Similar to 1997 AB Multiple Choice Solutions
Created by Students (No Calculators)

1. $\int_0^1 \left(6x^2 - 8x\right) dx$

$2x^3 - 4x^2 \Big]_0^1$

$[\,2(1)^3 - 4(1)^2\,] - [\,2(0)^3 - 4(0)^2\,]$

$= [2 - 4] - (0)$

$= -2$

* **The correct answer is A**

Created by Anna
Checked by Katie

Solution- Similar to 1997 AB #2

The correct answer is **C**

Here's Why:

$F(x) = x\sqrt{6x-7}$ Find $F'(x)$

1. $x\sqrt{6x-7} = x(6x-7)^{\frac{1}{2}}$

2. $F' = x\,\frac{1}{2}(6x-7)^{\frac{-1}{2}}(6) + (6x-7)^{\frac{1}{2}}$

3. $3x(6x-7)^{\frac{-1}{2}} + (6x-7)^{\frac{1}{2}}$

4. $\dfrac{3x}{\sqrt{6x-7}} + \dfrac{6x-7}{\sqrt{6x-7}}$

5. $\therefore \quad F' = \dfrac{9x-7}{\sqrt{6x-7}}$

Created By: Mary **Checked By:** Jenna

#3 Correct Answer: C

SOLUTION:
Separate $\int_c^d (f(x)-4)dx$ into $\int_c^d f(x)dx - \int_c^d 4dx$.
Since you already know that $\int_c^d f(x)dx$ is equal to 3c + 2d, you only have to integrate the second expression. When you integrate $\int_c^d 4dx$, your result is $4x\big|_c^d$, or 4d – 4c. You then subtract this answer from 3c + 2d. Your final result is 3c +2d – (4d – 4c), or 7c–2d.

Similar to 1997 AB **#4** No Calculator

Correct answer: **D**

Solution: $f'(x) = -8x^3 - 1 + \left(\frac{-1}{x^2}\right)$

After finding the derivative, just plug in the value at which you are trying to find the derivative value:

$-8(-1)^3 - 1 + \left(\frac{-1}{1}\right)$

8-1-1=6

Created by: Branden on May 15, 2003 Checked by: Laura

SOLUTION TO #5 Similar to 1997 AB

Correct answer: E

Solution: By taking the first derivative of the equation we get $y' = 4x^3 - 18x^2 - 48x + 2$. Then to find the zero points, we take the second derivative and get $y'' = 12x^2 - 36x - 48$. Factoring this equation gives us the zero points –1 and 4. Next we create a number line for the second derivative to see where the graph is positive and negative. In this case, the graph is positive before –1 and after 4, being negative in between. Therefore E is the best choice.

Problem Created By: Branden on May 16, 2003 Problem Checked By: Laura

Solutions for Similar to 1997 No Calculator

#6 SOLUTION:

The correct answer is (c)

Break the integral up into $\int e^{\frac{x}{3}}\,dx + \int 4\,dx$. In order to have the first integral in the form e^{u} du, you have to put a 3 outside the integral and a $\frac{1}{3}$ inside the integral. The integral is therefore equal to $3e^{\frac{x}{3}}$. The value of the 2nd integral is 4x.

<u>Check</u>:

The derivative of $3e^{\frac{x}{3}} + 4x + C = 3(\frac{1}{3})\, e^{\frac{x}{3}} + 4 + 0 = e^{\frac{x}{3}} + 4$

Problem By: Stephanie
Checked By: Glenn

#7 The Correct Answer is C

<u>SOLUTION</u>

$$\frac{d}{dx}\sin^2\left(x^3\right)$$

$$= \frac{d}{dx}\left(\sin\left(x^3\right)\right)^2$$

$$= 2\sin\left(x^3\right)\bullet\cos\left(x^3\right)\bullet 3x^2$$

$$= 6x^2\sin\left(x^3\right)\cos\left(x^3\right)$$

By Holly F.
Checked by Stephanie C.

Solutions for Similar to 1997 No Calculator

#8 ANSWER: c. 4

SOLUTION: the graph crosses the x-axis at (4,0), going from a positive to negative velocity and signaling a change in direction.

