Warm Up

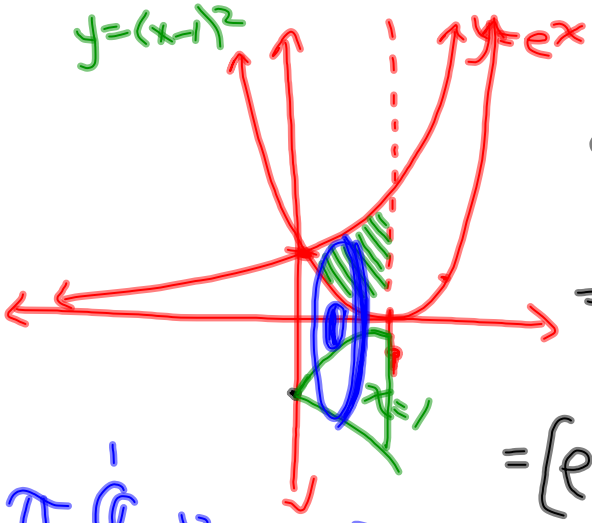
1990 AB3

Let R be the region enclosed by the graphs of $y = e^x$, $y = (x-1)^2$, and the line $x = 1$.

(a) Find the area of R.

(b) Find the volume of the solid generated when R is revolved about the x-axis.

(c) Set up, but do not integrate, an integral expression in terms of a single variable for the volume of the solid generated when R is revolved about the y-axis.



a) $\int_0^1 \left[e^x - (x-1)^2\right] dx$

$= e^x - \frac{1}{3}(x-1)^3 \Big|_0^1$

$= \left[e - 0\right] - \left[1 + \frac{1}{3}\right]$

$= e - \frac{4}{3}$

$= 1.384^2$

b) $\pi \int_0^1 \left[(e^x)^2 - (x-1)^4\right] dx$

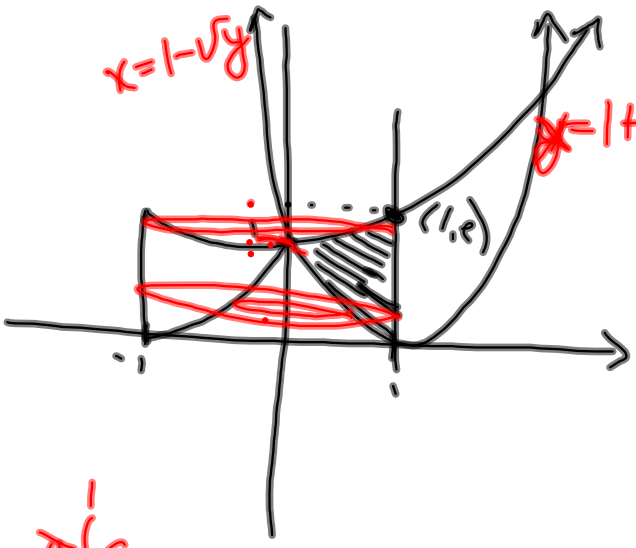
$\pi \int_0^1 e^{2x(2)} - (x-1)^4 \, dx$

$\pi\left[\frac{1}{2}e^{2x} - \frac{1}{5}(x-1)^5\right]\Big|_0^1$

$\pi\left[\left(\frac{1}{2}e^2 - 0\right) - \left(\frac{1}{2} + \frac{1}{5}\right)\right]$

$\pi\left(\frac{e^2}{2} - \frac{7}{10}\right)$

$= 9.408^3$



$y=e^x$

$\ln y = x \ln e$

$x = \ln y$

$\sqrt{y} = \sqrt{(x-1)^2}$

$\pm\sqrt{y} = x-1$

$x = 1 \pm \sqrt{y}$

$$\pi\int_0^1 \left[(1)^2-(1-\sqrt{y})^2\right]dy + \pi\int_1^e \left[(1)^2-(\ln y)^2\right]dy$$

B:

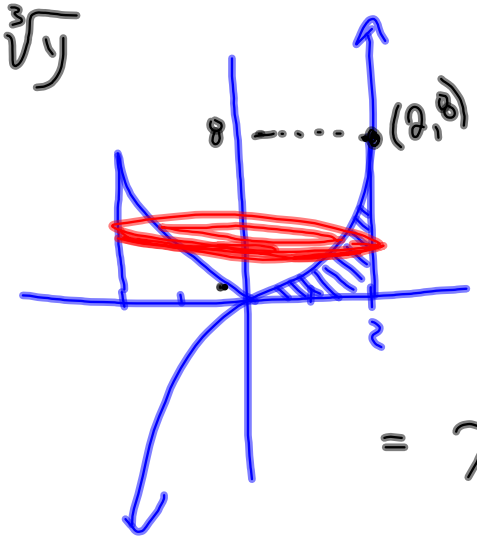
$$2\pi\int_0^1 x\left(e^x-(x-1)^2\right)dx$$

Sample Test Question...

