

Warm Up

Prove the following identities...

1. $\sec x - \tan x \sin x = \frac{1}{\sec x}$

$$\frac{1}{\cos x} - \frac{\sin x}{\cos x} \left(\frac{\sin x}{1} \right) \left\{ \begin{array}{l} \text{LS} \\ \text{RS} \end{array} \right. \frac{1}{\cos x}$$

$$\frac{1 - \sin^2 x}{\cos x} \quad \text{CS=RS}$$

$$\frac{\cos^2 x}{\cos x} = \cos x$$

2. $\frac{\sec \theta \sin \theta}{\tan \theta + \cot \theta} = \sin^2 \theta$

$$\frac{\frac{1}{\cos \theta} (\sin \theta)}{\frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta}} \left(\frac{\sin \theta}{\cos \theta} \right) \cdot \frac{\cos \theta \sin \theta}{1}$$

$$\frac{\sin^2 \theta}{\cos^2 \theta + \sin^2 \theta} \cdot \frac{\cos \theta \sin \theta}{1}$$

$$\frac{\sin^2 \theta}{1} \quad \text{CS=RS}$$

3. $\csc^4 x - \cot^4 x = \csc^2 x + \cot^2 x$

$$\frac{(\csc^2 x - \cot^2 x)(\csc^2 x + \cot^2 x)}{(1)(\csc^2 x + \cot^2 x)} \quad \text{CS=RS}$$

4. $\frac{\cos x + 1}{\sin^3 x} = \frac{\csc x}{1 - \cos x}$

$$\frac{\cos x + 1}{\sin x \sin^2 x} \cdot \frac{\sin x}{\sin x} \cdot \frac{1}{1 - \cos x}$$

$$\frac{(\cos x + 1)(1 - \cos x)}{\sin x (1 - \cos^2 x)}$$

$$\frac{1 - \cos^2 x}{\sin x (1 - \cos^2 x)} \cdot \frac{1}{1 - \cos x}$$

$$\frac{1}{\sin x (1 - \cos x)} \left(\frac{1}{\sin x} \right) \left(\frac{1}{1 - \cos x} \right)$$

$$\frac{\csc x}{1 - \cos x}$$

$$\frac{\csc x (1 + \cos x)}{1 - \cos x (1 + \cos x)}$$

$$\frac{\csc x (1 + \cos x)}{1 - \cos^2 x} \left(\frac{1}{\sin x} \right) \left(\frac{1 + \cos x}{1} \right)$$

$$\frac{1 + \cos x}{\sin x} \cdot \frac{1}{\sin^2 x}$$

$$\frac{1 + \cos x}{\sin^3 x}$$

New Identities

$$\cos \theta = \sin(90 - \theta)$$

$$\sin \theta = \cos(90 - \theta)$$

$$\sin(x + y) = \sin x \cos y + \cos x \sin y$$

$$\sin(x - y) = \sin x \cos y - \cos x \sin y$$

$$\cos(x + y) = \cos x \cos y - \sin x \sin y$$

$$\cos(x - y) = \cos x \cos y + \sin x \sin y$$

Double Angle Identities

If $\theta = \alpha$:

$$\begin{aligned}
 &= \sin 2\theta \\
 \sin(\theta + \alpha) &= \sin(\theta + \theta) \\
 &= \sin\theta \cos\theta + \cos\theta \sin\theta \\
 &= \sin\theta \cos\theta + \sin\theta \cos\theta \\
 \sin(2\theta) &= 2\sin\theta \cos\theta
 \end{aligned}$$

$$\begin{aligned}
 &= \cos(2\theta) \\
 \cos(\theta + \alpha) &= \cos(\theta + \theta)
 \end{aligned}$$

$$\begin{aligned}
 &= \cos\theta \cos\theta - \sin\theta \sin\theta \\
 \cos 2\theta &= \cos^2\theta - \sin^2\theta \\
 &= 1 - 2\sin^2\theta \\
 &= 2\cos^2\theta - 1
 \end{aligned}$$

Double Angle Identities

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta$$

$$= 1 - 2\sin^2 \theta$$

$$= 2\cos^2 \theta - 1$$

Prove...

$$\frac{1 - \sin 2\theta}{\cos 2\theta} = \frac{\cos 2\theta}{1 + \sin 2\theta}$$

LS

$$\frac{1 - 2\sin\theta\cos\theta}{\cos^2\theta - \sin^2\theta}$$

$$\frac{\cos^2\theta - 2\sin\theta\cos\theta + \sin^2\theta}{\cos^2\theta - \sin^2\theta}$$

$$\frac{(\cos\theta - \sin\theta)^2}{(\cos\theta - \sin\theta)(\cos\theta + \sin\theta)}$$

$$\frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta}$$

RS

$$\frac{\cos^2\theta - \sin^2\theta}{1 + 2\sin\theta\cos\theta}$$

$$\frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta + 2\sin\theta\cos\theta + \sin^2\theta}$$

$$\frac{(\cos\theta - \sin\theta)(\cos\theta + \sin\theta)}{(\cos\theta + \sin\theta)^2}$$

$$\frac{1 - \sin 2\theta}{\cos 2\theta} = \frac{\cos 2\theta}{1 + \sin 2\theta}$$

RS

$$\frac{\cos 2\theta (1 - \sin 2\theta)}{(1 + \sin 2\theta)(1 - \sin 2\theta)}$$

$$\frac{\cos 2\theta (1 - \sin 2\theta)}{1 - \sin^2 2\theta}$$

$$\frac{\cancel{\cos 2\theta} (1 - \sin 2\theta)}{\cos^2 2\theta}$$

Prove...

$$\frac{\cos \theta - \sin \theta}{\cos \theta + \sin \theta} = \sec 2\theta - \tan 2\theta$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

RS

$$\frac{1}{\cos 2\theta} - \frac{\sin 2\theta}{\cos 2\theta}$$

$$\frac{1}{\cos^2 \theta - \sin^2 \theta} - \frac{2 \sin \theta \cos \theta}{\cos^2 \theta - \sin^2 \theta}$$

$\cos^2 \theta + \sin^2 \theta \rightarrow 1$

$$\frac{1 - 2 \sin \theta \cos \theta}{\cos^2 \theta - \sin^2 \theta}$$

$$\frac{\cos^2 \theta - 2 \sin \theta \cos \theta + \sin^2 \theta}{\cos^2 \theta - \sin^2 \theta}$$

$$\frac{(\cos \theta - \sin \theta)^2}{(\cos \theta - \sin \theta)(\cos \theta + \sin \theta)}$$

$$\frac{\cos \theta - \sin \theta}{\cos \theta + \sin \theta}$$