**This leads to the following differentiation formula...**

**Derivative of the Natural Exponential Function**

$$\frac{d}{dx}(e^x) = e^x$$

**This is the ONLY function that is its own derivative**

$$f(x) = e^x$$
$$f'(x) = e^x(1)$$

# In General...

$$\frac{d(e^u)}{dx} = e^u \bullet du$$

$$y = e^{x^2}$$
$$y' = e^{x^2}(2x)$$

$$y = e^{\cos x^3}$$
$$y' = e^{\cos x^3}(-\sin x^3(3x^2))$$

# Practice Exercises

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#4, 5, 6, 8, 9, 10,

# Warm Up

Differentiate: $e^{xy^2} = 2x - 3xy + e^{\tan x}$

$$e^{xy^2}\left(y^2 + x(2y)\frac{dy}{dx}\right) = 2 - \left(3y + 3x\frac{dy}{dx}\right) + e^{\tan x}(\sec^2 x)$$

$$e^{xy^2}y^2 + 2xye^{xy^2}\frac{dy}{dx} = 2 - 3y - 3x\frac{dy}{dx} + e^{\tan x}\sec^2 x$$

$$\left(2xye^{xy^2} + 3x\right)\frac{dy}{dx} = 2 - 3y + e^{\tan x}\sec^2 x - e^{xy^2}y^2$$

$$\frac{dy}{dx} = \frac{2 - 3y + e^{\tan x}\sec^2 x - e^{xy^2}y^2}{2xye^{xy^2} + 3x}$$

Find the equation of the tangent line to the curve $y = 1 + xe^{2x}$ at the point where $x = 0$.

**Point**

$y = 1 + (0)e^0$

$y = 1$

$(0, 1)$

**Slope**

$y' = e^{2x} + xe^{2x}(2)$

at $x = 0$

$m = e^0 + 0(e^0)(2)$

$m = 1$

$y - 1 = 1(x - 0)$

$y = x + 1$

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#2. $f(x) = x\tan^{-1}x$ at $x = 1$

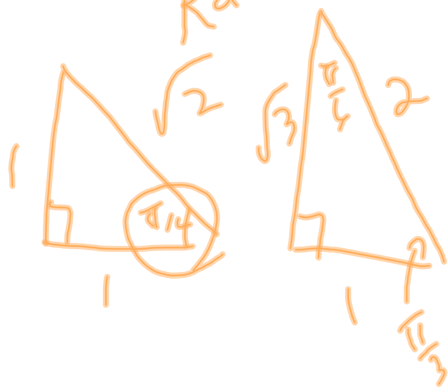
$$f'(x) = \tan^{-1}x + x\left(\frac{1}{1+x^2}\right)$$

$$f'(1) = \tan^{-1}(1) + \frac{1}{1+(1)^2}$$

Must be in Radians

$$= \frac{\pi}{4} + \frac{1}{2}$$

$$= \frac{\pi + 2}{4}$$



## Logarithms

$3^x = W \iff \log_3 W = x$

exponential Notation

$\log_3 17 = \quad 3^{?} = 17$

## 3 Basic Properties

$b > 0, b \neq 1$

① $\log_b 1 = 0$

$\log_7 7^3 = 3$

② $\log_b b^x = x$

$\log 36 = ?$

Assumed 10

③ $b^{\log_b x} = x$

$3^{\log_3 7} = 7$

## Laws of logarithms

Product: $\log_b(MN) = \log_b M + \log_b N$

Quotient: $\log_b\left(\frac{M}{N}\right) = \log_b M - \log_b N$

$\cancel{\frac{\log_b M}{\log_b N}}$

Power Law: $\log_b x^N = N(\log_b x)$

Common Log: Base 10, ex. $\log 17$

Natural Logarithm: Base "e", $\ln 7$

Base "e"

## Change of Base Formula

$\log_3 17$

$= \frac{\log 17}{\log 3}$ or $\frac{\ln 17}{\ln 3}$

$\doteq 2.57$

$\log_b M = \frac{\log_w M}{\log_w b}$

## Derivatives of Logarithmic Functions

Let's work from the known...

- At this point you should know how to differentiate $y = e^x$.

What other function could this model?

$$\ln y = x$$

Try to differentiate → $y = \ln x$.

$$e^y = x$$

$$e^y\left(\frac{dy}{dx}\right) = 1$$

$$\frac{dy}{dx} = \frac{1}{e^y} = \frac{1}{x}$$

Differentiate: $y = \ln x^3$

$$e^y = x^3$$

$$e^y \frac{dy}{dx} = 3x^2$$

$$\frac{dy}{dx} = \frac{3x^2}{e^y} = \frac{3x^2}{x^3}$$

Rule: $d(\ln u) = \frac{1}{u}\,\underline{du}$

ex. $y = \ln(7x^3)$

$y' = \frac{21x^2}{7x^3}$

$y = \ln^5 \sqrt{x}$

$y = (\ln \sqrt{x})^5$

$y' = 5(\ln\sqrt{x})^4 \left(\frac{1}{\sqrt{x}}\left(\frac{1}{2}x^{-\frac{1}{2}}\right)\right)$

3/

$y = \ln(\ln(\ln(\tan^{-1}x^2)))$

$y' = \frac{1}{\ln(\ln(\tan^{-1}x^2))}\left(\frac{1}{\ln(\tan^{-1}x^2)}\right)\left(\frac{1}{\tan^{-1}x^2}\right)\left(\frac{2x}{1+x^4}\right)$

$y = \ln(\overbrace{\ln x^2}^{u})$

$y' = \frac{1}{\ln x^2}\left(\frac{1}{x^2}(2x)\right)$