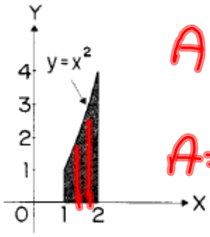
Warm-Up

x	y
1	1
$\frac{4}{3}$	$\frac{16}{9}$
$\frac{5}{3}$	$\frac{25}{9}$
2	4



$A = \frac{1}{2}\left(\frac{1}{3}\right)\left[\left(1+\frac{16}{9}\right)+\left(\frac{16}{9}+\frac{25}{9}\right)+\left(\frac{25}{9}+4\right)\right]$

$A = \frac{1}{6}\left(\frac{127}{9}\right) = \frac{127}{54}$

Calculate the approximate area of the shaded region in the figure by the trapezoidal rule, using divisions at $x = \frac{4}{3}$ and $x = \frac{5}{3}$.

(A) $\frac{50}{27}$ (B) $\frac{251}{108}$ (C) $\frac{7}{3}$ (D) $\frac{127}{54}$ (E) $\frac{77}{27}$

Let $F(x)$ be an antiderivative of $\frac{(\ln x)^3}{x}$. If $F(1) = 0$, then $F(9) =$

(A) 0.048 (B) 0.144 (C) 5.827 (D) 23.308 (E) 1,640.250

$(\ln x)^3\left(\frac{1}{x}\right)$

$\frac{1}{4}(\ln x)^4 + C = F(x)$

$\frac{1}{4}(\ln 1)^4 + C = 0$

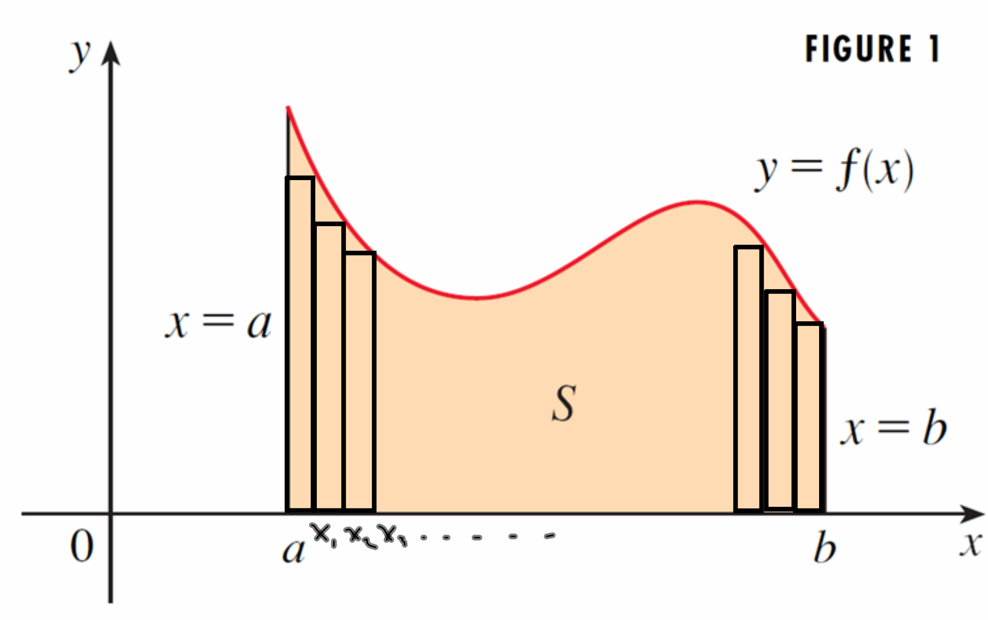
$0 + C = 0$

$C = 0$

$F(9) = \frac{1}{4}(\ln 9)^4$

$=$

Develop a general formula for the area below a curved surface using "n" rectangles.



"n" → # of Rectangles

width Δx

$\boxed{\Delta x = \frac{b-a}{n}}$

height x_k

$\boxed{x_k = a + (\Delta x)k}$

area

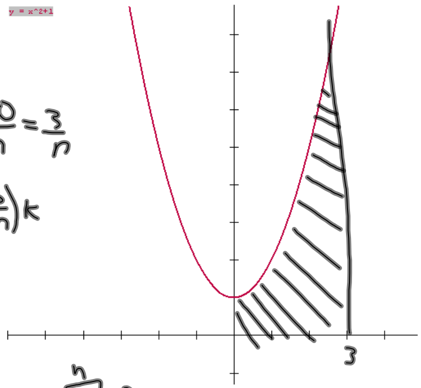
$A = \sum_{k=1}^{n} (\Delta x) f(x_k)$

$\boxed{A = \Delta x \sum_{k=1}^{n} f(x_k)}$

$$A = \Delta x \sum_{k=1}^{n} f(x_k)$$

$$\Delta x = \frac{b-a}{n} \qquad x_k = a + (\Delta x)k$$

Use a Riemann Summation to determine the area below the curve $y = x^2 + 1$, between $x = 0$ and $x = 3$.



