Integration Review...

$\int x^2 \sin x \, dx$

$= -x^2\cos x + 2x\sin x + 2\cos x + C$

$\int x2^{x^2}\, dx$

$= \frac{1}{2\ln 2} 2^{x^2} + C$

$\int \frac{4x^3 - x^2 + 16x}{x^2+4}\, dx$

$= 2x^2 - x + 2\tan^{-1}\left(\frac{x}{2}\right) + C$

$\int \frac{x+3}{2x(x-2)(x+2)}\, dx$

$= -\frac{3}{8}\ln|x| + \frac{5}{16}\ln|x-2| + \frac{1}{16}\ln|x+2| + C$

$\int \sin^3 7x \, dx$

$= -\frac{\cos 7x}{7} + \frac{\cos^3 7x}{21} + C$

$\int \frac{\sqrt{x^2-25}}{x^4}\, dx$

$= \frac{1}{75}\left(\frac{\sqrt{x^2-25}}{x}\right)^3 + C$

$$\begin{array}{r} 3 \\ 7\overline{)23} \\ \underline{21} \\ 2 \end{array} \qquad = \frac{23}{7}$$

$$3 + \frac{2}{7}$$

Common factor

$$\frac{x^2}{4} + \frac{4}{\cancel{4}}$$

$$u^2 + 1$$

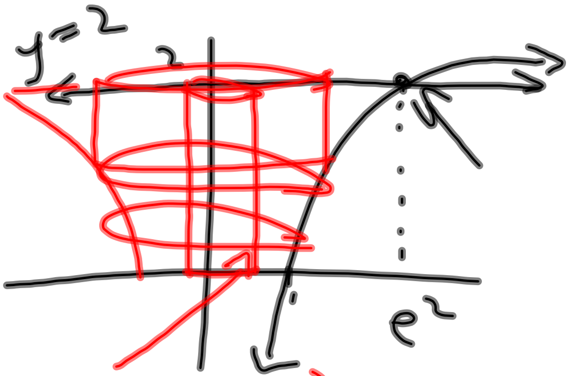
$$4\left(\frac{1}{4}x^2 + 1\right)$$

$$\left(\frac{1}{2}x\right)^2 + 1$$

$$2\int \frac{\left(\frac{1}{2}\right)dx}{\left(\frac{1}{2}x\right)^2 + 1}$$

$$2\tan^{-1}\left(\frac{1}{2}x\right)$$

4. b)



$y = \ln x$

$\ln x = 2$

$e^2 = x$

$$2\pi \int_0^1 x(2)\,dx + 2\pi \int_1^{e^2} (2 - \ln x)(x)\,dx$$

$y = \ln x$

$e^y = x$

OR

Disks

$$\pi \int_0^2 (e^y)^2\,dy$$

(b) $\int x^2 \sin x \, dx$ Use parts 2 times: $u = x^2$ $dv = \sin x \, dx$

$du = 2x \, dx$ $v = -\cos x$.

$\hookrightarrow = -x^2 \cos x - \int -2x \cos x \, dx$. Now for second integral:

$u = x$ $dv = \cos x \, dx$

$du = dx$ $v = \sin x$.

$= -x^2 \cos x + 2\left[x \sin x - \int \sin x \, dx\right]$

$= -x^2 \cos x + 2x \sin x + 2\cos x + C$.

3. Evaluate the following integrals.

- $\int x2^{x^2}dx$ $= \frac{1}{2}\int 2x\,2^{x^2}dx = \frac{1}{2}\int 2^u\,du$

let $u = x^2$

then $du = 2x\,dx$

sub. $= \frac{1}{2} 2^u \cdot \frac{1}{\ln 2}$

$= \frac{1}{2\ln 2} 2^{x^2} + C$

2. $\int \frac{4x^3-x^2+16x}{x^2+4}\,dx \quad = \int 4x-1 + \frac{4}{x^2+4}\,dx$

$$\begin{array}{r}
4x \quad -1 \qquad\qquad\qquad \\
x^2+4\,\big)\overline{4x^3 - x^2 + 16x + 0} \\
-(4x^3 \qquad + 16x) \\
\hline
-x^2 \qquad + 0 \\
(-x^2 \qquad\quad -4) \\
\hline
4
\end{array}$$

$$= \frac{4x^2}{2} - x + 4\tfrac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + C$$

1 $= 2x^2 - x + 2\tan^{-1}\left(\frac{x}{2}\right) + C$

$$\frac{x+3}{2x(x-2)(x+2)} = \frac{A}{2x} + \frac{B}{x-2} + \frac{C}{x+2}$$

$$x+3 = Ax^2 - 4A + 2Bx^2 + 4Bx + 2Cx^2 - 4Cx$$

Equating coefficients

$$\left.\begin{aligned} 0 &= A + 2B + 2C \\ 1 &= 4B - 4C \\ 3 &= -4A \Rightarrow A = -\tfrac{3}{4} \end{aligned}\right\} \Rightarrow \begin{aligned} \tfrac{3}{4} &= 2B + 2C \\ 1 &= 4B - 4C \end{aligned}$$

$$\Rightarrow \begin{aligned} \tfrac{3}{2} &= 4B + 4C \\ 1 &= 4B - 4C \end{aligned} \Rightarrow \frac{5}{2} = 8B \Rightarrow B = \frac{5}{16} \Rightarrow 1 = 4\left(\frac{5}{16}\right) - 4C$$

$$-\frac{1}{4} = -4C \Rightarrow C = \frac{1}{16}$$

So the integration becomes

$$\int \frac{-3/4}{2x}\,dx + \int \frac{5/16}{x-2}\,dx + \int \frac{1/16}{x+2}\,dx$$

$$= -\frac{3}{8}\ln|x| + \frac{5}{16}\ln|x-2| + \frac{1}{16}\ln|x+2| + C$$

$$\int \sin^3 7x\, dx = \int \sin^2 7x \cdot \sin 7x\, dx$$

$$= \int (1-\cos^2 7x) \sin 7x\, dx$$

$$= \int \sin 7x\, dx + \int -\sin 7x \cos^2 7x\, dx$$

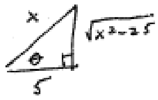
let $u = \cos 7x$

$du = -7 \sin 7x\, dx$

$$= -\frac{1}{7}\cos 7x + \frac{1}{7}\frac{\cos^3 7x}{3} + C$$

Let $x = 5\sec\theta$, $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

then $dx = 5\sec\theta\tan\theta\, d\theta$



Then $$\int \frac{\sqrt{x^2-25}}{x^4}\,dx = \int \frac{\sqrt{25\sec^2\theta - 25}}{625\sec^4\theta} \cdot 5\sec\theta\tan\theta\, d\theta$$

$$= \int \frac{\sqrt{25\tan^2\theta}}{125\sec^3\theta}\tan\theta\, d\theta = \frac{1}{25}\int \frac{\tan^2\theta}{\sec^3\theta}\,d\theta$$

$$= \frac{1}{25}\int \sin^2\theta\cos\theta\, d\theta = \frac{1}{25}\,\frac{\sin^3\theta}{3} + C$$

4

$$= \frac{1}{75}\left(\frac{\sqrt{x^2-25}}{x}\right)^3 + C$$