Asymptotes

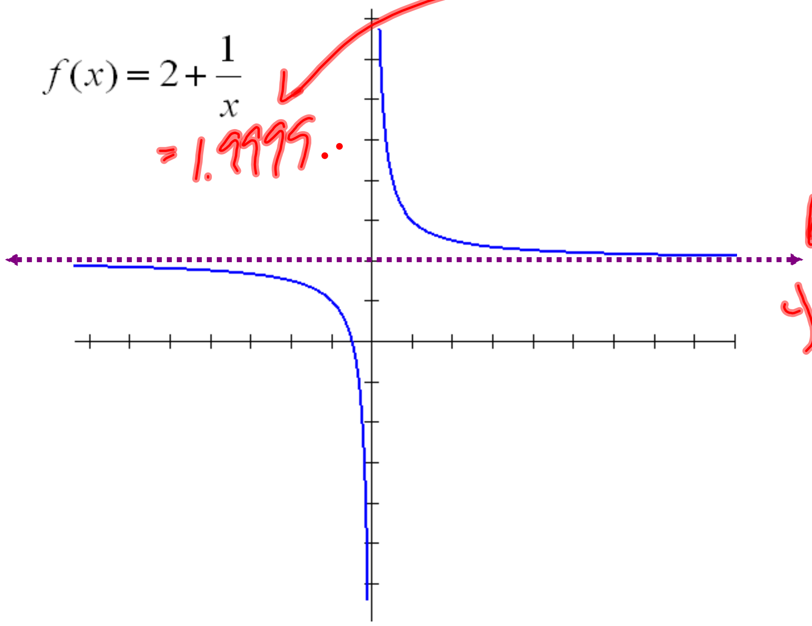
Horizontal Asymptote

The line $y = b$ is a horizontal asymptote of the graph of a function $y = f(x)$ if either

$$\lim_{x \to \infty} f(x) = b \quad \text{or} \quad \lim_{x \to -\infty} f(x) = b$$

$$f(x) = 2 + \frac{1}{x}$$

$= 1.9995..$

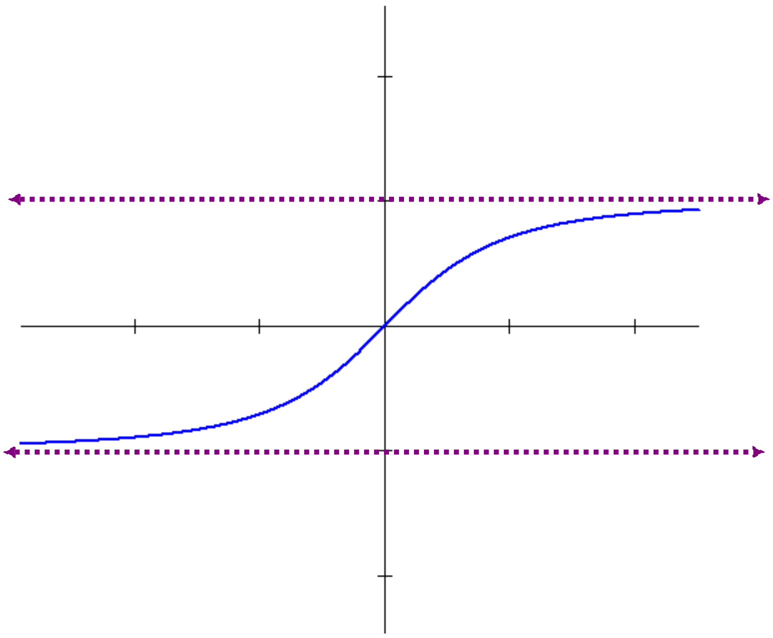


$y=2$ Horizontal Asymptote

Examine the limits of $f(x)$ as x approaches $\pm\infty$

There can be more than one horizontal asymptote.

Examine the function $f(x) = \dfrac{x}{\sqrt{x^2+1}}$



Examine the limits of $f(x)$ as x approaches $\pm\infty$

Vertical Asymptote

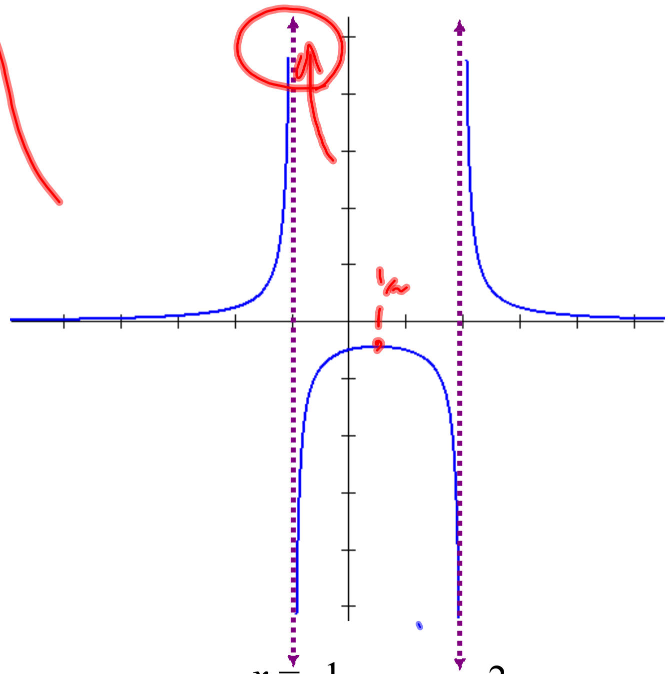
The line $x = a$ is a vertical asymptote of the graph of a function $y = f(x)$ if either

$$\lim_{x \to a^-} f(x) = \pm\infty \qquad \text{or} \qquad \lim_{x \to a^+} f(x) = \pm\infty$$

