

We can use the exponent laws to simplify expressions that contain rational number bases. It is a convention to write a simplified power with a positive exponent.

Example 1

Simplifying Numerical Expressions with Rational Number Bases

Simplify by writing as a single power. Explain the reasoning.

a) $0.3^{-3} \cdot 0.3^5$

$= 0.3^2$

c) $\frac{(1.4^3)(1.4^4)}{1.4^{-2}}$

$\frac{(1.4)^7}{(1.4)^{-2}}$

$(1.4)^9$

b) $\left[\left(-\frac{3}{2} \right)^{-4} \right]^2 \cdot \left[\left(-\frac{3}{2} \right)^2 \right]^3$

d) $\left(\frac{7^{\frac{2}{3}}}{7^{\frac{1}{3}} \cdot 7^{\frac{5}{3}}} \right)^6$

$$d) \left(\frac{7^{\frac{2}{3}}}{7^{\frac{1}{3}} \cdot 7^{\frac{5}{3}}} \right)^6$$

$$\begin{aligned} & \left(\frac{7^{\frac{2}{3}}}{7^{\frac{1}{3}} \cdot 7^{\frac{5}{3}}} \right)^6 \quad -\frac{4}{3} \cdot 6 \\ & \left(7^{-\frac{4}{3}} \right)^6 \\ & = 7^{-\frac{24}{3}} \\ & = 7^{-8} \\ & = \frac{1}{7^8} \end{aligned}$$

$$\begin{aligned} & \frac{7^4}{7^2 \cdot 7^{10}} \\ & = \frac{7^4}{7^{12}} \\ & = 7^{-8} = \frac{1}{7^8} \end{aligned}$$

$$\text{b) } \left[\left(-\frac{3}{2} \right)^{-4} \right]^2 \cdot \left[\left(-\frac{3}{2} \right)^2 \right]^3$$

$$\left(-\frac{3}{2} \right)^{-8} \cdot \left(-\frac{3}{2} \right)^6$$

$$\left(-\frac{3}{2} \right)^{-2}$$

$$\left(-\frac{2}{3} \right)^2 = \left(\frac{2}{3} \right)^2$$

Another important skill is writing expressions to a new base...let's take a look

$$2^3 = 8$$

$$2^3 = 8$$

$$3^4 = 81$$

$$3^4$$

$$\left(\frac{1}{16}\right) = 2^?$$

$$\frac{1}{2^4} = 2^{-4}$$

$$4^3 = 2^?$$

$$(2^2)^3 = 2^6$$

$$\left(\frac{1}{64}\right)^2 = 4^?$$

$$\left(\frac{1}{4^3}\right)^2$$

$$\frac{1}{4^6}$$

$$4^{-6}$$

$$125^x = 5^?$$

$$(5^3)^x$$

$$5^{3x} = 5^?$$

$$5^{3x} = 5^1$$

$$3x = 1$$

$$x = \frac{1}{3}$$

$$32^{3x+1} = 2^?$$

$$(2^5)^{3x+1}$$

$$2^{5(3x+1)}$$

Which is the largest?

A/ 16^{300}
 $(2^4)^{300}$
 2^{1200}

B/ 4^{555}

$$(2^2)^{555}$$

$$2^{1110}$$

C/ 8^{312}

$$(2^3)^{312} = 2^{936}$$

D/ 32^{210}

$$(2^5)^{210}$$

$$2^{1050}$$

Should also be comfortable solving equations involving Rational Exponents...

$$a) x^{\frac{5}{2}} = 32$$
$$(x^{\frac{2}{5}})^{\frac{25}{2}} = 32^{\frac{2}{5}}$$
$$x = 4$$

$$b) (a^{\frac{2}{3}})^{\frac{3}{2}} = 49^{\frac{3}{2}}$$
$$a = 7^3$$
$$\underline{a = 343}$$

$$c) 2a^{-\frac{3}{2}} = \frac{128}{2}$$
$$(a^{-\frac{2}{3}})^{\frac{2}{3}} = 64^{-\frac{2}{3}}$$
$$a^1 = \frac{1}{64^{\frac{2}{3}}}$$
$$a = \frac{1}{(\sqrt[3]{64})^2}$$
$$= \frac{1}{16}$$

Simplify...

$$a) -27^{-\frac{2}{3}}$$

$$b) a^{\frac{4}{5}} = 16 \quad (\text{Find } a)$$

$$c) \frac{3^{\frac{7}{12}}}{3^{\frac{1}{4}} \cdot 3^{\frac{1}{3}}}$$

$$d) \left(\frac{2a}{b}\right)^{-3} (2ab^2)^2$$