Check-Up...

$\times 3$

Given that (-2 , 5) is a point on the graph of $y = f(x)$, determine the coordinates of this point once the following transformations are applied...

$x(-3)$

(1) $y = 3f(x)$

$\Rightarrow$ Vertical stretch by a factor of 3

$(-2, 15)$

$(x, y) \rightarrow (x, 3y)$

(2) $y = f\left(-\frac{1}{3}x\right)$

$\Rightarrow$ Horizontal stretch by a factor 3

$\Rightarrow$ Reflected in y-axis

$(6, 5)$

$(x, y) \rightarrow (-3x, y)$

$(-2, 5) \rightarrow ??$

(3) $y = 4f\left[\frac{1}{2}(x+5)\right] - 3$ $(-9, 17)$

V.st. 4 H.st. 2 left 5 Down 3

$(x, y) \rightarrow (2x-5, 4y-3)$

$(-2, 5) \rightarrow (2(-2)-5, 4(5)-3)$

$\rightarrow (-9, 17)$

(4) $y - 5 = -2f(-2x+6)$ $(4, -5)$

$y = -2f(-2(x-3)) + 5$

Ref. in x-axis V.st by 2 Ref. in y-axis R+3 up 5

H.st. by $\frac{1}{2}$

$(x, y) \rightarrow \left(-\frac{1}{2}x+3, -2y+5\right)$

$(-2, 5) \rightarrow \left(-\frac{1}{2}(-2)+3, -2(5)+5\right)$

$\rightarrow (4, -5)$

Summary of Transformations...

Transformations of the graphs of functions	
$f(x)+c$	shift $f(x)$ up c units
$f(x)-c$	shift $f(x)$ down c units
$f(x+c)$	shift $f(x)$ left c units
$f(x-c)$	shift $f(x)$ right c units
$f(-x)$	reflect $f(x)$ about the y-axis (Multiply x by -1)
$-f(x)$	reflect $f(x)$ about the x-axis (Multiply y by -1)
$cf(x)$	When $0 < c < 1$ – vertical shrinking of $f(x)$ When $c > 1$ – vertical stretching of $f(x)$ **Multiply the y values by c**
$f(cx)$	When $0 < c < 1$ – horizontal stretching of $f(x)$ When $c > 1$ – horizontal shrinking of $f(x)$ **Divide the x values by c**

$$y = f(x) \longrightarrow y = \overset{-}{a} f\left(\overset{-4}{b}\left(x - \overset{+7}{c}\right)\right) + d$$

x-values

Mapping Rule:

$$(x, y) \to \left(\frac{1}{b}x + c,\ ay + d\right)$$

Important note for sketching...

Transformations should be applied in following order:

1. Reflections
2. Stretches
3. Translations

Remember....RST

The function $y = f(x)$ is transformed to the function $g(x) = -3f(\overset{4(x-4)}{4x - 16}) - 10$. Copy and complete the following statements by filling in the blanks.

The function $f(x)$ is transformed to the function $g(x)$ by a horizontal stretch about the $\boxed{a}$ by a factor of $\boxed{b}$. It is vertically stretched about the $\boxed{c}$ by a factor of $\boxed{d}$. It is reflected in the $\boxed{e}$, and then translated $\boxed{f}$ units to the right and $\boxed{g}$ units down.

a) y-axis
b) $\frac{1}{4}$
c) x-axis
d) 3
e) x-axis
f) 4
g) 10

$$(x,y) \rightarrow \left(-3x+7,\ \tfrac{3}{4}y-2\right)$$

$$g(x) = \tfrac{3}{4} f\left(-\tfrac{1}{3}(x-7)\right) - 2$$

$$(x,y) \rightarrow \left(-\tfrac{2}{5}x-1,\ -7y+5\right)$$

$$g(x) = -7 f\left(-\tfrac{5}{2}(x+1)\right) + 5$$

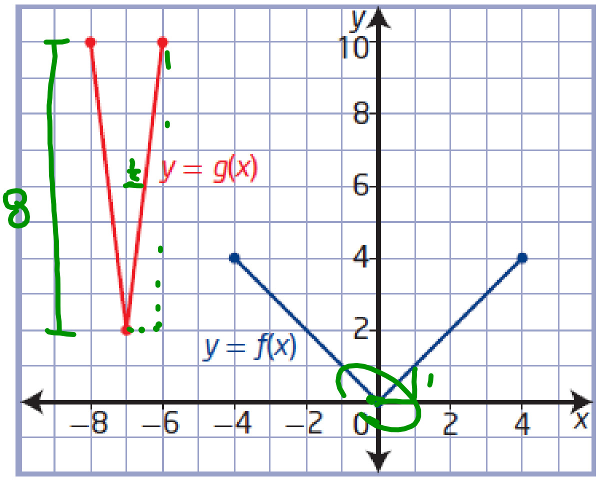
Write the Equation of a Transformed Function Graph