5. (a) Determine the volume of the solid generated by rotating the region bound by the curves $y = x^3$, $x = 2$, and $y = 0$ about the *y-axis*. [6]

   (b) Determine the volume of the solid generated by rotating the region bound by functions $y = x^2 + 4x$ and $y = x + 4$ about the line $y = 6$. [6]

(a) $x = \sqrt[3]{y}$



$$\pi \int_0^8 (2)^2 - (\sqrt[3]{y})^2 \, dy$$

$$\pi \int_0^8 \left(4 - y^{2/3}\right) dy$$

$$= \pi \left(4y - \frac{3}{5}y^{5/3}\right)\Big|_0^8$$

$$= \pi \left[\left(32 - \frac{96}{5}\right) - (0)\right]$$

$$= \frac{64\pi}{5} u^3$$

(b) Determine the volume of the solid generated by rotating the region bound by functions $y = x^2 + 4x$ and $y = x + 4$ about the line $y = 6$. [6]

$y = x^2 + 4x \quad y = x + 4$

$x^2 + 4x = x + 4$

$x^2 + 3x - 4 = 0$

$(x+4)(x-1) = 0$

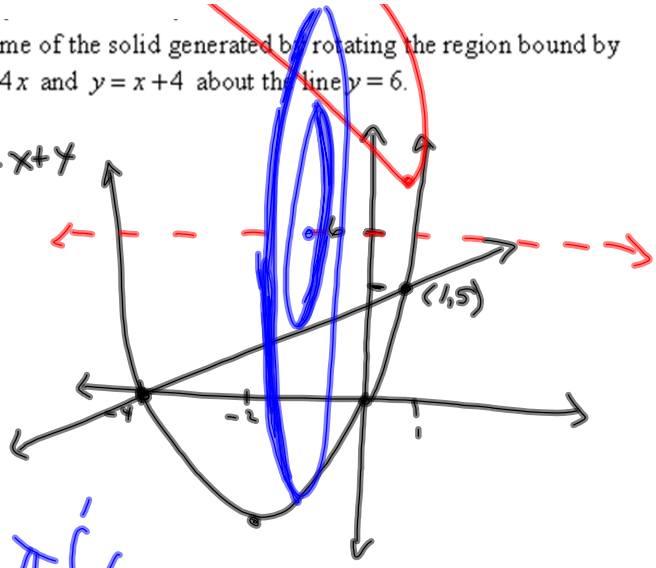
$x = -4, 1$

Interrupts

$0 = x^2 + 4x$

$0 = x(x+4)$

$x = 0, -4$



$$\pi \int_{-4}^{1} \left(6 - (x^2 + 4x)\right)^2 - \left(6 - (x+4)\right)^2 dx$$

$$\pi \int_{-4}^{1} (-x^2 - 4x + 6)^2 - (-x + 2)^2 dx$$

$(-x^2 - 4x + 6)(-x^2 - 4x + 6)$

$x^4 + 4x^3 - 6x^2 + 4x^3 + 16x^2 - 24x - 6x^2 - 24x + 36$

$x^4 + 8x^3 + 4x^2 - 48x + 36$

$$\pi \int_{-4}^{1} (x^4 + 8x^3 + 4x^2 - 48x + 36) - (x^2 - 4x + 4)\, dx$$

$$\pi \int_{-4}^{1} (x^4 + 8x^3 + 3x^2 - 44x + 32)\, dx$$

$$\pi \left(\frac{x^5}{5} + 2x^4 + x^3 - 22x^2 + 32x \right) \Big|_{-4}^{1}$$

$$\pi \left[\left(\frac{1024}{5} + 512 - 64 - 352 - 128 \right) - \left(\frac{1}{5} + 2 + 1 - 22 + 32 \right) \right]$$