjugate

$$\lim_{h\to 0} \frac{\sqrt{4+h}-2}{h}\left(\frac{\sqrt{4+h}+2}{\sqrt{4+h}+2}\right)$$

$$\lim_{h\to 0} \frac{(\cancel{4}+h)\cancel{-4}}{h\left(\sqrt{4+h}+2\right)}$$

$$\lim_{h\to 0} \frac{\cancel{h}\ 1}{\cancel{h}\left(\sqrt{4+h}+2\right)}$$

$$= \frac{1}{\sqrt{4+0}+2}$$

$$= \frac{1}{4}$$

Try these...remember to use your algebra skills
to try and eliminate the **indeterminate form**.

$$\lim_{x\to 0}\frac{x^2+3x}{(x+2)^2-(x-2)^2}$$

x^3+8

$$\lim_{x\to 0}\frac{x(x+3)}{[(x+2)-(x-2)][(x+2)+(x-2)]}$$

$$\lim_{x\to 0}\frac{\cancel{x}(x+3)}{(4)(2\cancel{x})}=\frac{3}{8}$$

$$\lim_{x\to -2}\frac{x^4-16}{x^3+8}$$

$$\lim_{x\to -2}\frac{(x^2-4)(x^2+4)}{(x+2)(x^2-2x+4)}$$

$$\lim_{x\to -2}\frac{(x-2)\cancel{(x+2)}(x^2+4)}{\cancel{(x+2)}(x^2-2x+4)}$$

$$=\frac{(-4)(8)}{(4+4+4)}=\frac{-8}{3}$$

$$\lim_{x\to 2}\frac{(x+2)^2-16}{x^2-4}$$

$[(x+2)-4][(x+2)+4]$

$$\lim_{x\to 2}\frac{x^2+4x+4-16}{x^2-4}$$

$$\lim_{x\to 2}\frac{x^2+4x-12}{x^2-4}$$

$$\lim_{x\to 2}\frac{(x+6)\cancel{(x-2)}}{\cancel{(x-2)}(x+2)}$$

$$=\frac{8}{4}=2$$

$$\lim_{x\to 2}\frac{\frac{1}{x}-\frac{1}{2}}{x-2}$$

$$\lim_{x\to 2}\left(\frac{\overset{-1}{\cancel{2-x}}}{2x}\right)\cdot\frac{1}{\cancel{x-2}}$$

$$=\frac{-1}{4}$$

Homework...

Page 18 - 19 $= \infty$

#1, 4, 5, 6, 9 $\longrightarrow \infty$

DNE