#9 ANSWER: d. 8.5

SOLUTION: to find the total distance traveled you must find the area under the graph

A= $\frac{1}{2}bh$, so for four triangles use this formula

1+1+0.5+1=3.5 (area of all triangles)

A= s^2, so for two squares use this formula

4+1=5 (area of all squares)

$\therefore$ 3.5+5=8.5 total distance traveled by bug

#10 Similar to 1997 AB

SOLUTION! Correct Answer: C

Y = sin (4x), so y' = 4cos(4x) ……WATCH CHAIN RULE!

Evaluate y' at x = $\frac{\pi}{2}$: y' = 4cos(4 * $\frac{\pi}{2}$)

y' = 4

Plug $\frac{\pi}{2}$ into original equation (y = sin (4x)) to find the y value!

y=0 …. 0=4($\frac{\pi}{2}$) + b

b= –2π

Equation of tangent line…. y=4x – 2π ☺

Problem By: Stephanie
Checked By: Holly

Solutions for Similar to 1997 No Calculator

Problem #11

Solution: E

Reason: On the left of point a, the curve must have a negative slope. Between a and b, the curve must have a positive slope. At b, the curve must have a critical value and on the right of b, the curve must still have a positive slope.

By Eric , checked by Katie

Problem #12

Correct Answer: B

SOLUTION:

For lines to be perpendicular, their slopes must be opposite reciprocals.

Take the derivative of $y = -3x^2$. Since we are trying to find the tangent line, this derivative will give us the slope of the tangent line.

Find the slope of $5x - 2y = 7$ by getting the equation into "y =" form.

The derivative $y = -3x^2$ must be equal to the opposite reciprocal of the slope of $5x - 2y = 7$.

(Therefore, set $-6x$ [the derivative] equal to $\frac{-2}{5}$ [the opposite reciprocal])

The derivative is equal to $\frac{-2}{5}$ when $x = \frac{1}{15}$

Because the question asks for a specific "point," you must find the y-coordinate when $x = \frac{1}{15}$.

Plug $x = \frac{1}{15}$ into the graph's equation of $y = -3x^2$ to find the y-coordinate.

Your final answer is $\left(\frac{1}{15}\ ,\ -\frac{3}{225}\right)$.

Problem created by: Emily

Problem checked by: Glenn

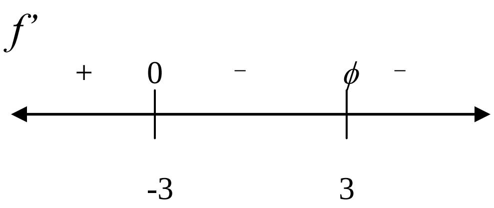
Solutions for Similar to 1997 No Calculator

Problem #13

Correct Answer: C

SOLUTION: This problem gives the first derivative and we know that the sign on the first derivative chart shows increasing and decreasing for the function. To solve, we set the first derivative equal to zero and solve for X as follows.

$9 - x^2 = 0$ => $x^2 = 9$ => $x = +3, -3$



Plug in values from the intervals of the critical numbers.

$f'(-4) = 1$
$f'(0) = -3$
$f'(4) = -7$

Therefore, the interval is decreasing on $(-3, \infty)$, $x \neq 3$.

Problem Created By: Nathan on May 15, 2003 Problem Checked By: Greg

---

Problem #14:

Correct Answer: C

SOLUTION:

Use the coordinates for the point at x = 4, which are (4,1) and the slope at x = 4, which is 4. Generate an equation of the tangent line in point slope form, then solve for y.

Point (4,1)
Slope = m = 4

$$y - y_1 = m(x - x_1)$$
$$y - 1 = 4(x - 4)$$
$$y = 4x - 16 + 1$$
$$y = 4x - 15$$

Solutions for Similar to 1997 No Calculator

#14 (Cont.)

To find a zero approximation for the graph of *f*, use the tangent line and set y = 0 and solve for x.

$0 = 4x - 15$

$15 = 4x$

$4x = 15$

$x = \frac{15}{4}$

$x \approx 3.8$

You cannot find an exact answer for x because this is not a calculator problem. Instead, eliminate answers by estimating 15/4. It cannot be lower than 3 and it cannot be more than 4, so your only answer choice is C.

Problem Created by: Glenn Problem Checked by: Colin

Solution for #15

Correct Answer: A

SOLUTION: We can find the answer to this problem by finding out why some of the answer choices are wrong. Choice B is wrong because $f(2) = 1$, whereas $f(4) = 2$. Choice C is also wrong for basically the same reason: $\lim_{x \longrightarrow 4} f(x) = 2$, which is not equal to $f(2) = 1$.