Example: $f(x) = \dfrac{1}{x^2 - x - 2}$ $(x-2)(x+1)=0$

Locate Vertical Asymptote ...

$f(x)$ undefined



Use limits to examine the behaviour of the function near the asymptotes

$$f(x)=\frac{8(x-2)}{x^2} \qquad f'(x)=\frac{-8(x-4)}{x^3} \qquad f''(x)=\frac{16(x-6)}{x^4}$$

Examine for asymptotes:

Horizontal:

$$\lim_{x\to\pm\infty}\frac{\frac{8x}{x^2}-\frac{16}{x^2}}{\frac{x^2}{x^2}} = \frac{0-0}{1}$$

$$\boxed{y=0}$$

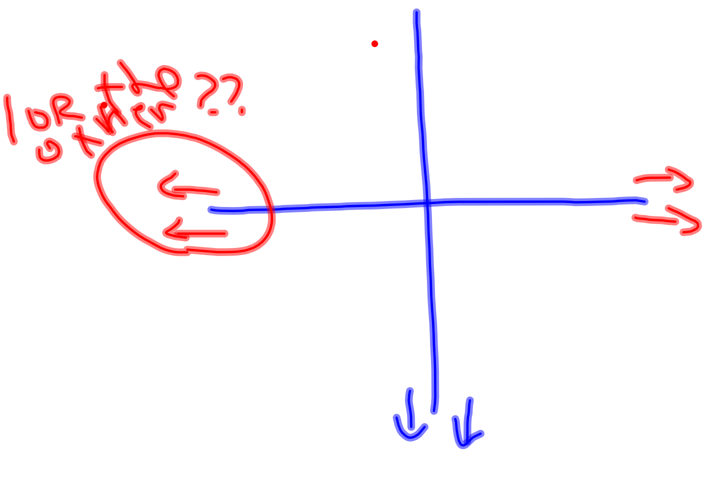
Vertical: f(x) undefi

$$x^2=0$$

$$x=0$$

$$\lim_{x\to0^-}\frac{8(x-2)}{x^2} = \frac{8(-2)}{(0^-)^2} = \frac{-16}{\text{small}(+)} \to -\infty$$

$$\lim_{x\to0^+}\frac{8(x-2)}{x^2} = \frac{-16}{(0^+)^2} = -\infty$$



Sketch the following function:

$$f(x)=\frac{8(x-2)}{x^2} \qquad f'(x)=\frac{-8(x-4)}{x^3} \qquad f''(x)=\frac{16(x-6)}{x^4}$$

Be sure to examine...

- Intercepts
- Asymptotes (*vertical and horizontal*)
- Regions of increase/decrease
- Local extrema
- Regions where concave up/down
- Inflection points → Sketch

Intercepts

x-Int.

$0=\frac{8(x-2)}{x^2}$

$x=2$

$(2,0)$

y-Int.

$y=\frac{8(0-2)}{0^2}$

undefined

$\therefore$ None

Asymptotes

Horizontal

$\lim_{x\to\pm\infty}\frac{\frac{8x}{x^2}-\frac{16}{x^2}}{\frac{x^2}{x^2}}=\frac{0-0}{1}$

$\boxed{y=0}$

Vertical

$x^2=0$

$x=0$

$\lim_{x\to 0^-}\frac{8(x-2)}{x^2}\to-\infty$

$\lim_{x\to 0^+}\frac{8(x-2)}{x^2}\to-\infty$

Inc/Dec.

$f'(x)=\frac{-8(x-4)}{x^3}$

Critical Values:

$x=4,0$

	-8	$x-4$	x^3	f'	f
$(-\infty,0)$	$-$	$-$	$-$	$-$	Dec
$(0,4)$	$-$	$-$	$+$	$+$	Inc
$(4,\infty)$	$-$	$+$	$+$	$-$	Dec

Local Max.

$(4,1)$

Local Min.

