

EXERCISE...

1. Evaluate: $= a+bi$

(a) $(1-i)^{20}$

$= -1024$

$(1, -1) \Rightarrow Q4$

$r = \sqrt{1+1}$

$r = \sqrt{2}$

$\tan \alpha = -1$
(Ref $\alpha = 45^\circ$)



$\therefore \alpha = 315^\circ$ (Quad. 4)

$= (\sqrt{2} \text{ cis } 315^\circ)^{20}$

$= \left[\sqrt{2} (\cos(315^\circ) + i \sin(315^\circ)) \right]^{20}$

$= (\sqrt{2})^{20} [\cos(20 \times 315^\circ) + i \sin(20 \times 315^\circ)]$

$= 2^{10} (\text{cis } 6300^\circ)$

$= 2^{10} (\cos 6300^\circ + i \sin 6300^\circ)$

$= 2^{10} (-1 + i(0))$

$= -2^{10} + 0i$

$= -1024$

~~(b) $(5-4i)^{17}$~~

$=$

(c) $(-1+i\sqrt{3})^{15}$

$= 32768$

$(-1, \sqrt{3})$ Quad. 2

$r = \sqrt{1+3}$

$r = 2$

$\tan \alpha = \sqrt{3}$
(Ref $\alpha = 60^\circ$)

$\therefore \alpha = 180 - 60^\circ$

$\alpha = 120^\circ$

$= (2 \text{ cis } 120^\circ)^{15}$

$= 2^{15} \text{ cis } (120 \times 15)$

$= 2^{15} \text{ cis } (1800^\circ)$

$= 2^{15} (\cos 1800^\circ + i \sin 1800^\circ)$

$= 2^{15} (1 + i(0))$

$= 32768$

Example:

$$\frac{(-1+i\sqrt{3})^9 (4i)^8 (3+3i)}{(-4\sqrt{3}-4i)^6 (1-i)^8}$$

SOLUTION:
-24 - 24i

$(-1, \sqrt{3}) \Rightarrow Q2$

$r = \sqrt{1+3}$
 $r = 2$

$\tan \theta = \frac{\sqrt{3}}{1}$
 $(\text{Ref } \theta = 60^\circ)$

$\theta = 120^\circ$
 $2 \text{ cis } 120^\circ = 2^9 \text{ cis } 1080^\circ$

$(0, 4) \Rightarrow Q1$

$r = 4, \theta = 90^\circ$

$= 4 \text{ cis } 90^\circ$

$= 4^8 \text{ cis } 720^\circ$

$(3, 3) \Rightarrow Q1$

$r = \sqrt{9+9}$
 $r = \sqrt{18}$
 $r = 3\sqrt{2}$

$\tan \theta = \frac{3}{3}$
 $(\text{Ref } \theta = 45^\circ)$
 $\theta = 45^\circ$
 $= 3\sqrt{2} \text{ cis } 45^\circ$

$(-4\sqrt{3}, -4) \Rightarrow Q3$

$r = \sqrt{48+16}$
 $r = 8$

$\tan \theta = \frac{4}{4\sqrt{3}}$
 $(\text{Ref } \theta = 30^\circ) \rightarrow Q3$

$\theta = 210^\circ$

$= (8 \text{ cis } 210^\circ)^6$

$= 8^6 \text{ cis } 1260^\circ$

$(1, -1) \Rightarrow Q4$

$r = \sqrt{1+1}$
 $r = \sqrt{2}$

$\tan \theta = 1$
 $(\text{Ref } \theta = 45^\circ) \rightarrow Q4$

$\therefore \theta = 315^\circ$

$= (\sqrt{2} \text{ cis } 315^\circ)^8$

$= 2^4 \text{ cis } 2520^\circ$

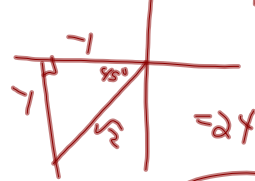
$$= \frac{(2^9 \text{ cis } 1080^\circ)(4^8 \text{ cis } 720^\circ)(3\sqrt{2} \text{ cis } 45^\circ)}{(8^6 \text{ cis } 1260^\circ)(2^4 \text{ cis } 2520^\circ)}$$

$$= \frac{(2^7 \cdot 4^8 \cdot 3\sqrt{2}) \text{ cis } (1080+720+45^\circ)}{(8^6 \cdot 2^4) \text{ cis } (1260+2520^\circ)}$$

$$= \frac{(2^7 \cdot 4^8 \cdot 3\sqrt{2}) \text{ cis } 1845^\circ}{(8^6 \cdot 2^4) \text{ cis } 3780^\circ}$$

$$= \frac{2^7 \cdot 4^8 \cdot 3\sqrt{2}}{8^6 \cdot 2^4} \text{ cis } (-1935^\circ)$$

$$= 24\sqrt{2} (\cos(-1935^\circ) + i \sin(-1935^\circ))$$



Principal Angle = 225°

$$= 24\sqrt{2} \left(-\frac{1}{\sqrt{2}} + i\left(-\frac{1}{\sqrt{2}}\right)\right)$$

$$= -24 - 24i$$

$24\sqrt{2} \left(\frac{\sqrt{2}}{2} - \frac{48}{2} - 24i\right)$

Practice Sheet: Solutions

19.

a) $3/25$

b) -2^{50}

c) -8192

d) $5/2$

e) 1

f) $-8\sqrt{3} + 8i$

BONUS

Evaluate:

$$\frac{\left(-\frac{3}{2} + \frac{3i\sqrt{3}}{2}\right)^4 (2\sqrt{3} + 2i)^3}{(9\sqrt{2} + 9i\sqrt{2})^2}$$

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(-3/2+3i*sqrt(3)/2)^4  
4(2*sqrt(3)+2i)^3/(9  
sqrt(2)+9i*sqrt(2))^2  
-8+13.85640646i
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