Choice D is wrong because the limit of the function does exist as x approaches two. Although the function has removable discontinuity at 2, the limit as x approaches two is two. Finally Choice E is wrong because the derivative at $f(3)$ D.N.E. because there could be an infinite number of tangent lines to that point on the graph. Therefore, we are left with Choice A as the correct answer.

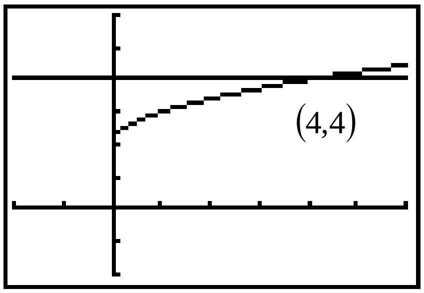
Problem Created By: Nathan
Problem Checked By: Greg May 19, 2003

Solutions for Similar to 1997 No Calculator

#16 Correct Answer: B

SOLUTION:

The formula for the area of the region between two given functions is:



$$A = \int_a^b f(x)dx - \int_a^b g(x)dx$$

a = lower x-intercept of the 2 functions
b = upper x intercept of the 2 functions
f(x) = the function that is on the tope
g(x) = the lower of the 2 functions

This formula can be rewritten as:

$$A = \int_a^b f(x) - g(x)dx$$

First, figure which function is the top function and which is the bottom function. Sketching a graph may be useful. From the graph you can see that $y = 4$ is above $y = \sqrt{x} + 2$. Next, find the limits of integration, aka, the bounds. You know that a = 0, because the second function has a domain of $x \geq 0$. To find the upper limit, set the two functions equal to each other.

$$y = y$$
$$\sqrt{x} + 2 = 4$$
$$\sqrt{x} = 2$$
$$x = 4$$

Now you know that a = 0, b = 4, *f*(x) = 4, and *g*(x) = $\sqrt{x} + 2$. Set up the integral using the formula from above and solve it to find the answer.

$$\mathrm{A} = \int_0^4 4 - (\sqrt{\mathrm{x}} + 2)\mathrm{dx}$$
$$\mathrm{A} = \int_0^4 2 - \sqrt{\mathrm{x}}\mathrm{dx}$$
$$\mathrm{A} = 2\mathrm{x} - \frac{2}{3}\mathrm{x}^{\frac{3}{2}} \Big]_0^4$$
$$\mathrm{A} = 8 - \frac{16}{3} - (0)$$
$$\mathrm{A} = \frac{8}{3}$$

Problem Created by: Glenn
Problem Checked by: Colin

Solutions for Similar to 1997 No Calculator

#17 Correct Answer: A

Solution:

First, the first derivative needs to be found.

$$4x - 2ydy = 0$$
$$4x = 2ydy$$
$$dy = \frac{4x}{2y}$$
$$dy = \frac{2x}{y}$$

Next, the second derivative must be found

$$d^2y = \frac{y(2) - (2x)(dy)}{y^2}$$
$$d^2y = \frac{2y - (2x)\frac{2x}{y}}{y^2}$$
$$d^2y = \frac{2y - \frac{4x^2}{y}}{y^2}$$

Now evaluate

$$d^2y = \frac{4 - \frac{16}{2}}{4}$$
$$d^2y = \frac{4-8}{4}$$
$$d^2y = \frac{-4}{4}$$
$$d^2y = -1$$

Problem Created by: Colin Problem checked by: Glenn

 # 18 Correct Answer: D

Solution:

Let u = sec (x). So, du = sec (x) tan (x) dx = $\frac{1}{\cos(x)} \times \frac{\sin(x)}{\cos(x)} dx = \frac{\sin(x)}{\cos^2(x)} dx$.

So, $\int_0^{\frac{\pi}{3}} e^u du = [e^u]_0^{\frac{\pi}{3}} = [e^{\sec x}]_0^{\frac{\pi}{3}} = e^{\sec\frac{\pi}{3}} - e^{\sec 0} = e^2 - e^1 = e(e-1)$

Problem by Bernie Checked by Peter

Solutions for Similar to 1997 No Calculator

Problem #19 Correct Answer: C

Remember that the formula for differentiating the natural log function is $f'(ln\ u) = \frac{1}{u}du$. In this problem, u = $x^4 - 1$, and after using the chain rule, the derivative is $\frac{4x^3}{x^4 - 1}$.