The graph of the function $y = g(x)$ represents a transformation of the graph of $y = f(x)$. Determine the equation of $g(x)$ in the form $y = af(b(x - h)) + k$. Explain your answer.

RST

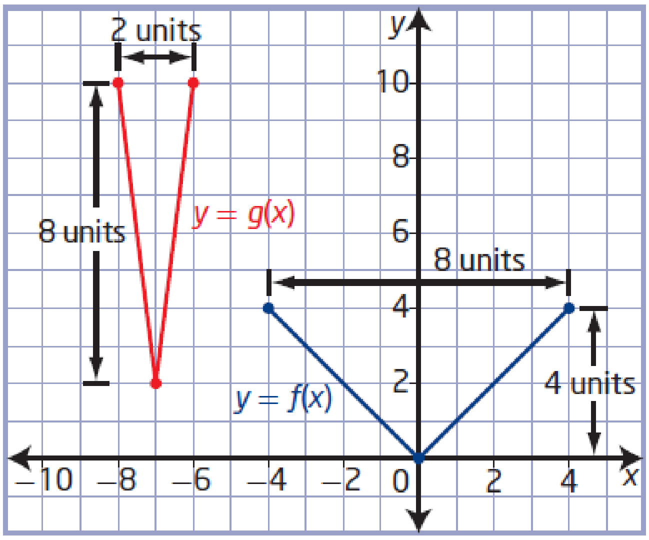
v.st. ⇒ 8

$y = 8f(x+7)+2$



Solution

The equation of the transformed function is $g(x) = 2f(4(x + 7)) + 2$.

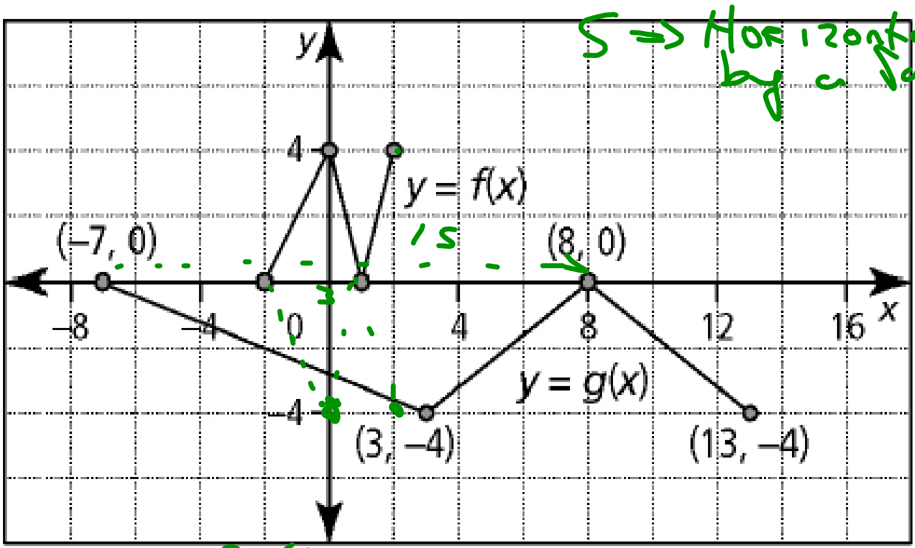


How could you use the mapping $(x, y) \rightarrow \left(\frac{1}{b}x + h, ay + k\right)$ to verify this equation?

The graph of the function $y = g(x)$ represents a transformation of the graph of $y = f(x)$. Determine the equation of $g(x)$ in the form $y = af(b(x - h)) + k$.

R ⇒ in x-axis

S ⇒ Horizontally by a factor of 5



$y = -f\left(\frac{1}{5}(x-3)\right)$

Practice Problems...

Pages 39 - 41

#3, 4, 6, 7, 8, 10, 13, 14

Inverse of a Relation

An inverse function is a second function which undoes the work of the first one.

