Try this one...

$$f(x)=\begin{cases}2-x^2 & \text{if } x<1\\ 3 & \text{if } x=1\\ 2x-1 & \text{if } 1<x\le 3\\ (x-4)^2 & \text{if } x>3\end{cases}$$

Evaluate:

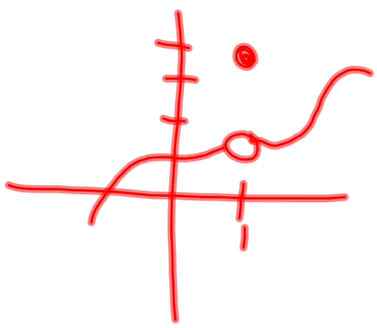
$\lim_{x\to 1} f(x) = 1$ $\qquad$ $\lim_{x\to 3} f(x) = \text{DNE}$

$\lim_{x\to 1^-} f(x) = 2-(1)^2 = 1$

$\lim_{x\to 1^+} f(x) = 2(1)-1 = 1$

$\lim_{x\to 3^-} f(x) = 2(3)-1 = 5$

$\lim_{x\to 3^+} f(x) = (3-4)^2 = 1$



Continuity

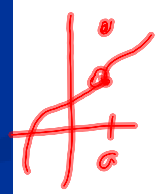
Definition

- We noticed in the preceding section that…
  - the limit of a function as x approaches a

  can often be found simply by…
  - calculating the value of the function at a .
- Functions with this property are called *continuous* at a :

1 Definition A function f is **continuous at a number a** if

$$\lim_{x \to a} f(x) = f(a)$$



- This definition implicitly requires three things if f is continuous at a :

1. $f(a)$ is defined
   - That is, a is in the domain of f
2. $f(x)$ has a limit as x approaches a
3. This limit is actually equal to $f(a)$.

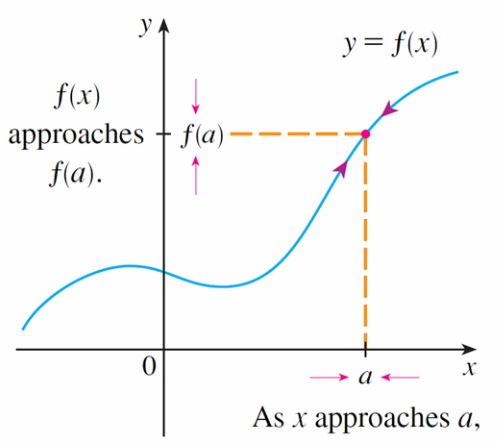


FIGURE 1

Examine the graph shown below
for points of discontinuity...

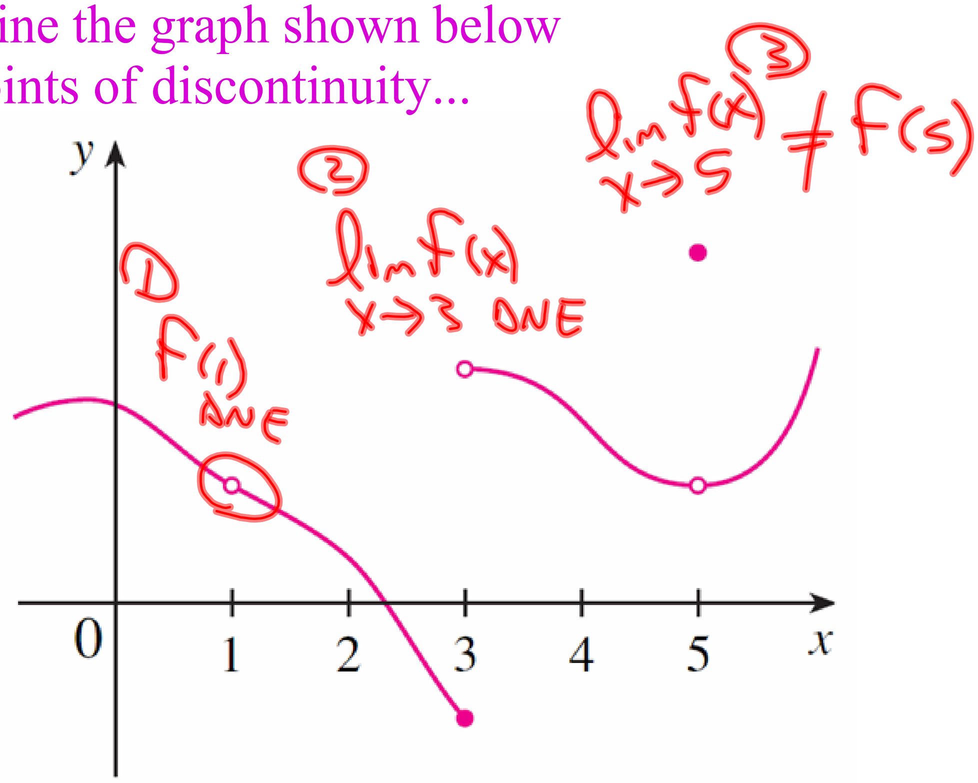


FIGURE 2

- f is discontinuous at 1 because $f(1)$ is not defined…
  - …despite the fact that f has a limit at $a = 1$
- f is also discontinuous at 3, but for a different reason:
  - $f(3)$ is defined, but f has no limit at $a = 3$.
- f has both a value and a limit at 5, but they are different; thus f is discontinuous at 5.

Let's simplify things...

A function whose graph has holes or breaks is considered discontinuous at these particular points.

If you have to lift your pencil from the page to sketch the graph, it is discontinuous anywhere you lift your pencil

Examples:

Given the function $f(x) = \begin{cases} 3-x & , \quad if \quad x < -1 \\ 4 & , \quad if \quad -1 \leq x < 2 \\ 1 & , \quad if \quad x = 2 \\ 8-x^2 & , \quad if \quad x > 2 \end{cases}$

(a) Check $f(x)$ for any points of discontinuity. Provide a mathematical reason to validate any point(s) where the function is discontinuous.

(b) Sketch $f(x)$.

$\underline{x=-1}$

$f(-1)=4$

$\lim_{x\to -1^-} f(x) = 3-(-1) = 4$

$\lim_{x\to -1^+} f(x) = 4$

$\lim_{x\to -1} f(x) = 4$

$\lim_{x\to 1} f(x) = f(1)$

$\therefore f(x)$ is continuous at $x=1$

$\underline{x=2}$

$f(2)=1$

$\lim_{x\to 2^-} f(x) = 4$

$\lim_{x\to 2^+} f(x) = 8-(2)^2 = 4$

$\lim_{x\to 2} f(x) \neq f(2)$

$\therefore$ discontinuous at $x=2$

Homework

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