Problem Created by: Shilpi on May 15, 2003
Problem Checked by: Emily on May 15, 2003

Problem #20 Correct Answer: A

The formula for finding the average value of a function is: AV= $\frac{1}{b-a} * \int_a^b f(x)\,dx$.

In this problem, a = -1 and b = 4. Therefore, $\frac{1}{4--1}\int_{-1}^{4} \sin x\, dx$ when integrated, equals $\frac{1}{5}\left[-\cos x\right]_{-1}^{4}$. When evaluating the function, it equals $\frac{-\cos(4)--\cos(-1)}{5}$ and after simplifying, it equals $\frac{-\cos(4)+\cos(-1)}{5}$, which is the same as answer choice A.

Problem Created by: Shilpi on May 15, 2003
Problem Checked by: Emily on May 15, 2003

#21 Correct Answer: E

Solution:

$\frac{x}{\tan x} = \frac{\pi}{0}$ = nonexistent

L'Hop cannot be used because the limit is not $\frac{0}{0}$ *or* $\frac{\infty}{\infty}$, so the one sided limits must be used.

$\lim_{x\to\pi^+} \frac{x}{\tan x} = \frac{\pi}{0^+} = \infty$ and $\lim_{x\to\pi^-} \frac{x}{\tan x} = \frac{\pi}{0^-} = -\infty$

The one sided limits are not equal so the limit does not exist.

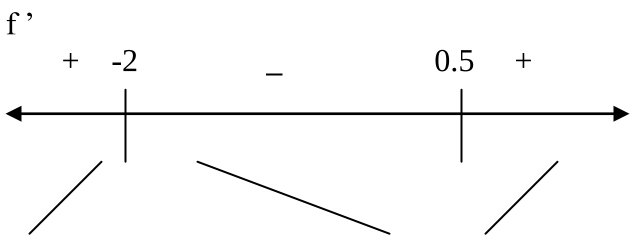
Problem Created By: Ryan May 15, 2003
Checked by: Noah

Solutions for Similar to 1997 No Calculator

#22 Correct Answer: C

Solution: In order to find where the function is decreasing one must find where the derivative is negative.

$f(x) = e^x(2x^2 - x - 1)$
$f'(x) = e^x(4x - 1) + e^x(2x^2 - x - 1)$
$f'(x) = e^x(2x^2 + 3x - 2)$
$f'(x) = e^x(2x - 1)(x + 2)$
$e^x \neq 0$
So $f'(x) = 0$ when $x = -2, 0.5$
Make a sign chart



Therefore $f(x)$ is decreasing on $[-2, 0.5]$

Created By: Hannah Checked By: Kristin

#23 Correct Answer: A

Using the equation for a disk set up an integral and solve:

$V = \int_a^b x^2 dx$ y = x^2

$V = \pi \int_0^2 (x^2)^2 dx$

$V = \pi \int_0^2 x^4 dx$

$V = \pi \left[\frac{1}{5} x^5 \right]_0^2$

$V = \pi \left[\frac{32}{5} - 0 \right]$

$V = \frac{32\pi}{5}$

Problem created by: Colin
Problem checked by: Glenn

Solutions for Similar to 1997 No Calculator

#24 C is the correct answer.

Solution:

According to the definition of a Reimann's Sum, the constant is defined by $\left(\frac{b-a}{n}\right)$. In this problem the constant equals $\frac{1}{15}$, showing that the upper bound (b) minus the lower bound (a) must equal 1. Therefore, choice A and D can be ruled out because the difference between their bounds are greater than 1. The remaining choices have the correct function, (x^2), and the correct change in bounds. However, choice B is incorrect because it has an extra constant in front of the integral.

C IS THE CORRECT ANSWER
Created by: *Jenna* Checked by: Allison

#25 The Correct Answer is A

SOLUTION (integration by parts)

$\int x\sin(3x)dx =$

Tabular:

X	↘	sin (3x)
1	↘	$-\frac{1}{3}\cos(3x)$
	-	
0		$-\frac{1}{9}\sin(3x)$

*Multiply the 'x' in the first row by the $-\frac{1}{3}\cos(3x)$ in the second row; keep the sign.

*Multiply the 1 in the first row by the $-\frac{1}{9}\sin(3x)$ in the second row; change the sign

*Add these two products together to get:

$-\frac{1}{3}x\cos(3x)+\frac{1}{9}\sin(3x)+C$

Made by Holly
Checked by Stephanie