Warm Up

Evaluate the following integrals:

$$\int \sqrt{x^3 - 3x}\,(x^2 - 1)\,dx$$

$$\frac{1}{3}\int (x^3-3x)^{1/2}(3x^2-3)$$

$$\frac{2}{9}(x^3-3x)^{3/2} + C$$

OR

$$u = x^3 - 3x$$

$$du = 3x^2 - 3\,dx$$

$$du = 3(x^2-1)dx$$

$$\frac{du}{3} = (x^2-1)dx$$

$$\Rightarrow \frac{1}{3}\int u^{1/2}du$$

$$\frac{2}{9}u^{3/2} + C$$

$$\frac{2}{9}(x^3-3x)^{3/2} + C$$

$$\int_1^4 \frac{dx}{\sqrt{x}(1+\sqrt{x})^3}$$

$$\int_1^4 (1+\sqrt{x})^{-3}\,x^{-1/2}dx$$

$$u = 1+\sqrt{x}$$

$$du = \frac{1}{2}x^{-1/2}dx$$

$$2\,du = x^{-1/2}dx$$

$$x=1:\ u = 1+\sqrt{1} = 2$$

$$x=4:\ u = 1+\sqrt{4} = 3$$

$$2\int_2^3 u^{-3}du$$

$$-u^{-2}\Big|_2^3$$

$$= -\frac{1}{9} - \left(-\frac{1}{4}\right)$$

$$= -\frac{4}{36} + \frac{9}{36}$$

$$= \frac{5}{36}$$

Integration by Parts

- Used to integrate an integral that is the derivative of a product

Derivation of integration by parts formula...

$$\frac{d[f(x)g(x)]}{dx} = f'(x)g(x) + f(x)g'(x)$$

Therefore...

$$\int [f'(x)g(x) + f(x)g'(x)]dx = f(x)g(x)$$

$$\int f'(x)g(x)dx + \int f(x)g'(x)dx = f(x)g(x)$$

$$\int f(x)g'(x)dx = f(x)g(x) - \int g(x)f'(x)dx$$

$$Let\ u = f(x) \quad and \quad v = g(x)$$

$$du = f'(x)dx \qquad dv = g'(x)dx$$

Integration by Parts Rule becomes...

$$\int udv = uv - \int vdu$$

The goal of integration by parts is to end up with an integral in the formula that is LESS complicated than the original integral...

This means that you must let "u" equal something that when differentiated becomes less complicated,or at least stays about the same.

Examples:

$$\int x \underbrace{\ln x}_{u} \, dx$$

$u = \ln x \qquad \int dv = \int x \, dx$

$du = \frac{1}{x} dx \qquad v = \frac{x^2}{2}$

$= \frac{x^2}{2} \ln x - \int \frac{x^2}{2}\left(\frac{1}{x}\right) dx$

$= \frac{x^2}{2} \ln x - \frac{1}{2} \int x \, dx$

$= \frac{x^2}{2} \ln x - \frac{1}{4} x^2 + C$

$$= \frac{x^2 \ln x}{2} - \frac{x^2}{4} + C$$

$$\int \tan^{-1} x\,dx$$

$$= x\tan^{-1} x - \frac{1}{2}\ln\left(1+x^2\right) + C$$

$u = \tan^{-1} x \qquad dv = dx$

$\frac{du}{dx} = \frac{1}{1+x^2}dx \qquad v = x$

$= x\tan^{-1}x - \int x\left(\frac{1}{1+x^2}\right)dx$

$= x\tan^{-1}x - \frac{1}{2}\int \frac{2x\,dx}{1+x^2} \quad \frac{du}{u}$

$= x\tan^{-1}x - \frac{1}{2}\ln(1+x^2) + C$

$$\int x^2 e^x dx \qquad = x^2e^x - 2xe^x + 2e^x + C$$

$u = x^2 \quad du = e^x dx$

$du = 2x\,dx \quad v = e^x$

$$= x^2e^x - 2\boxed{\int xe^x dx}$$

$u = x \quad \int e^x dx = \int dv$

$du = 1 dx \quad e^x = v$

$$= xe^x - \int e^x dx$$

$$= xe^x - e^x$$

$$= x^2e^x - 2(xe^x - e^x) + C$$

$$= x^2e^x - 2xe^x + 2e^x + C$$

By Parts !!

Twice ..

Tabular Integration

- This is a shortcut for integration by parts IF the integral is of the form

$$\int f(x)g(x)dx$$

where either f(x) or g(x) can be continuously differentiated to eventually result in 0.

$$ie.\ f(x) = x^3$$

Example:

$$\int x^2 e^x dx$$

1st product positive, then alternate signs.

$$f(x) = x^2 \ and \ g(x) = e^x$$

Derivatives of $f(x)$		Integrals of $g(x)$
x^2	+	e^x
$2x$	-	e^x
2	+	e^x
0		e^x

$$= x^2 e^x - 2xe^x + 2e^x + C$$

$$\int x^5 \cos x dx$$

x^5	$+$	$\cos x$
$5x^4$	$-$	$\sin x$
$20x^3$	$+$	$-\cos x$
$60x^2$	$-$	$-\sin x$
$120x$	$+$	$\cos x$
120	$-$	$\sin x$
0		$-\cos x$

$$= x^5 \sin x + 5x^4 \cos x - 20x^3 \sin x - 60x^2 \cos x + 120x \sin x$$
$$+ 120 \cos x + C$$

Practice...

Page 498

#3 - 24 (odd numbers)

Bonus

$$\int \frac{12t^2 + 36}{\sqrt[5]{3t + 2}}\, dt$$

Answer....NOT A CHANCE I move this box today!!!