Check-Up

Given the function $f(x) = 4x^4 - 8x^2 + 1$ determine ...

(a) the absolute maximum and minimum values on the interval [0 , 3].

(b) the intervals of increase/decrease and local extreme values on the interval (-∞ , ∞).

$f'(x) = 16x^3 - 16x$

Critical Values: $16x(x^2-1) = 0$

$16x(x-1)(x+1) = 0$

$x = 0, \pm 1$

(a)

x	y
0	1
1	-3
3	253

Abs. Max. = 253

Abs. Min. = -3

b)

	16x	x-1	x+1	f'	f
$(-\infty, -1)$	−	−	−	−	Dec
$(-1, 0)$	−	−	+	+	Inc
$(0, 1)$	+	−	+	−	Dec
$(1, \infty)$	+	+	+	+	Inc

$\overbrace{16x \cdot (x-1)(x+1)}^{16x^3-16x}$

Local Minimum

$(-1, -3)$ $(1, -3)$

Local Maximum

$(0, 1)$

Optimization Problems

In optimization problems, in general, we are looking for the largest and/or smallest that a function can be. We saw how to one kind of optimization problem in the Absolute Extrema section where we found the largest and smallest value that a function would take on an interval.

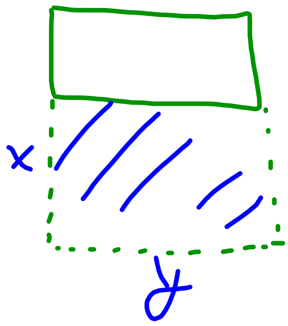
In this section we are going to look at another type of optimization problem. Here we will be looking for the largest or smallest values of a function subject to some kind of constraint. It's usually easiest to see how these work with some examples.

Example 1:

We need to enclose a field with a fence. We have 500 feet of fencing material and a building is on one side of the field and so won't need any fencing. Determine the dimensions of the field that will enclose the largest area.

constraint

- Draw a sketch
- Determine the constraint
- Determine a function in terms of a single variable
- Determine the absolute maximum value of this function
- Test solution with second derivative



$2x + y = 500$

$y = 500 - 2x$

$A = xy$

$A = x(500 - 2x)$

$A = 500x - 2x^2$

$A' = 500 - 4x$

Critical Values $\rightarrow$ $0 = 500 - 4x$

$\frac{4x}{4} = \frac{500}{4}$

$x = 125$

$y = 500 - 2(125)$

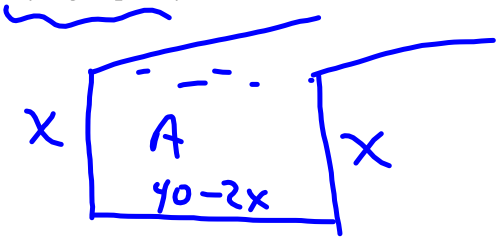
$= 250$

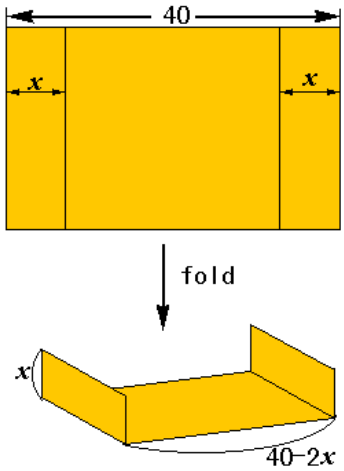
250 feet by 125 feet

Example 2:

A piece of tin 40 cm wide is to be folded up as shown.

How deep will this gutter be if it is to have maximum carrying capacity?





Maximize Area of Face

$A = x(40-2x)$

$A = 40x - 2x^2$

$A' = 40 - 4x$

$0 = 40 - 4x$

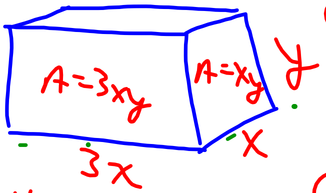
$4x = 40$

$x = 10$

10 cm deep

Example 3:

We are going to construct a box whose base length is 3 times the base width. The material used to build the top and bottom cost \$10/ft² and the material used to build the sides cost \$6/ft². If the box must have a volume of 50ft³ determine the dimensions that will minimize the cost to build the box.



$$\text{Cost} = 6(2xy) + 6(3xy)(2) + 10(3x^2)(2)$$

$$V = 50$$

$$3x^2y = 50$$

$$y = \frac{50}{3x^2}$$

$$C = 12xy + 36xy + 60x^2$$

$$C = 48xy + 60x^2$$

$$C = \overset{16}{\cancel{48}}x\left(\frac{50}{\cancel{3}x^2}\right) + 60x^2$$

$$C = \frac{800}{x} + 60x^2$$

$$\longrightarrow C = 800x^{-1} + 60x^2$$

$$C' = -800x^{-2} + 120x$$

$$(x^2)\,0 = -\frac{800}{\cancel{x^2}}\cancel{(x^2)} + 120x(x^2)$$

$$\frac{-120x}{1} = \frac{-800}{x^2}$$

$$-120x^3 = -800$$

$$\sqrt[3]{x^3} = \sqrt[3]{\frac{800}{120}}$$

$$x = 1.88 \text{ feet}$$

length: $3(1.88)$
$= 5.64$ feet

height: $y = \frac{50}{3(1.88)^2}$

$y = 4.7$ feet

Continue practice problems...

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Attachments

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