$\Delta x = \frac{3-0}{n} = \frac{3}{n}$

$x_k = 0 + \left(\frac{3}{n}\right)k$

$x_k = \frac{3k}{n}$

$A = \Delta x \sum_{k=1}^{n} f(x_k)$

$A = \frac{3}{n} \sum_{k=1}^{n} f\left(\frac{3k}{n}\right)$ Now expand this sigma Notation

$A = \frac{3}{n}\left[\underbrace{f\left(\frac{3}{n}\right)}_{k=1} + \underbrace{f\left(\frac{6}{n}\right)}_{k=2} + \underbrace{f\left(\frac{9}{n}\right)}_{k=3} + \cdots + \underbrace{f\left(\frac{3n}{n}\right)}_{k=n}\right]$

Remember: $f(x) = x^2 + 1$

$A = \frac{3}{n}\left[\left[\left(\frac{3}{n}\right)^2 + 1\right] + \left[\left(\frac{6}{n}\right)^2 + 1\right] + \left[\left(\frac{9}{n}\right)^2 + 1\right] + \cdots + \left[\left(\frac{3n}{n}\right)^2 + 1\right]\right]$

$A = \frac{3}{n}\left[\frac{9}{n^2} + 1 + \frac{36}{n^2} + 1 + \frac{81}{n^2} + 1 + \cdots + \frac{9n^2}{n^2} + 1\right]$

$A = \frac{3}{n}\left[\frac{9}{n^2}\left(1 + 4 + 9 + \cdots + n^2\right) + \underbrace{\left(1 + 1 + 1 + \cdots + 1\right)}_{n \text{ of these}}\right]$

Summation Rule!!

$A = \frac{3}{n}\left[\frac{9}{n^2}\left(\frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6}\right) + 1n\right]$

$A = \frac{3}{n}\left[3n + \frac{9}{2} + \frac{3}{2n} + n\right]$

$A = 9 + \frac{27}{2n} + \frac{9}{2n^2} + 3$

$A = 12 + \frac{27}{2n} + \frac{9}{2n^2}$

Want # of Rectangles $\longrightarrow \infty$

$\lim_{n\to\infty}\left(12 + \overset{\to 0}{\frac{27}{2n}} + \overset{\to 0}{\frac{9}{2n^2}}\right)$

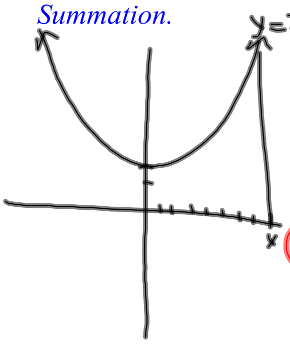
$= 12u^2$

Example:

Suppose ***R*** is the area in the first quadrant below the curve
$y = 2x^2 + 2$ between $x = 0$ and $x = 4$.

(a) Approximate the area of "R" using 8 rectangles with height determined by the left-hand endpoint

(b) Determine the exact area of "R" using a Riemann Summation.



X	Y1
0	2
.5	2.5
1	4
1.5	6.5
2	10
2.5	14.5
3	20

X=3

X	Y1
0	2
.5	2.5
1	4
1.5	6.5
2	10
2.5	14.5
3.5	26.5

X=3.5

(a) $A = \frac{1}{2}(2+2.5+4+6.5+10+14.5+20+26.5)$

$A = \frac{1}{2}(86) \doteq 43\,u^2$

b) $\Delta x = \frac{4}{n}$ $\quad x_k = 0 + \frac{4k}{n}$

$x_k = \frac{4k}{n}$

$A = \frac{4}{n}\sum_{k=1}^{n} f\left(\frac{4k}{n}\right)$

$f(x) = 2x^2+2$

$A = \frac{4}{n}\left[\underbrace{\left[2\left(\frac{4}{n}\right)^2+2\right]}_{k=1} + \underbrace{\left[2\left(\frac{8}{n}\right)^2+2\right]}_{k=2} + \cdots + \underbrace{\left[2\left(\frac{4n}{n}\right)^2+2\right]}_{k=n}\right]$

$A = \frac{4}{n}\left[\frac{32}{n^2} + 2 + \frac{128}{n^2} + 2 + \cdots + \frac{32n^2}{n^2} + 2\right]$

$A = \frac{4}{n}\left[\frac{32}{n^2}\left(\overbrace{1+4+\cdots+n^2}^{\sum k^2}\right) + \left(\overbrace{2+2+\cdots+2}^{\text{"n" of those}}\right)\right]$

$A = \frac{4}{n}\left[\frac{32}{n^2}\left(\frac{n^3}{3}+\frac{n^2}{2}+\frac{n}{6}\right) + 2n\right]$

$A = \frac{152}{3}$

$A = \frac{4}{n}\left[\frac{32n}{3} + 16 + \frac{16}{3n} + 2n\right]$

$A = \frac{128}{3} + \frac{64}{n} + \frac{64}{3n^2} + 8$

$\lim_{n\to\infty}\left(\frac{128}{3} + \frac{8}{1} + \frac{64}{n} + \frac{64}{3n^2}\right)$ ($\frac{64}{n} \to 0$, $\frac{64}{3n^2} \to 0$)

$= \frac{128}{3} + \frac{24}{3}$

$= \frac{152}{3}\,u^2$

Example:

Use a Riemann Summation to determine the area below the curve $y = x^2 + x + 1$ and above the x-axis, between x = -2 and x = 1.

$A = \frac{9}{2} u^2$