None

Concavity

$0=\frac{16(x-6)}{x^4}$

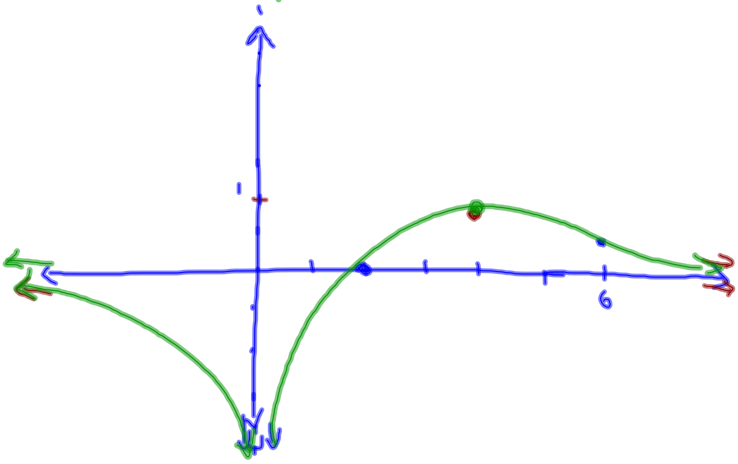
Critical Values

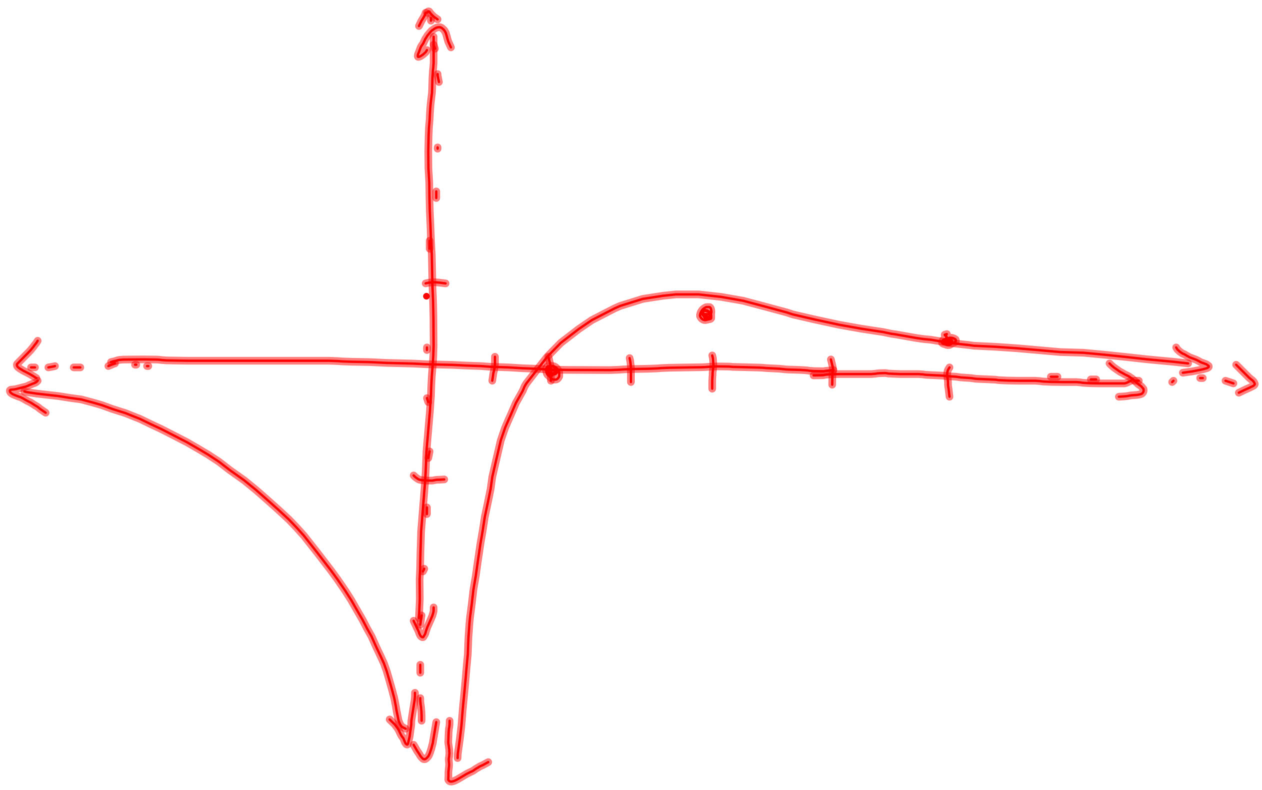
$x=6,0$

	16	$x-6$	x^4	f''	f
$(-\infty,0)$	$+$	$-$	$+$	$-$	Down
$(0,6)$	$+$	$-$	$+$	$-$	Down
$(6,\infty)$	$+$	$+$	$+$	$+$	Up

Inflection Point(s)

$(6,\frac{8}{9})$





Given $f(x) = x^{1/3}(4 + x)$,

Intercepts

$$f'(x) = \frac{4x + 4}{3x^{2/3}} \qquad \text{and} \qquad f''(x) = \frac{4x - 8}{9x^{5/3}}.$$

(a) Find and specify all intervals where f is increasing; decreasing; concave up; and concave down.

(b) Determine the coordinates of any relative extreme values and any points of inflection.

(c) Sketch a graph of f, showing all information obtained in parts (a) and (b).