1. Introduction

Suppose we have a function f that takes x to y, so that

$$f(x) = y.$$

An inverse function, which we call f^{-1}, is another function that takes y back to x. Sc

$$f^{-1}(y) = x.$$

For f^{-1} to be an inverse of f, this needs to work for every x that f acts upon.

Did You Know?

The -1 in $f^{-1}(x)$ does not represent an exponent; that is $f^{-1}(x) \neq \frac{1}{f(x)}$.

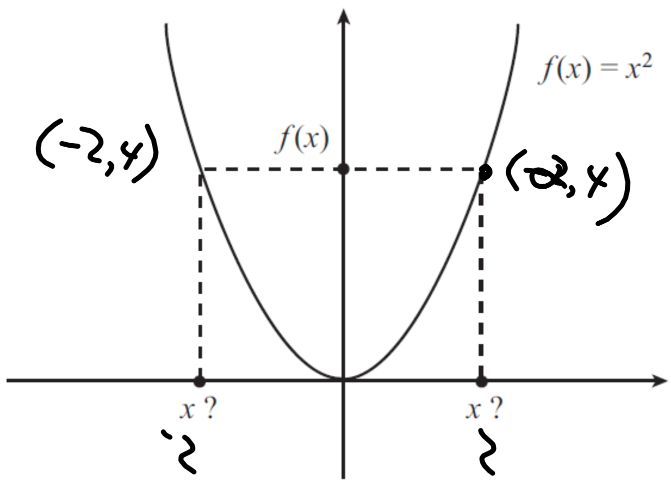
Not all functions have inverses. For example, let us see what happens if we try to find an inverse for $f(x) = x^2$.

$f(2) = 4$

$f(-2) = 4$

$f(x) = 4$

$x = ??$



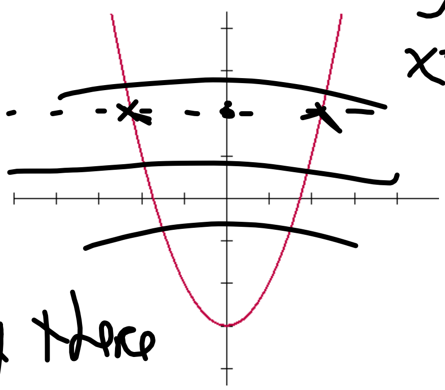
A function is said to be a one-to-one function if it never takes on the same value twice.

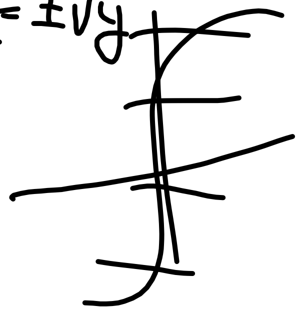
$\sqrt{y} = \sqrt{x^2}$

$x = \pm\sqrt{y}$

Look at this function...

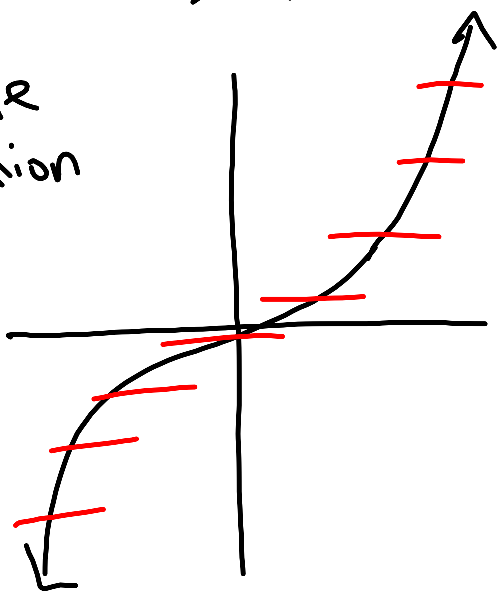
Not a one-one function





$\Rightarrow$ for every y there is more than one x

one-one function



① Sketch:

