

Connecting Radicals and Exponents:

Time to continue our development of the properties of radicals...

What is the value of each of the following:

$$\sqrt{5} \cdot \sqrt{5} = \sqrt{25} = 5$$

$$\sqrt{7} \times \sqrt{7} = 7^1$$

$$\sqrt[3]{7} \times \sqrt[3]{7} \times \sqrt[3]{7} = \sqrt[3]{7^3} = 7^1$$

$$\sqrt[3]{3} \times \sqrt[3]{3} \times \sqrt[3]{3} = \sqrt[3]{3 \times 3 \times 3}$$

How about the following:

$$x^{\frac{1}{2}} \cdot x^{\frac{1}{2}} = x^1$$

$$\sqrt{x} \cdot \sqrt{x} = x^1$$

$$7^{\frac{1}{3}} \cdot 7^{\frac{1}{3}} \cdot 7^{\frac{1}{3}} = 7^1$$

$$\sqrt[3]{27} = 3$$

$$3^{\frac{1}{2}} = \sqrt{3}$$

QUIZ

$$16^{\frac{1}{2}} = 4 = \sqrt{16}$$

Based on the previous slide it would seem that...

$$\sqrt{x} = x^{\frac{1}{2}}$$

$$\sqrt[3]{x} = x^{\frac{1}{3}}$$

Generally then... $\sqrt[n]{x} = x^{\frac{1}{n}}$

Powers with Rational Exponents with Numerator 1

When n is a natural number and x is a rational number, $x^{\frac{1}{n}} = \sqrt[n]{x}$

ex. $\sqrt[4]{7} = 7^{\frac{1}{4}}$

What about when numerator is NOT a 1??

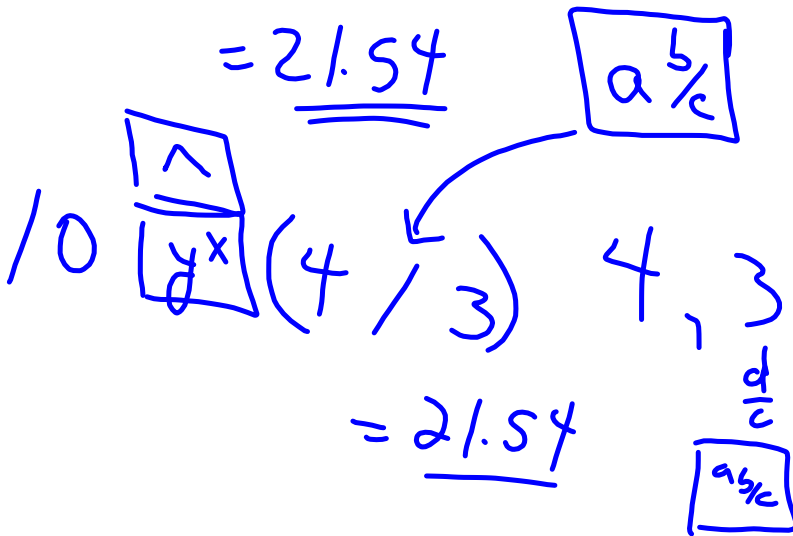
ie. Something like $8^{\frac{5}{3}}$?? $\frac{5}{1}(\frac{1}{3})$
 $= \frac{5}{3}$

Let's relate back to exponent laws...which one would help?

$$8^{\frac{1}{3}} = \sqrt[3]{8} = 2 \quad \text{---OR---} \quad (8^{\frac{1}{3}})^5 = (\sqrt[3]{8})^5 = 2^5 = 32$$

$$(8^5)^{\frac{1}{3}} = \sqrt[3]{32768} = 32$$

$$10^{\frac{4}{3}} = (\sqrt[3]{10})^4 = 21.54$$



$$\frac{3}{4} + \frac{2}{5} = \frac{15}{20} + \frac{8}{20} = \frac{23}{20}$$

$$1\frac{3}{20} = \frac{23}{20}$$

Important Property!!

Powers with Rational Exponents

When m and n are natural numbers, and x is a rational number,

$$\begin{aligned}x^{\frac{m}{n}} &= \left(x^{\frac{1}{n}}\right)^m \\ &= \left(\sqrt[n]{x}\right)^m\end{aligned}$$

and

$$\begin{aligned}x^{\frac{m}{n}} &= \left(x^m\right)^{\frac{1}{n}} \\ &= \sqrt[n]{x^m}\end{aligned}$$

ex. $x^{7/8} = \left(\sqrt[8]{x}\right)^7$ or $\sqrt[8]{x^7}$

Practice Problems...

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