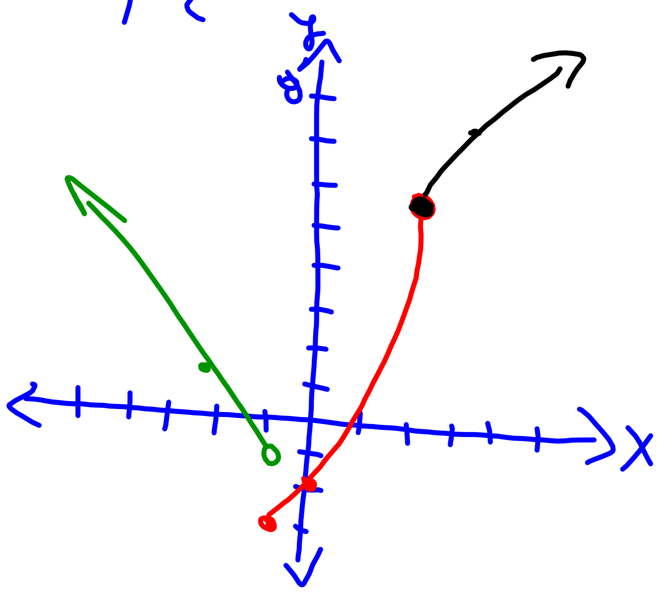
$$f(x)=\begin{cases} -2x-3, & \text{if } x<-1 \\ (x+1)^2-3, & \text{if } -1\leq x<2 \\ 2x+2, & \text{if } x\geq 2 \end{cases}$$

$V(-1,-3)$

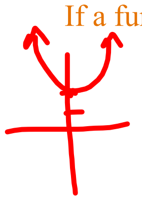
x	y
-1	-1
-2	1

x	y
-1	-3
2	6
0	-2

x	y
2	6
3	8



If a function is a one-to-one function then it will posses what is called an inverse function.



If f is a one-to-one function with domain A and range B. Then its **inverse function,** f^{-1} has domain B and range A and is defined by

$$f^{-1}(y) = x \Leftrightarrow f(x) = y$$

for any y in B.

$F(x) = x^2 + 2$ D: $x \in \mathbb{R}$
R: $y \geq 2$

$\boxed{\begin{array}{l} \text{domain of } f^{-1} = \text{range of } f \\ \text{range of } f^{-1} = \text{domain of } f \end{array}}$

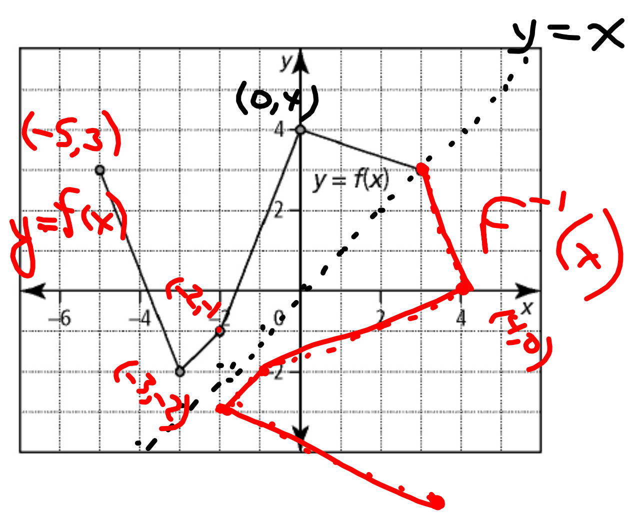
In plain english....the x and y coordinates will just switch places

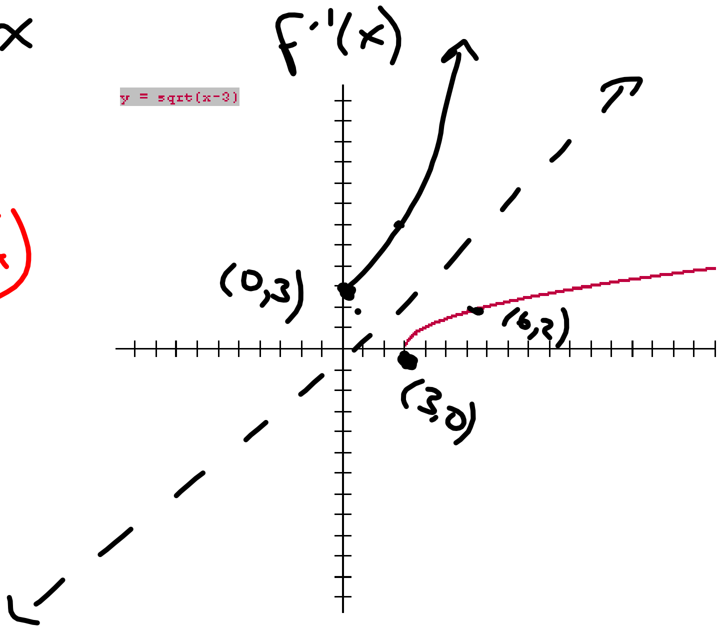
$F^{-1}(x)$: D: $x \geq 2$
R: $y \in \mathbb{R}$

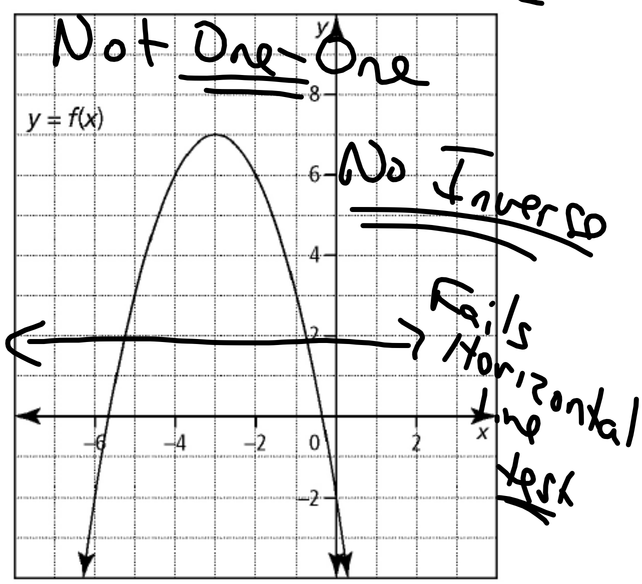
The inverse of a relation is found by interchanging the x-coordinates and y-coordinates of the ordered pairs of the relation. In other words, for every ordered pair (x, y) of a relation, there is an ordered pair (y, x) on the inverse of the relation. This means that the graphs of a relation and its inverse are reflections of each other in the line $y = x$.

$(x, y) \rightarrow (y, x)$

How does this play out graphically?







What if given the function algebraically?

Determine algebraically the equation of the inverse of each function. switch x & y !!!

a) $f(x) = 3x - 6$ b) $f(x) = \frac{1}{2}x + 5$

c) $f(x) = \frac{1}{3}(x + 12)$ d) $f(x) = \frac{8x + 12}{4}$

(a) $\overset{f(x)}{y} = 3x - 6$

$x = 3y - 6$

$\frac{3y}{3} = \frac{x}{3} + \frac{6}{3}$

$y = \frac{x}{3} + 2$

$f^{-1}(x) = \frac{x}{3} + 2$

$= \frac{x+6}{3}$

d) $f(x) = \frac{8x+12}{4}$

$4\,x = \frac{8y+12}{\cancel{4}}\ \cancel{(4)}$

$4x = 8y + 12$

$4x - 12 = 8y$

$y = \frac{4x-12}{8}$

$f^{-1}(x) = \frac{4x-12}{8}$

Determine the inverse for each of the following functions:

1. $\cancel{f(x) = 2x - 5}$

$f^{-1}(x) = 12$

$x = ?$

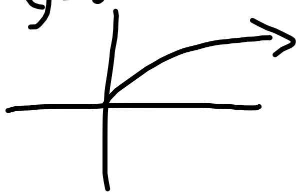
$f(x) = 7$

$(x, 12)$ on inverse

$\therefore (12, y)$ is on $f(x)$

2. $f(x) = \sqrt{x-3} + 4$

$y = \sqrt{x}$



$x = \sqrt{y-3} + 4$

$x - 4 = \sqrt{y-3}$

$(x-4)^2 = y - 3$

$y = (x-4)^2 + 3$

$f^{-1}(x) = (x-4)^2 + 3$

$f(12) = \sqrt{9} + 4$

$= 7$

$12 = (x-4)^2 + 3$

$\sqrt{9} = \sqrt{(x-4)^2}$

$3 = x - 4$

$7 = x$

$g(x) = -2f[3(x-5)]+7$

If $(3,-1)$ is on $f(x)$

What are it's coordinates on $g^{-1}(x)$?

switch x's y for $g(x)$

$(x,y) \to \left(\frac{1}{3}x+5, -2y+7\right)$

$(3,-1) \to \left(\frac{1}{3}(3)+5, -2(-1)+7\right)$

$(x,y) \to (6, 9)$

$g^{-1}(x)$

$\to (9, 6)$

Test Thursday

Practice Problems...

Pages 51 - 55
#2, 3, 5, 6, 8, 9, 11, 15, 18, 20, 21